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Analysis of the electromagnetic wave propagation in an electrooptic multimode wavequide

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In the paper the analysis of optical properties in multimode electrooptic waveguide was carried out for the case when some voltage is applied to the electrodes located in opposite sides of a plane parallel plate of the crystal $LiNbO_3$ ($LiTaO_3$) of c-cut. The analysis was performed by the method of effective index of refraction which allowed us to split the problem into two parts, i.e., the propagation in two types of waveguides: (i) uniform planar waveguide with metal coating, and (ii) nonuniform planar waveguide. Due to the existing anisotropy the modes of E^x- and E^z-type propagating in the waveguide depend upon the changes of ordinary (n_o) and extraordinary (n_e) refractive indices, respectively. It has been shown that several hundreds of modes of E^x- and E^z-types may propagate in the waveguide, depending on the conditions of preparation.

1. Introduction

The electrooptic multimode waveguides are one of the light controlling system being actually developed in lightguide technique. These waveguides result from interaction of electric field with the lithium niobate (lithium tantalate) the geometry of which is shown in Fig. 1a. The waveguide parameters described in work [1] are presented in the Table. The distribution of the changes of the refractive indices Δn_e and Δn_o in the waveguide core was approximated by a steady function in the z-direction and by the Gauss distribution in the x-direction (Fig. 1b).

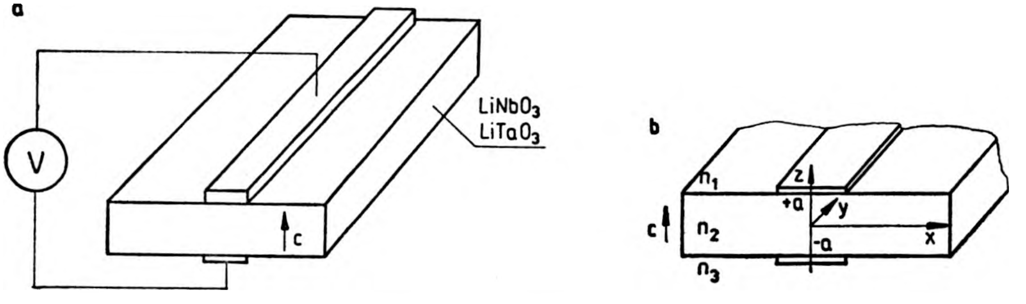

Fig. 1. Geometry of the electrooptic waveguide (a), and assumed coordinate system (b)

Similar problem was numerically analysed in the paper [2] by the method of geometric optics. The isotropic waveguides with a uniform distribution of the refractive index are often analysed by the Marcetili [3] method, which, in the case under study, is of little use due to nonuniformity appearing in the x-direction.

Parameters σ and $\Delta n^0_{e,o}$ of the changes in refractive index of $LiNbO_3$ and $LiTaO_3$ crystals for $2a = 100$ [μm], $U = 800$ [V]

Electrode width	σ [μm]		$\Delta n^0_e \times 10^{-3}$		$\Delta n^0_o \times 10^{-3}$	
w [um]	$LiNbO_3$	$LiTaO_3$	$LiNbO_3$	$LiTaO_3$	$LiNbO_3$	$LiTaO_3$
20	47	33	1.07	1.34	0.33	0.31
100	102		1.17		0.37	
200	168		1.17		0.37	

The most effective method for the analysis of a two-dimensional waveguide is that of effective refractive index. It was applied in paper [4] to the analysis of (two-dimensional) diffusion waveguides of both 1D (diffusion in one direction) and 2D (diffusion in two directions) types. A similar way to that used in the analysis of a waveguide of 1D-type has been applied to the analysis of a waveguide channel in [5]. However, both the papers [4] and [5] deal with isotropic waveguides.

The present paper includes a complete analysis of the waveguide properties of the electrooptic waveguide model shown in [1]. It is two-dimensional in the cross-section, anisotropic with nonuniform distribution of the refractive index in the direction of x and with a metallic layer (electrodes). Because of the nonuniform distribution of the refractive index the analysis of a waveguide was made by the method of effective index of refraction. It consists in successive solving the planar waveguides in the plane respectively restricted in the z- (Fig. 2b) and x- (Fig. 2c) directions, the effective index of refraction in the waveguide with the restriction in the z-direction (Fig. 2b) being used when solving the waveguide restricted in the x-direction (Fig. 2c). In this work we have

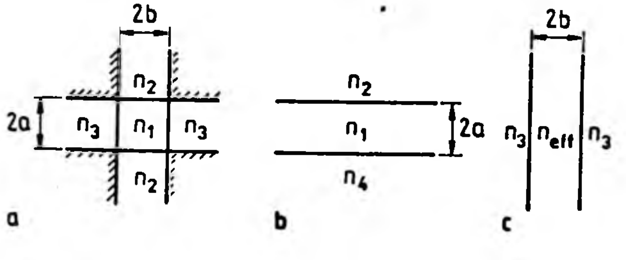

Fig. 2. Rectangular waveguide (a), planar waveguide unlimited in the x-direction (b), planar waveguide unlimited in the z-direction (c) with the effective refractive index of the waveguide core calculated from (b)

assumed as electrodes a group of metals for which the optical properties of thin deposited layers may be treated as lossy dielectrics. Such an assumption allows us to consider the waveguide, like in the case of a waveguide with a dielectric coat, provided that all the describing equations and magnitudes are complex [6].

The equations derived for waveguide modes E^z and E^x are presented in Sec. 2. In Section 3 the waveguide equations are then divided by the effective index method into two one-dimensional problems, i.e., the planar waveguide with the metallic coat and the nonuniform planar waveguide. The planar waveguide equation is analysed in Sec. 4 taking a special account of the influence of the metallic coating on the mode attenuation. The nonuniform equation is solved by the WKB method in Sec. 5. In Section 6 the results of the numerical calculations are presented and also the number of modes which may propagate in the electrooptic channel is determined. The paper is completed with final conclusions concerning the analysis and the results obtained.

