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DECOMPOSITION ANALYSIS OF INCOME INEQUALITY IN POLAND BY SUBPOPULATIONS AND FACTOR COMPONENTS

The Gini index of concentration is considered to be the best synthetic measure of income inequality. The desirable feature of an inequality measure is its decomposability by income sources and by subpopulations. It is known that the indices based on entropy can be decomposed simply into within-group and between-group components, while the Gini concentration index is decomposable only into the sum of the inequality within groups and the inequality between groups, plus a crossover term that takes into account the overlapping across subpopulations. The presence of the third component resulted in interpretational difficulties and induced some scholars to reject the Gini coefficient as a decomposable inequality measure. On the other hand, the crossover term can be treated as an advantage of the Gini index and can be used to complete the inequality analysis by subgroups. In the paper the selected decomposition methods were discussed and then applied to the analysis of income distribution in Poland. The aim of the analysis was to verify to what extent the inequality in different subgroups (and different income sources) contributes to the overall income inequality in Poland. To provide the decomposition of the Gini index by subgroups the population of households was partitioned into several socio-economic groups on the basis of the exclusive or primary source of maintenance. Moreover, the households were divided by economic regions using the Eurostat classification units NUTS1. To complete the analysis the Gini index decomposition by factor components was conducted. This helped to find out which sources of household income have a significant influence on income inequality in Poland.
Keywords: income distribution, income inequality, decomposition.

1. INTRODUCTION

Among the numerous works on income distribution and income inequality there are attempts to assign inequality contributions to various components of income (such as labour income or property income) or to various population subgroups. Both these approaches may contribute to a better understanding of the influence of various socio-economic determinants on income levels and income inequality.

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The Gini index is a well known and widely used synthetic inequality measure usually expressed in terms of the area under the Lorenz curve. It measures income differences between each and every pair of individuals, in contrast to many other inequality coefficients measuring only the deviations from the mean and thus interlinking the concept of location with the concept of variability. It has a clear economic interpretation and thus has been applied in numerous empirical studies and policy research. Because of its good statistical properties the Gini coefficient is considered to be the best single measure of income inequality (Morgan 1962, Gastwirth 1970). On the other hand, the Gini index, being the function of both income and the ranking of economic units, cannot be decomposed easily by population subgroups and by factor components. Regardless of these difficulties a great effort has been made to specify the conditions under which the decomposition of the Gini coefficient by subgroups and by income components is feasible. Lerman and Yitzhaki (1984) proposed the clear decomposition by income components based on their covariance formula for the Gini index, providing a good tool for income inequality analysis. Decomposition by subpopulations proved to be more complicated. Since Shorrocks (1984) characterized the indices of inequality that are decomposable by population subgroups, the Gini index has been considered to be decomposable only when the subpopulations do not overlap. In fact, when the distributions overlap the third component called "overlapping" or "interaction term", difficult to interpret, has to be taken into consideration. That "third component" was discussed by Pyatt (1976), Silber (1989), Yitzhaki (1994), Deutsch and Silber (1999), to name only a few, what resulted in some interesting decomposition formulas. Unfortunately, they are computationally cumbersome and it is not always clear which meaningful interpretation each of the components has. An interesting approach to the decomposition of the Gini index was proposed by Dagum (1997). It introduces the concept of economic distance between subpopulations as an important element in the Gini index decomposition by subpopulation groups. Moreover, the interaction term can be viewed as a measure of income stratification or the degree to which the incomes of different social groups cluster.

The aim of this paper is to discuss selected decomposition procedures, as well as to show that their application can be useful in the analysis of income distribution in Poland in different divisions.

2. THE GINI INDEX DECOMPOSITION BY SUBPOPULATIONS

The Gini index of inequality is usually defined by means of the geometric formula based on the well known Lorenz curve. It can be expressed as double the area between this curve and the straight line called the line of equal shares. Gini (1912) showed that the geometric approach is related to the statistical approach via a concept called the mean difference. The mean difference Δ, being the measure of dispersion, was introduced also by Gini (1912) and can be defined as the average absolute difference between all possible pairs of observations in a population of income receivers. The inequality decomposition proposed by Dagum (1997) is based on the mean difference formula where the Gini index can be regarded as a relative dispersion measure, being the ratio of a measure of dispersion, the mean difference, to the average value:

$$G = \frac{\Delta}{2\overline{Y}} = \sum_{r=1}^{n}\sum_{i=1}^{n} | Y_i - Y_r | \Big/ 2n^2\overline{Y} = \sum_{j=1}^{k}\sum_{h=1}^{k}\sum_{i=1}^{n_j}\sum_{r=1}^{n_h} | y_{ji} - y_{hr} | \Big/ 2n^2\overline{y} \tag{1}$$

