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Quasi-Wiener optimum spatial frequency filter for image restoration*

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A computer-aided holographic filter for image deblurring is presented in this paper. The solution of the nonstationary inverse problem is solved with the help of Tichonov regularization factor availing the experimentally measured power spectrum of the smeared image.

1. General

The paper is devoted to the problem of the recovery of message emitted by a source from data obtained in a receiver. The optical system is a lossy information channel. The losses arise because of different reasons such as aberration, spatial frequency filtering, a system noises, etc.

Let f be a function describing source data, g — a function describing the image data in a receiver. The reconstruction of unknown information consists in finding a solution of the inverse problem, it is in finding an operator k (Fig. 1), where

$$g = kf, \tag{1}$$

and

$$f = k^{-1}g. \tag{2}$$

We have assumed that there exists an inverse operator k^{-1}, thus the reconstruction operators form an Abelian group. The inverse problem realized experimentally is non-precise mathematically, because the data with the transmission system (channel)

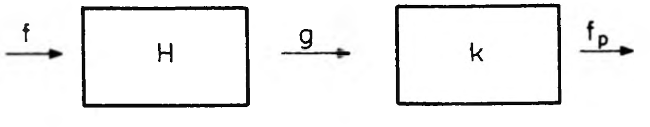

Fig. 1. Schematic arrangement of coherent system for informatic signals reconstruction by the quasi-Wiener filtration

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and the output signals are charged with errors. Thus, it is impossible to gain the explicit solution. So, we are to look for an approximate solution of Eq. (1) which allows us to obtain only a set of approximated f: f_a,

If $g \in G$, where G is a set of output signals, $f, f_a \in F$, where F is a space (class) of input signals in request, and F, G are metric spaces (e.g., of Euclidean matrices), then by finding the solution we mean that we obtain an approximate solution f_a, which fulfils the inequality $\varrho(f_a, f) < \Delta$, Δ being the fixed accuracy of the solution, i.e., the reconstruction of the input image.

Often, during experimental realization in radioastronomy, optics and electron microscopy we have the data a priori concerning the class F_p of the possible input images (admissible solutions of the Eq. (1)). Thus, we can treat the problem as finding the optimal f_{op} [1] which fulfils the equation

$$\varrho_{opt} = \inf_{i}[\varrho(f_i, k^{-1}g_i)] \tag{3}$$

where $f_i \in F_p$, g_i – measured output signal.

The reconstruction of the input signal f_a is henceforth no explicit, moreover, the solution of the equation is unstable, because in general a low variation of g induces a high variation of f_a. Note that the solution of the inverse problem is unstable because k must be, and often it is not, a continuous operator, thus k^{-1} becomes undetermined in discontinuity points. The inverse solution is also uncertain since in reality we do not know the proper input function, but only in simulated experiments. We can only fix the class from the a priori data, the uncertainess of inverse is of mathematical and physical origin.

If the optical system is linear and isoplanatic with the transfer function $H(\omega)$, which can be determined experimentally with the accuracy ΔH, then the inverse problem lies in the solution of the equation

$$\begin{gathered} \Gamma = PH, \ P = \mathscr{F}\{f\}, \ \Gamma = \mathscr{F}(g), \\ f \in F, \ g \in G. \end{gathered} \tag{4}$$

In such a case we can use Tichonov regularization factor $M(\omega)$ [2] for determining the function

$$K(\omega) = \frac{H^*(\omega)}{|H(\omega)|^2 + \beta M(\omega)}. \tag{5}$$

$K(\omega)$ is a prescription for construction of deblurring spatial filter. The filter should be inserted in the restoration system, where deconvolution of the blurred signal is performed.

If system noises are additive, then the optimization of the inverse problem solution leads to the transmittance function of the optimum Wiener filter

$$K(\omega)_W = \frac{H^*(\omega)}{|H(\omega)|^2 + \beta|S(\omega)|^2/|P(\omega)|^2} \tag{6}$$

where $|S(\omega)|^2$ is a spectrum of noise power. Wiener solution assumes the knowledge of the power spectrum of the input signal (in request) $|P(\omega)|^2$. Unfortunately, the knowledge is generally not attainable [3].

Taking into account the researches concerning the inverse problem with unstable equations, I propose here a new quasi-Wiener regularized solution $|K(\omega)_{\mathrm{qW}}$ in the following form:

$$K(\omega)_{\mathrm{qW}} = \frac{H^*(\omega)}{|H(\omega)|^2 + \beta |S(\omega)|^2 / |\Gamma(\omega)|^2} \tag{7}$$

where the measurable power spectrum of the output signal of blurred image replaces the power spectrum of the unknown input signal Γ, and β is a number matching factor.

2. Experimental

The presented idea has been corroborated experimentally. In this paper, the experimental results connected with making a quasi-Wiener spatial frequency filter for image restoration are given. The filter was made by the methods of digital holography and by using results of the measurement of Fourier spectrum modulus of the signal received at the system output Γ.