2. Waveguide model, waveguide equation and boundary conditions

The model of the electrooptic waveguide considered in this work is shown in Fig. 3. Since the parameter σ of the refractive index distribution is comparable with the electrode width w we assume that the electrodes are located on the

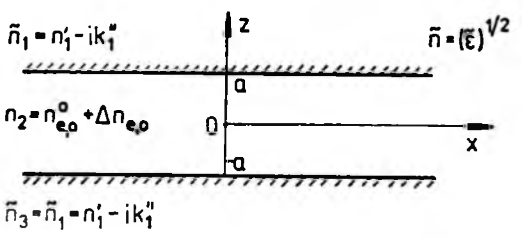

Fig. 3. Cross-section of the electrooptic waveguide

whole waveguide surface. The distribution of the refractive indices is thus described by the function

$$n_{e,o}(x, z) = \begin{cases} \tilde{n}_1, & z > a \\ n_{e,o}(x) = n^0_{e,o} + \Delta n^0_{e,o} e^{-(x/\sigma)^2}, & -a < z < a \\ \tilde{n}_3 = \tilde{n}_1, & z < -a \end{cases} \tag{1}$$

where: $2a$ – thickness of the waveguide plate,

$\tilde{n}_1, \tilde{n}_3$ – complex refractive index for the waveguide coating (the metal coating in the low lossy dielectric approximation),

$n^0_{e,o}$ – extraordinary and ordinary refractive indices for the crystal,

$\Delta n^0_{e,o}$ – maximal change in the refractive indices caused by the voltage applied,

σ – parameter of the refractive index distribution in the x-direction.

When considering the metal coating in the low lossy dielectric we must limit our interest to the materials of $\varepsilon_1' < n_2^2$, where $\tilde{n}_1 = (\varepsilon_1' - i\varepsilon_1'')^{1/2}$, i.e., to the group of such metals as Ag, Au, Al and Cr [7]. We assume that the dependence upon time t and the variable y is of the form: $\exp[i(\omega t - \beta y)]$, where β is the propagation constant, and ω – angular frequency. The small gradient of the refractive index distribution in the x-direction ($\Delta n_{e,o}^0 \ll n_{e,o}^0$) leads to small changes of the field amplitude which may be expressed by the relations:

$$\left|\frac{1}{k_0}\frac{\partial}{\partial x}\right| \ll 1 \quad \text{and} \quad \left|\frac{1}{k_0^2}\frac{\partial^2}{\partial x^2}\right| \ll 1 \tag{2}$$

where $k_0 = \frac{2\pi}{\lambda} = \omega\sqrt{\varepsilon_0\mu_0}$.

The full wave equations without simplifying assumptions have the forms:

$$\frac{n_e^2}{n_o^2}\frac{\partial^2 E_z}{\partial z^2} + \frac{\partial^2 E_z}{\partial x^2} + (k_0^2 n_e^2 - \beta^2)E_z = \frac{1}{n_o^2}\frac{\partial E_x}{\partial z}\frac{\partial n_o^2}{\partial x}, \tag{3}$$

$$\frac{\partial^2 E_x}{\partial z^2} + \frac{\partial^2 E_x}{\partial x^2} + (k_0^2 n_o^2 - \beta^2)E_x = \frac{n_o^2 - n_e^2}{n_o^2}\frac{\partial^2 E_z}{\partial x \partial z} - \Theta \tag{4}$$

where

$$\Theta = \frac{\partial}{\partial x}\left(\frac{1}{n_o^2}\right)\left[n_e^2\frac{\partial E_z}{\partial z} + n_o^2\frac{\partial E_x}{\partial x} + E_x\frac{\partial n_o^2}{\partial x}\right] + \frac{1}{n_o^2}\left[\frac{\partial n_e^2}{\partial x}\frac{\partial E_z}{\partial z} + \frac{\partial n_o^2}{\partial x}\left(\frac{\partial E_x}{\partial x} + \frac{\partial E_z}{\partial z}\right) + E_z\frac{\partial^2 n_o^2}{\partial x^2}\right]. \tag{5}$$

By taking advantage of the smallness of the refractive index gradient in the x-direction $\left|\frac{1}{n_{e,o}}\frac{dn_{e,o}(x)}{dx}\right| \ll 1$ the right-hand side in the formula (3) and the term Θ in (4) may be neglected. Moreover, in the formula (4) the term connecting the anisotropic components E_x and E_z may be omitted, since $(n_o - n_e)/n_o \ll 1$. The anisotropies of both the crystal and the solutions remain preserved since E_x and E_z depend on the ordinary and the extraordinary indices of refraction, respectively. Finally, the Eqs. (3) and (4) are separated and may be written in the forms:

$$\frac{n_e^2}{n_o^2}\frac{\partial^2 E_2}{\partial z^2} + \frac{\partial^2 E_z}{\partial x^2} + (k_0^2 n_e^2 - \beta^2)E_z = 0, \tag{3a}$$

$$\frac{\partial^2 E_x}{\partial z^2} + \frac{\partial^2 E_x}{\partial x^2} + (k_0^2 n_o^2 - \beta^2)E_x = 0. \tag{4a}$$

For the remaining components of the electromagnetic field we have:

$$H_y = \frac{1}{i\omega\mu_0}\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right), \tag{6}$$

$$H_z = \frac{1}{\mu_0\omega\beta}\left[\frac{\partial^2 E_z}{\partial x \partial z} - \left(k_0^2 n_o^2 + \frac{\partial^2}{\partial z^2}\right)E_x\right], \tag{7}$$

$$H_x = \frac{1}{\mu_0\omega\beta}\left[-\frac{\partial^2 E_x}{\partial x \partial z} + \left(k_0^2 n_e^2 + \frac{\partial^2}{\partial x^2}\right)E_z\right], \tag{8}$$

$$E_y = \frac{1}{i n_o^2 \beta}\left(n_e^2 \frac{\partial E_z}{\partial z} + n_o^2 \frac{\partial E_x}{\partial x}\right). \tag{9}$$

From Equations (3)–(9) it follows that the propagating modes are of hybrid type and consist of 6 components of electromagnetic field.