The last term of the formula given above presents the Gini index for a population divided into k subgroups (subpopulations). The Gini index for the sub-population j takes the form:

$$G_j = \frac{\Delta}{2\overline{Y}_j} = \frac{1}{2\overline{Y}_j}\sum_{r=1}^{n_h}\sum_{i=1}^{n_j} | y_{ji} - y_{jr} | / n_j^2 \tag{2}$$

where:
$\overline{Y}_j$ – the mean income in group j,
n_j – frequency.

The Gini index expressed in terms of the Gini mean difference can be also generalized for two populations case, measuring the between-populations (or intra-groups) inequality. Thus the **extended Gini index** between groups j and h can be written as follows:

$$G_{jh} = \frac{\Delta_{jh}}{\overline{Y}_j + \overline{Y}_h} = \frac{1}{\overline{Y}_j + \overline{Y}_h}\sum_{i=1}^{n_j}\sum_{r=1}^{n_h} | y_{ji} - y_{hr} | / n_j n_h \tag{3}$$

where:
Δ_{jh} – mean difference modified for two income distributions.

The Gini ratio G for a population of economic units partitioned into k subpopulations n_j ($j = 1,\ldots, k$), can be expressed as the average of the Gini ratios within each subpopulation G_{jj} and the extended Gini ratios between subpopulations G_{jh}, weighted by the products of the j-th subpopulation share p_j times the h-th population share s_h:

$$G = \sum_j \sum_h G_{jh} p_j s_h \tag{4}$$

where:

$$p_j = \frac{n_j}{n}, \qquad s_j = \frac{n_j \bar{Y}_j}{n \bar{Y}}.$$

It follows from equation (3) that: $G_{jj} = G_j$. Using the symmetry properties of G_{jh} and Δ_{jh} and the equation (4), the Gini index can be decomposed into two elements (Dagum, 1997):

$$G = \sum_{j=1}^{k} G_j p_j s_j + \sum_{j=1}^{k} \sum_{h=1}^{k} G_{jh} (p_j s_h + p_h s_j) \tag{5}$$

$$G = G_w + G_{gb}$$

As can be easily noticed, the Gini index provides an unusual "between-group" component. It measures the income inequality between each and every pair of subpopulations, whereas entropy and most of between-groups inequality measures yield only the income inequalities among the subpopulation. The first component of the decomposition given by the formula (5) (G_w) describes the ***contribution of the Gini inequality within subpopulations*** to the total inequality of a population described by the Gini ratio G. The second component (G_{gb}) measures the ***gross contribution of the extended Gini inequality between subpopulations*** to the total Gini G. This component depends on the differences between subpopulations coming from:

- ***differences in mean*** income levels.
- ***differences in shape*** (the populations differ in variance and asymmetry, which implies that they have different inequality measures).

The income differences between the elements coming from various subgroups can be of the same or of an opposite sign than the deviation in their corresponding means.

The interpretation of G_{gb} given above suggests the possibility of further decomposition of the Gini index by subgroups. The contribution of gross between-groups inequality can be divided into two separate parts: the first one consistent with the differences between the means and the remaining part, called transvariation. Such a decomposition enables a more precise analysis and interpretation of the sources of income inequality in a population partitioned into subpopulations more precisely. The total Gini ratio of a population of size n divided into k subpopulations can be decomposed as follows (Dagum, 1997) :

$$G = G_w + G_b + G_t$$

G_w – the contribution of ***within-groups inequality*** to the Gini index,

G_b – the contribution of ***net between-groups inequality*** to the Gini index which can be given by the following formula:

$$G_b = \sum_{j=1}^{k}\sum_{h=1}^{j-1} G_{jh}(p_j s_h + p_h s_j) D_{jh} \ , \tag{6}$$

G_t – the contribution of "***transvariation***" to the Gini index which can be written as:

$$G_t = \sum_{j=2}^{k}\sum_{h=1}^{j-1} G_{jh}(p_j s_h + p_h s_j)(1 - D_{jh}) \ . \tag{7}$$

where: D_{jh} – "economic distance" ratio (Dagum, 1980).