The object to be improved is a deblurred Siemens radial test pattern. It is placed at the input plane of the restoring system. The measured intensity distribution $\Gamma(\omega)$ of Fourier image of this pattern has been taken for computation of the restoring filter. We admit the following form of impulse response distribution:

$$h(x, y) = \mathrm{circ}\left[\frac{(x^2+y^2)^{1/2}}{r_0}\right], \quad r = (x^2+y^2)^{1/2} \tag{8}$$

with transfer function

$$H(\omega) = \mathscr{F}[h(r)] = \frac{2\pi J_0(2\pi\omega r_0)}{\omega r_0}, \qquad \omega = [u^2+v^2]^{1/2}. \tag{9}$$

The value of r_0 was matched with the qualitatively assessed smear of the Siemens's test and treated as a first approximation. We admit that the noises $S(\omega)$ are additive and have the form of Gaussian distribution

$$S(\omega) = \exp[-\alpha^2(u^2+v^2)] = \exp[-\alpha^2\omega^2], \tag{10}$$

We can formulate the desired filter transmittance function

$$K(\omega)_{\mathrm{qW}} = \frac{2\pi J_0(2\pi\omega r_0)/\omega r_0}{[2\pi J_0(2\pi\omega r_0)/\omega r_0]^2 + \beta[\exp(-\alpha^2\omega^2)]^2/|\Gamma(\omega)|^2}. \tag{11}$$

Here α and β are parameters to be chosen. $K(\omega)$ is a two-parameter family of functions with variable parameters α and β. The main purpose of this paper is to choose the best fitted values of α and β for the filter constructed from a fixed r_0.

The case with $\alpha = 1$ corresponds to white noises which being not of our interest will not be discussed. Since the second term in the denominator playes the role of a regularization factor, the parameter β should be a small fraction, ranging within 0.01–0.05.

Several functions $K(\omega)$ are computed for α equal to 0.5, 0.3, 0.05 and β equal to 0.01, 0.05, and 0.1. The appropriately restored filters are manufactured and tested in the restoring optical system. The best result, shown in Fig. 2, was otained when using a filter computed with the aid of parameters $\alpha = 0.3$ and $\beta = 0.01$.

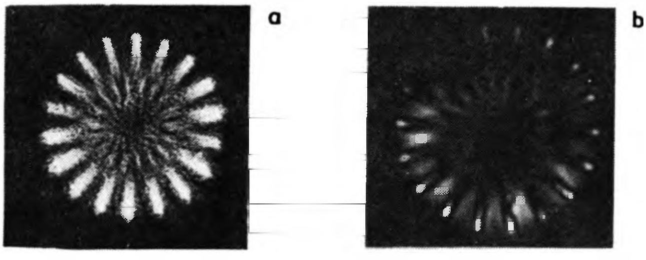

Fig. 2. Result of the restoration, using a quasi-Wiener filter with parameters $\alpha = 0.3$ and $\beta = 0.01$ (**a** – input signal, **b** – output signal)

Further development of this method needs the optimization of the value r_0. In this work, the filters have been computed with the aid of IBM-PC and plotted by Roland-DXY-880A.

The filter is realized by the technique of LOHMANN [4] using connected cells as proposed by STĘPIEŃ and GAJDA [5] (see Fig. 3).

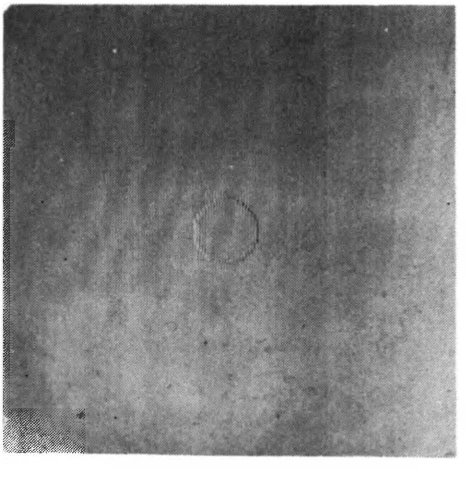

Fig. 3. Magnified part of the filter used

The whole filter-pattern was of A2 size, a single line was 0.25 mm thick. The pattern has been next about 100 × diminished and recorded at Agfa Gevaert Holomask plates.

3. Conclusion

A mask realized by the design proposed here can work as a deblurring spatial filter with some effort. The parameters α and β have been chosen optimally. In result, the frequency response of the image has been compensated (the reversal of contrast is removed). Obviously, the lacking parts of the image cannot be recovered. The future works should deal with the optimization of the assumed transfer function and the value r_0.

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Оптимальный квазивинеровский фильтр пространственных частот для реконструкции образа

Представлен компьютерно-голографический фильтр для компенсации аберрации. Решения нестационарного обратного вопроса достигли при помощи агента регуляризации Тихонова, использующего спектр силы размытого образа, измеренный экспериментально.