Aiming at determining the waveguide solution we distinguish the modes of E^x and E^z types. Mode of E^x type means that the electric field vector $\boldsymbol{E}$ is directed almost parallelly to the x-axis and has the components $E_x \neq 0$, $E_z = 0$ and $E_y \neq 0$. Modes of E^z-type have the vector electric field directed almost parallelly to the z-axis ($E_x = 0$, $E_z \neq 0$, $E_y \neq 0$). The expression *almost parallell* denotes the wave polarization, in which, besides one of the transversal components, there occurs also a longitudinal component, the absolute value of which is much smaller than that of the transversal component.

2.1. Modes of E^z type ($\boldsymbol{E}$ almost parallell to the z-axis)

Let $E_x = 0$, then the wave equation (3a) has the form

$$\frac{n_e^2}{n_o^2}\frac{\partial^2 E_z}{\partial z^2} + \frac{\partial^2 E_z}{\partial x^2} + (k_0^2 n_e^2 - \beta^2)E_z = 0. \tag{10}$$

When the assumptions (2) is fulfilled, the Eqs. (6)–(9) take the forms:

$$H_y = \frac{1}{i\mu_0\omega}\frac{\partial E_z}{\partial x}, \tag{11}$$

$$H_z = \frac{1}{\mu_0\omega\beta}\frac{\partial^2 E_z}{\partial x \partial z}, \tag{12}$$

$$H_x = \frac{k_0^2 n_e^2}{\mu_0\omega\beta}E_z, \tag{13}$$

$$E_y = \frac{n_e^2}{i n_o^2 \beta}\frac{\partial E_z}{\partial z}. \tag{14}$$

The boundary conditions of Eq. (10) follow from the continuity of the tangent components of the field at the plane $z = \pm a$. This continuity is automatically fulfilled for the component E_x, since $E_x = 0$. From Equations (11) and (13) it follows that $|H_y| \ll |H_x|$. There remains the condition following from the Eq. (13) which must be satisfied. Finally we obtain:

$$n_e^2 E_z - \text{continuous for } z = \pm a, \tag{15a}$$

and from Eq. (4)

$$\frac{n_e^2}{n_o^2}\frac{\partial E_z}{\partial z} - \text{continuous for } z = \pm a. \tag{15b}$$

2.2. Modes of E^x type (E almost parallel to the axis x)

Let $E_z = 0$, then Eq. (4a) remains unchanged and is expressed by the relation

$$\frac{\partial^2 E_x}{\partial z^2} + \frac{\partial^2 E_x}{\partial x^2} + (k_0^2 n_o^2 - \beta^2) E_x = 0. \tag{16}$$

On the other hand, when the condition (2) is satisfied, the Eqs. (6)–(9) have the forms:

$$H_y = -\frac{1}{i\mu_0\omega}\frac{\partial E_x}{\partial z}, \tag{17}$$

$$H_z = -\frac{1}{\mu_0\omega\beta}\left(k_0^2 n_o^2 + \frac{\partial^2}{\partial z^2}\right) E_x, \tag{18}$$

$$H_x = -\frac{1}{\mu_0\omega\beta}\frac{\partial^2 E_x}{\partial x \partial z}, \tag{19}$$

$$E_z = \frac{1}{i\beta}\frac{\partial E_x}{\partial x}. \tag{20}$$

The continuity of the tangent components of the field modes E^x should be preserved like for the modes E^z in the boundary plane $z = \pm a$. By taking advantage of the assumption (2) from Eq. (19) we obtain $|H_x| \ll |E_x|$ and from Eq. (20) $|E_y| \ll |E_x|$. The boundary conditions for H_x and E_y are fulfilled automatically if they are satisfied for E_x. Thus we have

$$E_x - \text{continuous for } z = \pm a, \tag{21a}$$

$$\frac{\partial E_x}{\partial z} - \text{continuous for } z = \pm a. \tag{21b}$$

3. Method of effective index of refraction

From the considerations carried out above it follows that the problem is reduced to the solution of Eq. (10) for the component E_z of the modes E^z and Eq. (16) for the component E_x of the modes E^x, provided that the conditions (15) and

(21) are fulfilled. Equations (10) and (16) may be generally written in the form

$$c_1 \frac{\partial^2 E}{\partial z^2} + \frac{\partial^2 E}{\partial x^2} + (k_0^2 n_{e,o}^2 - \beta^2) E = 0 \tag{22}$$

where

$$c_1 = \begin{cases} n_e^2/n_o^2 \\ 1 \end{cases}, \quad n_{e,o}^2 = \begin{cases} n_e^2(x,z) \text{ for modes } E^z,\ E = E_z, \\ n_o^2(x,z) \text{ for modes } E^x,\ E = E_x. \end{cases}$$

When looking for the solution in the form $E(x,z) = E_1(x)E_2(z)$ and applying the method of effective index of refraction we obtain the separation of Eq. (2) into two problems:

$$\frac{d^2E_1(x)}{dx^2} + \left(k_0^2 n_{\mathrm{eff}}^2(x) - \beta^2\right) E_1(x) = 0, \tag{23}$$

$$c_1 \frac{d^2E_2(z)}{dz^2} + k_0^2 \left(n_2^2(z) - n_{\mathrm{eff}}^2\right) E_2(z) = 0 \tag{24}$$

where

$$n_2(z) = \begin{cases} \bar{n}_1 & z > a, \\ n_{e,o} = n_{e,o}^0 + \Delta n_{e,o}^0, & -a < z < a \\ n_1, & z < -a \end{cases} \tag{25}$$

and

$$n_{\mathrm{eff}}^2(x) = n_{\mathrm{eff}}^2 + 2n_{e,o}^0 \Delta n_{e,o}^0 [e^{-(x/\sigma)^2} - 1]. \tag{26}$$