The concept of transvariation (*transvariazione*) was originally introduced by Gini (1916) and plays a crucial role in the Gini index decomposition by population subgroups. Transvariation between two populations exists when at least one income difference between individuals belonging to different groups has the sign opposite to the sign of the difference between their means. Obviously, the idea of transvariation is similar to the concept of distribution overlapping. The **probability of transvariation** can be simply defined (Gini, 1916) as the ratio of the actual number of transvarying pairs to its maximum. Thus, transvariation probability takes values in the interval [0,1] and the more the two groups overlap, the greater value it takes. In order to account not only for the frequency but also for the amount of income differences another measure called **intensity of transvariation** (Gini, 1959) was proposed. The term D_{jh} (eq. 6, 7) called economic distance ratio or REA (*relative economic affluence*) is related to the normalized intensity of transvariation which is simply 1- $D_{jh.}$ and can be regarded as the measure of ***relative economic affluence*** of the j-th subpopulation with respect to the h-th subpopulation. It can be defined as the weighted average of the income differences $y_{ji} - y_{hr}$ for all the members belonging to the population j-th with income greater than the income of the members belonging to population h, given that $\overline{Y}_j > \overline{Y}_h$ (for details see: Dagum, 1980).

3. DECOMPOSITION OF THE GINI INDEX BY FACTOR COMPONENTS

The Gini index of concentration can also be expressed by means of the covariance formula which is based on the covariance between incomes and their ranks (Lerman, Yitzhaki, 1984):

$$G = 2\frac{\operatorname{cov}[y, F(y)]}{\bar{y}} \tag{8}$$

where: $F(y)$ - cumulative distribution function of income.

The above expression can be a starting point to derive the Gini index decomposition by sources of income. Let y_k denote the k-th component of the total household income y ($y = \sum_k y_k$). Using the covariance formula given by equation (8) we obtain the decomposition formula expressing the

Gini index as the sum of contributions of various income components (Lerman, Yitzhaki 1985):

$$G = \sum \frac{\operatorname{cov}[y_k, F(y)]}{\operatorname{cov}[y_k, F(y_k)]} \cdot \frac{2\operatorname{cov}[y_k, F(y_k)]}{\bar{y}_k} \cdot \frac{\bar{y}_k}{\bar{y}} \tag{9}$$

$$G = \sum_k R_k \cdot G_k \cdot W_k$$

R_k is the ***Gini correlation coefficient*** taking values from the interval $\langle -1, 1 \rangle$. It takes value *0* when the variables y_k i y are independent. The value *1* is taken if y_k is an increasing function of *y*, which means that the households' ordering according to the *k*-th component y_k is identical with the ordering according to the total income *y* (similarly to the Spearman rank correlation coefficient). Negative values of R_k indicate negative covariance between income component *k* and the cumulative distribution function of income *F(y).*

G_k denotes the ***Gini index*** measuring inequality ***within the k-th income component*** while W_k stands for the ***k-th component share*** in the total income. $R_k G_k W_k$ can be considered to be the ***k-th component contribution*** to the overall income inequality of a population. If the correlation between a selected component and total income is negative or zero, a marginal income increase within this component will decrease inequality. When R_k is positive, then the impact upon inequality depends on the sign of $R_k G_k - G$ (see: Stark, Taylor, Yitzhaki, 1986).

4. APPLICATION

The methods mentioned above were applied to the analysis of family income in Poland by socio-economic groups and regions as well as by income components. The basis for the calculations was individual data obtained from the Household Budgets Survey (HBS) conducted monthly by the Polish Central Statistical Office. Household groups representing the basic socio-economic parts of the population were established on the basis of the exclusive or primary source of maintenance. The results of the estimation for the year 2006 are presented in tables and figures.

Table 1 describes in detail the results of income inequality decomposition by socio-economic groups while the results for the population divided by regions are presented in table 2. Special attention was devoted to the presentation of the estimation results for each pair of the subpopulations considered. To do this the extended Ginis and economic distance ratios for all the pairs were also presented. On the basis of the results the corresponding summary tables (1a and 2a) were prepared in order to show the breakdown of the Gini index by socio-economic groups and regions, respectively. The outcome of the Gini index decomposition by factor components is given in tables 3 and 4. Figure 1 presents the density curves describing income distributions of selected socio-economic groups in Poland. They were estimated by means of the maximum likelihood method. As a theoretical distribution the three-parameter Burr type III model (known as the Dagum distribution) was applied. Analyzing the figures one can easily notice the differences in the location and shape of the populations under consideration.