The parameter n_{eff} is the effective dielectric constant of the mode n with respect to the coordinate z. The boundary conditions for Eq. (24) are obtained from Eqs. (15) and (21). For the modes E^z and E^x they have the respective forms:

$$n_1^2 E_2^{(1)}(z)|_{z=\pm a} = n_e^2 E_2^{(2)}(z)|_{z=\pm a}, \tag{27a}$$

$$\left.\frac{dE_2^{(1)}(z)}{dz}\right|_{z=\pm a} = \frac{n_e^2}{n_o^2} \left.\frac{dE_2^{(2)}(z)}{dz}\right|_{z=\pm a} \tag{27b}$$

and

$$E_2^{(1)}(z)|_{z=\pm a} = E_2^{(2)}(z)|_{z=\pm a}, \tag{28a}$$

$$\left.\frac{dE_2^{(1)}(z)}{dz}\right|_{z=\pm a} = \left.\frac{dE_2^{(2)}(z)}{dz}\right|_{z=\pm a}. \tag{28b}$$

4. Planar waveguide with the metallic coating

Equation (24) with the boundary conditions (28) and (27) is the wave equation of the plane structure of TE modes, where $c_1 = 1$ or TM modes, when $c_1 = n_e^2/n_o^2$, respectively. The solution for the planar waveguides are well known. The metallic coating causes that all the magnitudes describing the wave propagation in such a waveguide become complex, thus the solution of the characteristic equation should be sought on the complex plane. This constitutes a serious difficulty of the numerical analysis. However, taking account of the anisotropy and proceeding like in [6] it may be shown that the relations

$$n'_{\text{eff}} = \sqrt{c_1} n_o \left\{1 - \frac{(n+1)^2\pi^2}{2n_o^2} \frac{1}{(2k_0 a)^2}\left[1 - \frac{n_o}{n_2 k_0 a} \operatorname{Re}\left(\frac{c_2}{n_2^2 - \tilde{n}_1^2}\right)\right]\right\}, \tag{29a}$$

$$n''_{\text{eff}} = \sqrt{c_1} \frac{1}{n_e} \frac{2(n+1)^2\pi^2}{(2k_0 a)^2} \operatorname{Im}\left(\frac{c_2}{\sqrt{n_2^2 - n_1^2}}\right) \tag{29b}$$

where $n_{\text{eff}} = n'_{\text{eff}} - i n''_{\text{eff}}$, and

$$c_1 = \begin{cases} 1 \\ n_e^2/n_o^2 \end{cases}, \quad c_2 = \begin{cases} 1 \\ n_1^2/n_o^2 \end{cases}, \quad \tilde{n}_2^2 = \begin{cases} n_o \\ n_e \end{cases}$$

are valid for the respective modes TE and TM.

The remaining wave propagation parameters are calculated from the formulae

$$\tilde{q}^2 = k_0^2 (n_{\text{eff}}^2 - n_1^2), \tag{30a}$$

$$\tilde{h}^2 = k_0^2 \left(n_o^2 - \frac{n_{\text{eff}}^2}{c_1}\right). \tag{30b}$$

The solution of the waveguide equation has a well-known form

$$E_2(z) = \left.\begin{matrix} A_e^{-\tilde{q}(z-a)}, & z \geqslant a \\ A\left\{\cos[\tilde{h}(a-2)] + \frac{\tilde{q}}{\tilde{h}} \frac{1}{c_2} \sin[\tilde{h}(a-z)]\right\}, & -a \leqslant z < a \\ A\left\{\cos(2\tilde{h}a) + \frac{\tilde{q}}{\tilde{h}} \frac{1}{c_2} \sin(2\tilde{h}a)\right\} e^{\tilde{q}(z-a)}, & z \leqslant -a \end{matrix}\right\}. \tag{31}$$

5. Nonuniformity wave equation

We rewrite the wave equation (23) in the normalized coordinates $u = x/\sigma$

$$\frac{d^2 E_1(u)}{du^2} + V^2(e^{-u^2} - b) E_1(u) = 0 \tag{32}$$

where: $V = k_0\sigma\sqrt{a_p}$, $b = \dfrac{(\beta/k_0)^2 - n_{\mathrm{ed}}^2}{a_p}$, $0 < b < 1$,

$$a_p = 2n_2^0 \Delta n_2^0,\quad n_{\mathrm{ed}}^2 = n_{\mathrm{eff}}^2 - a_p. \tag{33}$$

The imaginary part of the effective index of refraction n''_{eff} is transferred via the formula (33) to the imaginary part of the propagation constant β. Consequently, the wareguide modes are attenuated like the planar waveguide modes. Keeping this in mind, in further considerations we will assume that b is real. The solution of Eq. (32) which should disappear in infinity $E_1(\pm\infty) = 0$ may be obtained by using the WKB method. This solution is of oscillatory character within the turning points $|u| < |u_z|$ and diminishes exponentially outside $|u| > |u_z|$. The turning points are obtained from the equation

$$e^{-u} - b = 0. \tag{34}$$

Since the function e^{-u^2} is symmetric with respect to $u = 0$, the function $E_1(u)$ has either symmetric or antisymmetric solution. It suffices to writte the solution for a single half axis, for instance, for $u > 0$, it takes the form

$$E_1(u) = \begin{cases} 2A\dfrac{1}{\sqrt{\varkappa(u)}}\cos\left(\displaystyle\int_u^{u_z}\varkappa(u)\,du - \dfrac{\pi}{4}\right), & 0 \leqslant u < u_z, \\ A\sqrt{\dfrac{2\pi\int_u^{u_z}\varkappa(u)\,du}{3\varkappa(u)}}\left\{J_{1/3}\left[\displaystyle\int_u^{u_z}\varkappa(u)\,du\right] + J_{-1/3}\left[\displaystyle\int_u^{u_z}\varkappa(u)\,du\right]\right\}, & u \leqslant u_z, \\ A\sqrt{\dfrac{2\pi\int_{u_z}^{u}\chi(u)\,du}{3\chi(u)}}\left\{-I_{1/3}\left[\displaystyle\int_{u_z}^{u}\chi(u)\,du\right] + I_{-1/3}\left[\displaystyle\int_{u_z}^{u}\chi(u)\,du\right]\right\}, & u \geqslant u_z, \\ A\dfrac{1}{\sqrt{\chi(u)}}\exp\left[-\displaystyle\int_{u_z}^{u}\chi(u)\,du\right], & u > u_z \end{cases} \tag{35}$$

where A is a constant, J_ν and I_ν are the ordinary and modified Bessel functions of ν-order, respectively, while the parameters $\varkappa(u)$ and $\chi(u)$ are defined as follows:

$$\varkappa(u) = V(e^{-u^2} - b)^{1/2},$$

$$\chi(u) = V(b - e^{-u^2})^{1/2}.$$

Taking account of the asymptotic development of the function J_ν and I_ν and sewing them together at the point $u = u_z$ we may find the solution of the function E_1 at the turning point

$$E_1(u_z) = \mathrm{A}\frac{\sin(\pi/3)\Gamma(1/3)}{3^{2/3}\sqrt{\pi}\,(2u_z b)^{1/6}}$$

where $\Gamma(1/3)$ – gamma Euler function. The characteristic equation is given in the form

$$2V\int\limits_0^{u_z}\sqrt{e^{-u^2}-b}\,du = \left(m+\frac{1}{2}\right)\pi,\; m = 0,\,1,\,2\,\ldots. \tag{36}$$

Depending on the value of m, the function $E_1(u)$ may be symmetric or antisymmetric.

6. Calculations and discussion of results

All the calculations were carried out on the Odra computer, using the MALA programme for the crystal $LiNbO_3$ of $n_o^0 = 2.286$ and $n_e^0 = 2.192$. In Figures 4 and 5 the dispersive characteristics of the waveguide vs. the z-coordinate are presented. From Fig. 4 it may be seen that the dispersive characteristics of the TE and TM modes differ only slightly and are similar to each other independently of the kind of the metal coating (Fig. 5). The influence of the metal coating on the propagation of the electromagnetic wave is visible in the imaginary part on the effective complex index of refraction (Fig. 6). The silver, which is a metal of high reflectance, causes much lower attenuation of the same modes than aluminium and chromium and shows smaller difference in TE and TM modes attenuation than the other mentioned metals. The determination of the number of modes which may propagate in the planar waveguide with a metal coating is by no means an easy job due to the complexity of the formula (29a). However, for the thick waveguides ($a \gg \lambda$) the following condition is fulfilled

$$\frac{1}{k_0 a}\,\mathrm{Re}\left(\frac{c^2}{\sqrt{n_2^2-n_1^2}}\right) \ll 1.$$

This allows us to simplify considerably the expression (29a), for the cut-of conditions $n'_{\mathrm{eff}} \to 0$ the number of propagating modes may be determined from the formula

$$N = 2\,\frac{k_0 a n_0\sqrt{2}}{\pi} - 1. \tag{37}$$

From Figures 6 and 7 it may be seen that the modes of higher order are attenuated very strongly, and from this fact their presence in the waveguide should be inferred. Assuming the losses of order of 10 dB/cm as a criterion for the presence of a mode in the waveguide, it may be shown that in a $LiNbO_3$ waveguide of 2a = 50 μm with an Ag coating as much as $N = 240$ TE modes and $N = 110$ TE modes propagate simultaneously. The distributions of the field modes $\mathrm{Re}(H_x)$ and $\mathrm{Im}(H_x)$ are shown in Fig. 8. The $\mathrm{Re}(H_x)$ field distributions are similar to those in the waveguides with the dielectric coatings. It is worth noting that the imaginary part $\mathrm{Im}(H_x)$ is by two orders of magnitude less than

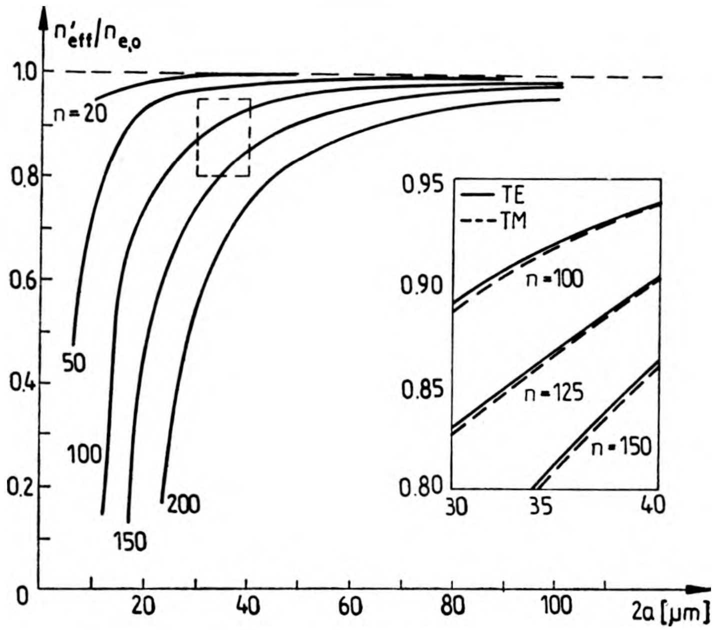

Fig. 4. Dispersive characteristics of TE and TM modes in the waveguides coated with silver $n' = 0.065$, $n'' = 3.9$, $n = n' - in''$