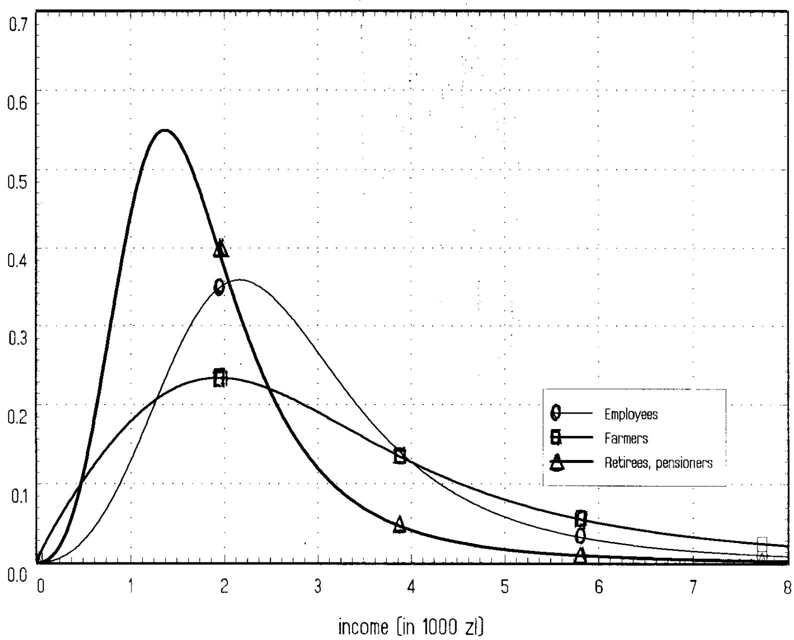

Fig.1. Distribution of non-equivalised income for selected socio-economic groups (ML estimates of the Burr type III model)
Source: author`s calculations

Table1. Gini index decomposition by socio-economic groups

No. j	Socio-economic group	Sample size n_j	Mean income $\overline{y}_j$	Sample share p_j	Income share S_j	Within-groups Gini G_j	Between-groups Gini G_{jh}					Between-groups distance D_{jh}				
							1	2	3	4	5	1	2	3	4	5
1	Employees	4413	2.944	0.4722	0.5336	0.29	0.29	0.36	0.34	0.33	0.41	0	0.34	0.42	0.65	0.78
2	Farmers	511	3.644	0.0547	0.0765	0.41		0.41	0.39	0.43	0.51		0	0.35	0.75	0.83
3	Self-employed	591	3.955	0.0632	0.0960	0.36			0.36	0.43	0.51			0	0.82	0.88
4	Retirees, pensioners	3364	1.907	0.3599	0.2635	0.29				0.29	0.34				0	0.32
5	Non-earned sources	467	1.585	0.0500	0.0304	0.35					0.35					0
6	Total	9346	2.605	1	1	0.34										

Source: author's calculations

Table 1a. Breakdown of income inequality in Poland by socioeconomic groups

1.	Contribution of the **between-groups** inequality		**0.1412**
2.	Contribution of the **within-groups** inequality		**0.1051**
	– contribution of *employees*	0.0731	
	– contribution of *farmers*	0.0017	
	– contribution of *self-employees*	0.0021	
	– contribution of *retirees*	0.0276	
	– contribution of *non-earned sources*	0.0005	
3.	Contribution of **transvariation**		**0.0910**
4.	Total inequality measured by the Gini index		**0.3373**

Source: author's calculations

Table 2. Gini index decomposition by regions

No. j	Region	Sample size n_j	Mean income $\overline{y}_j$	Sample share p_j	Income share S_j	Within-groups Gini G_j	Between-groups Gini G_{jh}						Between-groups distance D_{jh}					
							1	2	3	4	5	6	1	2	3	4	5	6
1	Central	1972	2.882	0.2110	0.2334	0.37	0.37	0.34	0.36	0.35	0.35	0.36	0	0.20	0.22	0.11	0.18	0.13
2	Southern	1916	2.516	0.2050	0.1980	0.31		0.31	0.32	0.32	0.31	0.33		0	0.04	0.09	0.01	0.06
3	Eastern	1679	2.431	0.1796	0.1677	0.34			0.34	0.33	0.32	0.34			0	0.12	0.05	0.09
4	North-western	1441	2.642	0.1542	0.1563	0.33				0.33	0.32	0.34				0	0.08	0.03
5	South-western	999	2.512	0.1069	0.1031	0.31					0.31	0.33					0	0.04
6	Northern	1339	2.572	0.1433	0.1415	0.35						0.35						0
7	Total	9346	2.605	1	1	0.34												