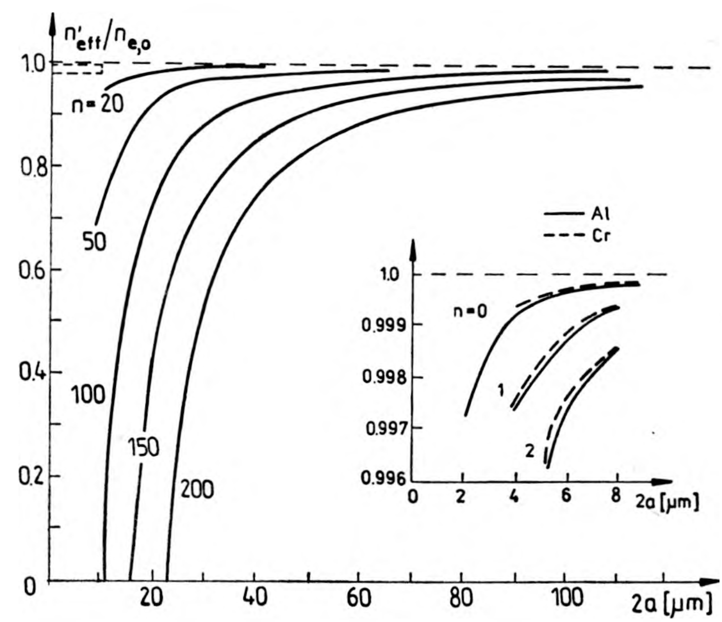

Fig. 5. Dispersive characteristics of waveguide for two different metal coatings (aluminium and chromium): $n_{Al} = 1.7 - i\ 7.0$, $n_{Cr} = 3.19 - i\ 2.26$

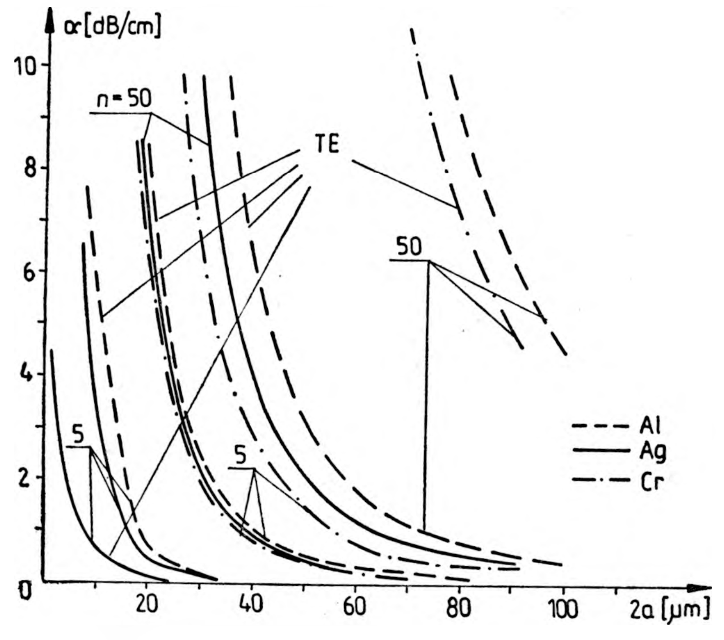

Fig. 6. Mode attenuation vs. the waveguide thickness for different metal coatings: $n_{Al} = 1.7 - i\ 7.0$, $n_{Cr} = 3.19 - i\ 2.26$, $n_{Ag} = 0.065 - i\ 3.9$ ($\alpha = -8.686\ n''_{eff} k_0 \times 10^4$ [dB/cm])

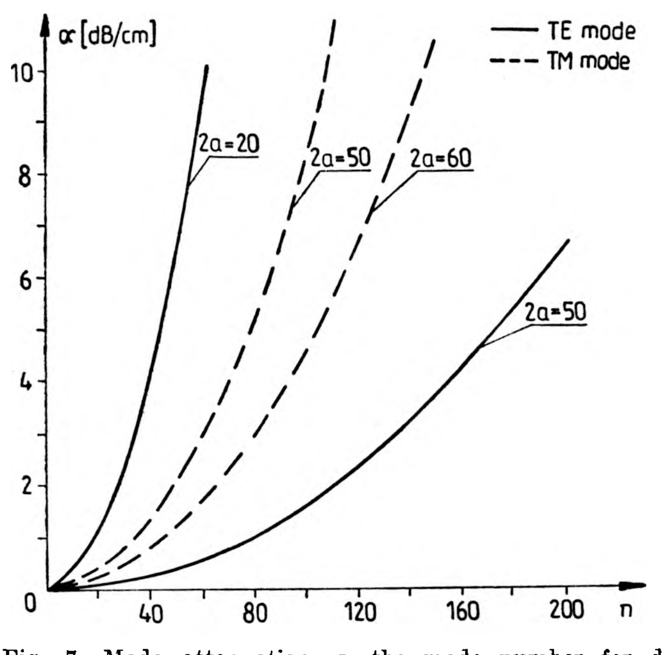

Fig. 7. Mode attenuation vs. the mode number for different thicknesses of the waveguide coated with silver

$\mathrm{Re}(H_x)$ which corresponds to the relation of the imaginary part h'' to the real part h' of the parameter h. The dispersive waveguide characteristics as the functions of the coordinate x (Fig. 9) differ considerably from the characteristics being the function of the coordinate z. This is the result of small changes of refractive indices in the direction of x. The shapes of the characteristics of E^x and E^z modes are similar, except the fact that for the same modes m the values V

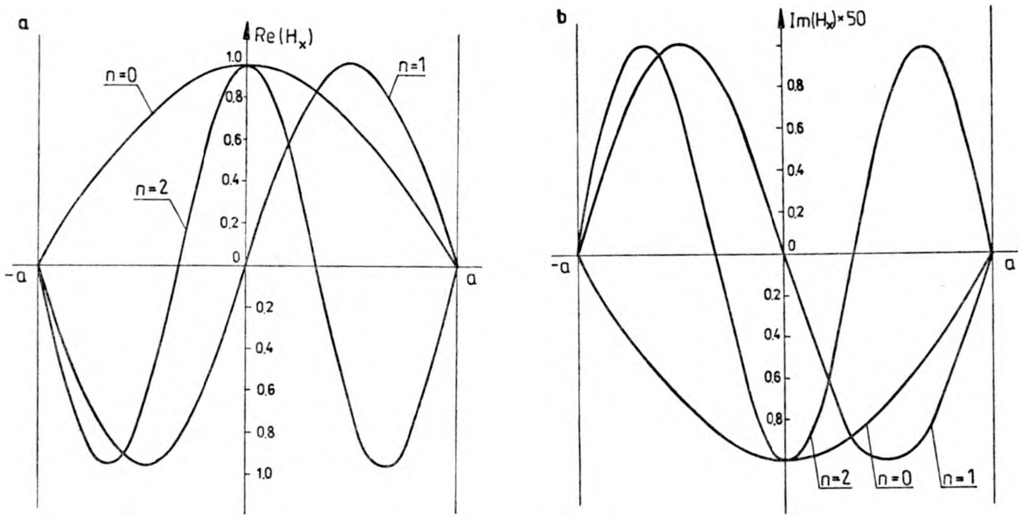

Fig. 8. Distributions of field H_x: (a) real part of $\mathrm{Re}(H_x)$ and (b) imaginary part of $\mathrm{Im}(H_x$

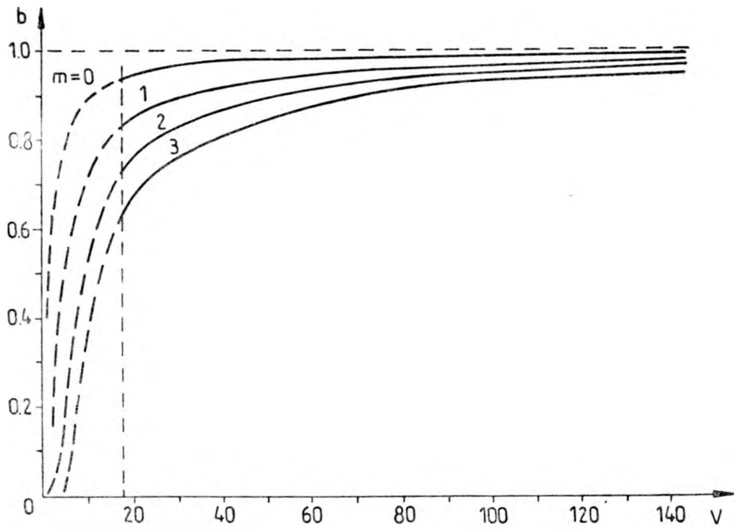

Fig. 9. Dispersive characteristics of E^z modes in waveguide with respect to x-coordinate. Graph for $V < 15$ after [4]

and b for modes E^x and E^z are different, since the modes E^z depend on the extraordinary refractive index n_e, while the modes E^x depend on the changes of extraordinary refractive index n_o.

Figure 10 shows the power distribution within the modes in the x-direction. The vertical line denotes the turning points of the modes. Low numbers of the modes m for high values of the parameter V cause a quick exponential decay. At the same time when V increases, for increasing σ, the region of oscillatory solutions is broadened. The number of modes M which may propagate within

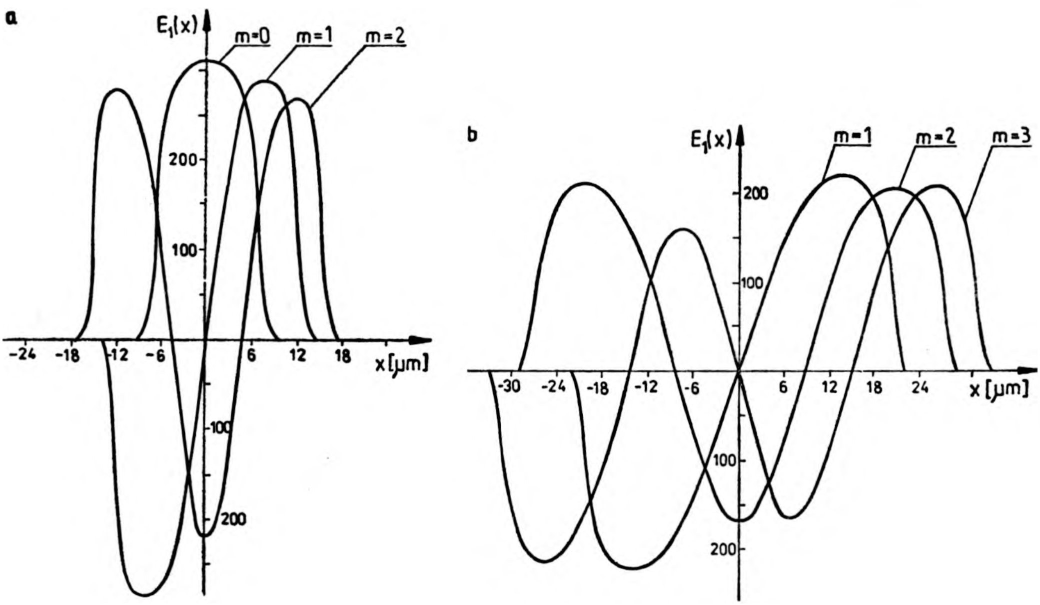

Fig. 10. Field distribution for E^z modes with respect to the x-coordinate: for $V = 25$ (a), and $V = 72$ (b)

a structure with respect to the x-coordinate is calculated by assuming the cut-off conditions $b \to 0$ and $x_z \to \infty$ and by integrating the formula (36). The latter is

$$M = V\frac{\sqrt{2}}{\sqrt{\pi}} - \frac{1}{2}. \tag{38}$$

Since $V \sim \sigma$ and $V \sim \sqrt{2n^0_{e,o}\Delta^0_{e,o}}$, the numbers of modes E^x and E^z propagating in the channel in the x-direction are different and directly proportional to the width of the electrodes w (since $\sigma \sim w$) and to the magnitude of the applied voltage U (since $\Delta n_{e,o} \sim u$). The complete distribution of the field modes $E^z_{0,0}$ and $E^z_{1,0}$ is a product of the distribution in the directions x and z and is presented in Fig. 11.