Source: author's calculations

Table 2a. Breakdown of income inequality in Poland by regions NUTS1

1.	Contribution of the **between-groups** inequality		**0.0306**
2.	Contribution of the **within-groups** inequality		**0.0592**
	– contribution of *central*	0.0182	
	– contribution of *southern*	0.0125	
	– contribution of *eastern*	0.0102	
	– contribution of *north-western*	0.0079	
	– contribution of *south-western*	0.0034	
	– contribution of *northern*	0.0071	
3.	Contribution of **transvariation**		**0.2475**
4.	Total inequality measured by the Gini index		**0.3373**

Source: author's calculations

Table 3
Inequality decomposition by income components (non-equivalized income)

No. k	Income source	Gini correlation R_k	Gini concentration G_k	Income share W_k	$R_k\ G_k\ W_k$	$\frac{R_k\ G_k\ W_k}{G}$
1	Wages and salaries	0.6976	0.6323	0.4684	0.2066	0.6130
2	Self-employment	0.6635	0.9482	0.0822	0.0517	0.1534
3	Property income	0.6156	0.9963	0.0029	0.0018	0.0054
4	Social insurance	0.1354	0.6409	0.2719	0.0236	0.0700
5	Social services	-0.0105	0.8230	0.0532	-0.0005	-0.0015
6	Farm produce	0.6263	0.9495	0.0704	0.0419	0.1243
7	Other income	0.2704	0.8596	0.0510	0.0119	0.0354
8	Disposable income	1	0.3370	1	0.3370	1

Source: author's calculation

Table 4

Inequality decomposition by income components (equivalized income)

No. k	Income source	Gini correlation R_k	Gini concentration G_k	Income share W_k	$R_k\ G_k\ W_k$	$\frac{R_k\ G_k\ W_k}{G}$
1	Wages and salaries	0.6124	0.6382	0.4460	0.1743	0.5658
2	Self-employment	0.6345	0.9493	0.0790	0.0476	0.1545
3	Property income	0.6563	0.9962	0.0030	0.0020	0.0064
4	Social insurance	0.2486	0.6514	0.3096	0,0501	0.1628
5	Social services	-0.2059	0.8200	0.0482	-0.0081	-0.0264
6	Farm produce	0.4984	0.9470	0.0592	0.0279	0.0907
7	Other income	0.2993	0.8654	0.0550	0.0143	0.0463
8	Disposable income	1	0.3081	1	0.3081	1

Source: author's calculation

The overall income inequality in Poland in 2006 estimated on the basis of the Polish HBS was G_{2006}=0.34. This value confirms a rather high level of income inequality in Poland as compared with other European countries (see: www.eurofound.europa.eu/areas/qualityoflife/). It is worth mentioning that the value of the Gini index for HBS increased significantly during the last few years (G_{1999}=0.25) and is still increasing (G_{2007}=0.35; G_{2008}=0.37).

The decomposition of the Gini index by subgroups (tables 1, 1a) takes into account the splitting up into households of self-employed, households of employees (managers, office workers, blue-collar workers, school teachers etc), households of not employed (retirees and pensioners) and households of other not employed (mainly unemployed). The households of farmers constitute a separate group. In 2006 the intragroup inequality (that is the within-group component G_w) accounted for 31% of the overall inequality. The within-group component reflects the inner polarization of all the groups which gives rise to remarkable differentials in average income between managers and blue-collar workers within the group of *employees*, between entrepreneurs and the others within the group of *self-employed* or between retirees and pensioners within the fourth group. The households of self-employed are the wealthiest group, the one with the highest mean income. The income distribution of self-employed also shows a very high level of inequality (G=0.36), similarly to the households of farmers (G=0.41). Table 1a can be helpful to answer the question: to what extent do particular groups contribute to the overall inequality? According to formula (5), the contributions of within-group components depend on the Gini index among the units of each group, on income share and population share of the group. Because of the very small income and population shares, the income disparities among the *self-employed* weigh only 0.6% on the total inequality, while the contribution of *farmers* is even smaller being 0.5%. The group with the highest share (7%) in the overall Gini index is the group of *employees.*

Turning to the inequality between socio-economic groups, the net between-groups component G_{nb} contributes 42% of the total Gini coefficient. The highest value of economic distance ratio (REA) was observed for *non-earned sources* and *self-employed* (D=0.88) means that the economic situation of *self- employed* is by 88% better as compared to the *non-earned sources).* The transvariation component G_t, describing the overlapping of the subpopulations, accounts for the remaining 27% of the total income inequality in Poland. This value suggests that the income source of the head does not represent the only revenue of the household.