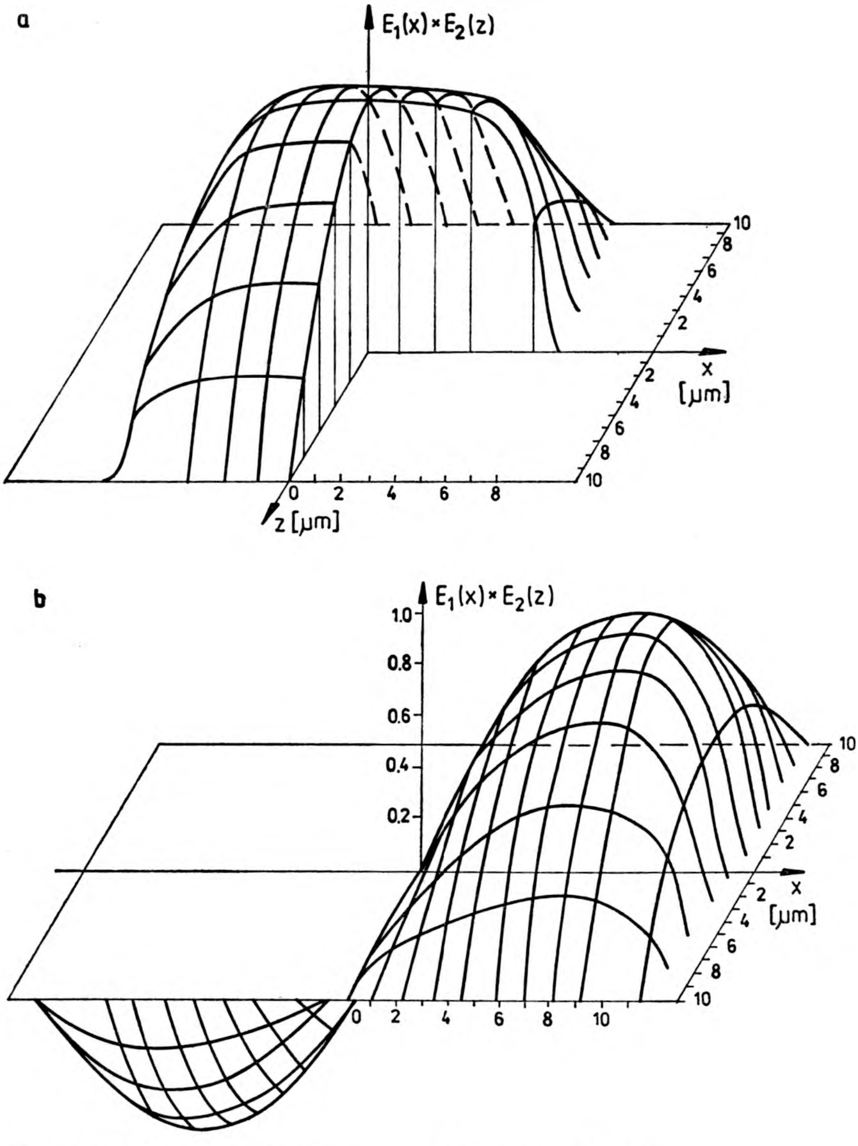

Fig. 11. Field distribution for modes: $E^z_{0,0}$(a), and $E^z_{1,0}$ (b)

7. Conclusions

In the paper the properties of the electrooptical channel have been analysed. The analysis was based on the method of effective refractive index, solving two basic problems:

– planar waveguide with a metallic coating,

– waveguide with nonuniform distribution of the refractive index.

It has been pointed out that the modes E^z and E^x are the products of the solutions of modes TM or TE in the uniform planar structure, and nonuniform waveguide equation with respect to the x-coordinate. The nonuniformity of the waveguide equations for modes E^z and E^x follows respectively from the nonuniformity of the changes in extraordinary (n_e) and ordinary (n_o) refractive index distributions in the direction of x. The number of modes propagating in the structure should be considered for the x- and z-coordinates, separately. The prevailing factors determining the number of propagating modes are: the mode attenuation for the z-coordinate and the cut-off conditions, following from the changes in ordinary and extraordinary refractive indices, for the x-coordinate. Since Δn_e is greater than Δn_o the number of modes E^z propagating in the waveguide is higher than that of propagating modes E^x.

In the work, the influence of the metal coating on the attenuation of the propagating electromagnetic wave has been shown. Besides the waveguide thickness, the predominant factors for the attenuator are also the kind of metal coating and the number of the propagating mode. The results of this work are useful for analysing the coupling between two electrooptic waveguides which will be the subject of the next publication.

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Анализ распространения электромагнитной волны в многомодовом электрооптичес ком волноводе

Проведен анализ оптических свойств многомодового электрооптического волновода, возникающего в результате приложения напряжения к электродам, помещенным по противоположным сторонам плоскопараллельной пластинки кристалла $LiNbO_3$ ($LiTaO_3$) со срезом z. Для анализа применен метод эффективного коэффициента преломления, причем получилось подразделение проблемы на два вопроса: распространение в волноводах – однородном, плоском, с металлическим покрытием, а также в плоском, неоднородном. Ввиду анизотропии распространяющиеся в волноводе моды типа E^x и E^z зависят соответственно от изменений чрезвычайного n_e и обыкновенного n_o коэффициентов преломления. Показано, что в волноводе, в зависимости от условий генерирования, может распространяться несколько сот модов типа E^x и E^z.