Analyzing the results of calculations presented in tables 2 and 2a one can easily notice that the regional income disparities in Poland are rather small: the between regions inequality is only 9% of the total Gini. The substantial contribution of transvariation G_t , equal to 74%, is evidence of the notable overlapping of income distributions for NUTS1. To analyze the problem more thoroughly one can observe the economic distance ratios *D* measuring the relative economic affluence of one region with respect to another (table 2). It can be easily noticed that only the *central* region is significantly more affluent than the others. As a result, the transvariation component is dominated mainly by the overlapping between the distributions of central region and the other regions. The highest value of *D* was observed for the *central* and *eastern* regions. It is equal to 0.22 which means that the economic situation of *central* voivodeships in Poland is better by 22% than the situation of the eastern ones, taking into consideration the differences in mean incomes as well as in the shapes of the compared distributions.

The Gini ratios and means within regions do not differ significantly (table 2) so the contributions of particular subpopulations to the overall inequality are determined mainly by their sizes (table 2a). On the whole, the inequality within groups is responsible for only 17% of the total inequality.

Tables 3 and 4 depict in detail the Gini index decomposition by sources of income. Table 3 shows the estimation results for the distribution of household total income, while the results for the corresponding equivalent income are presented in table 4. To convert the household income into equivalent income the OECD equivalence scale has been applied. *Wages and salaries* is the income source having the biggest influence on income inequality in Poland (table 3). Its contribution to the overall inequality measured by the Gini ratio is 62%. Negative value of R_k correlation coefficient for *social services* (social benefits, unemployment benefits etc.), accompanied by a relatively high population shares of this component in *disposable income,* clearly suggest that the proportional income increase within these income sources can reduce overall inequality. The last columns of tables 3 and 4 show relative contributions of particular income components to the overall Gini index. Should an income source's contribution to the overall Gini be less than its share in the overall income, then a marginal income increase within that source will reduce inequality (see: page 8). Such a situation occurs not only for *social services* mentioned above but also for *social insurance* and *other income*.

The results of source decomposition presented in table 4 take into account the size and composition of households. It can be noticed that the overall inequality calculated for equivalent income is smaller (G=0.31) due to the equivalizing effect of family size. The most notable influence of family size was observed for income from social insurance and social services which resulted mainly in different Gini correlation values. For *social insurance* the Gini correlation (R_k=0.25) was much higher than the corresponding value for non-equivalized income (R_k=0.13), while for *social services* the value of this coefficient was significantly below zero (R_k= - 0.21). As a consequence, the relative influence of these sources of income on the overall Gini index also increased, as compared to the non-equivalized income. The last column of table 4, giving the weighted shares of each income source in the overall Gini, shows it in detail: the relative contribution of *social insurance* to the Gini index is 17% while the influence of *social services* is negative and equal to 3%.

5. CONCLUDING REMARKS

Taking into account the above theoretical considerations and empirical results obtained on the basis of the Polish income data, one can formulate some conclusions. Decomposition of income inequality measures by sub-populations can be useful in comparing income distributions by assessing the contributions of between-group and within-group inequalities to the overall inequality of a population. It can also be useful in stratification and market segmentation by including the concept of overlapping.

Decomposition of inequality measures by factor components can be helpful in recognizing the main sources of income inequality (wages and salaries, property income etc) and evaluating their contributions to the overall inequality. This approach can also be used in order to examine the impact upon inequality of marginal income changes in particular components. As a consequence, source decomposition can be a useful tool for social policy makers.

The interesting results of the decomposition of income inequality in Poland obtained on the basis of household budgets' data suggest that this approach can be helpful for the better understanding of the problem and can be used in many further economic analysis, including poverty and social welfare investigations. However, one should be conscious that the estimation results can be biased mainly because of a very high non-response rate being an immanent feature of household budgets surveys all over the world.

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