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Analysis of the thermal focusing effect in a cw Nd:YAG laser

Z. JANKIEWICZ, M. MINDAK, J. SZYDLAK, J. WÓJCIK

Institute of Optoelectronics, Military Technical Academy, Warsaw, Poland.

In the paper an analysis of the thermal focusing effect in the cw Kr lamp-pumped Nd : YAG laser rod has been presented as well as results of measurements of the thermal focal length as a function of the lamp power have been given for two Nd : YAG crystals. A theoretical analysis has been performed on a basis of Koechner's work [1] taking additionally into account the change in the lamp spectral characteristics with increasing current density. It has been shown that the heat generated in continuously excited Nd : YAG crystal is proportional to $M_p^{3/2}$, where M_p denotes the electrical input to the lamp, while the thermal focal length f is reversely proportional to $M_p^{3/2}$ ($f = \eta M_p^{-3/2}$, η is a constant parameter for a given laser head and a given lamp). The theoretical results have been confirmed experimentally by measurements of focal length of the lens formed in Nd : YAG rods. In a wide range of the pumping powers (1.5–6 kW) a good agreement between the theory and the experiment has been obtained, even a better one than that in [1] and [2]. The measurement results are in agreement with those given by FOSTER and OSTERINK [3]. Basing on the empirically determined value of the parameter and on the material constants it has been calculated that the amount of heat dissipated in the Nd : YAG laser rod pumped by Kr arc lamp is from 3.5% (for $M_p = 1.5$ kW) to 6.5% (for $M_p = 6$ kW) of electrical input power.

1. Introduction

In continuously pumped high-power solid laser materials appreciable amounts of heat are generated. Radiationless transitions, being a result of the quantum efficiency less than unity and absorption of the lamp radiation beyond the excitation bands, are the heat sources.

The necessary, in this situation, intensive cooling of the outer surface of the lasing material leads to formation of temperature gradient along the rod radius, being the source of mechanical strains. Due to the temperature-induced and strain-induced changes of refractive index, both the mentioned factors cause the laser-radiation focusing by the lasing medium which behaves as a thick converging lens.

The next sources of radiation distortion are the following: deformation of the front surfaces of the laser rod due to the temperature gradient and mechanical strain-induced birefringence of the material.

The further considerations will be restricted only to an analysis of thermal focusing due to temperature-induced refractive-index changes, since this phenomenon has a crucial significance for cw Nd : YAG laser operation.

2. Thermal focusing

Assuming the heat generation rate, A (Wcm^{-3}), to be constant in the whole volume of the active material, the temperature profile along the laser-rod radius is given by the equation [1]

$$T(r) = T_0 - Ar^2/4K \tag{1}$$

where T_0 denotes temperature along the crystal axis, and K is the crystal thermal conductivity.

The total heat power P (W) generated in the laser rod depends upon the power of the incident radiation and its spectral characteristics.

Radiation power of Kr lamp widely used for pumping cw Nd : YAG lasers is directly proportional to the electric input power to the lamp, while the radiation spectrum depends upon current density flowing through the lamp.

With increasing current density both the continuum of the Kr lamp spectrum and short-wavelength radiation intensity increase more quickly than intensity of the Kr emission lines coinciding with Nd : YAG excitation bands [4]. It means that with the increasing lamp input power an absorption beyond the excitation bands will increase and the thermal load of the lasing medium will also increase.

Relation between the heat power P generated in the active material and the lamp electric input power M_p can be written in the following way:

$$P = \eta_k \eta_\lambda M_\mathrm{p} \tag{2}$$

where factor η_k is a constant for a given type of the lamp and for a given laser head construction, whereas η_λ takes account of the lamp spectrum change with increasing current density.

Let us assume in the first approximation that the factor η_λ is direct proportional to the current density, i.e., $\eta_\lambda \propto \varrho_I$. Since $\varrho_I \propto M_\mathrm{p}^{1/2}$, then also $\eta_\lambda \propto M_\mathrm{p}^{1/2}$. Substituting the latter relation into Eq. (1) we obtain

$$P = \eta M_\mathrm{p}^{3/2} \tag{3}$$

where η is a constant depending upon the laser head construction (mainly upon its reflector parameters) and upon a type of the used pumping lamp.

Knowing the dependence of P upon the lamp input, the heat generation rate A may be also expressed as a function of M_p

$$A = \frac{P}{\pi R^2 l} = \frac{\eta}{\pi R^2 l} M_\mathrm{p}^{3/2} \tag{4}$$

where l is the pumped length of the laser rod, and R denotes its radius. Substituting Eq. (4) into Eq. (1) we obtain

$$T(r) = T_0 - \frac{\eta r^2}{\pi R^2 l 4K} M_{\mathrm{p}}^{3/2}. \tag{5}$$

The radial temperature gradient produced in the rod induces refractive index changes along the rod radius. Changes of n can be divided into changes directly due to temperature and changes due to the mechanical strains introduced to the crystal

$$n(r) = n_0 + \Delta n_T(r) + \Delta n_M(r)$$

(n_0 – refractive index along the rod axis).

In the further considerations small changes of n due to elastooptic effects will be neglected.

Refractive-index distribution in a presence of the temperature gradient can be expressed by the relation

$$n(r) = n_0 - \frac{\partial n}{\partial T} \times \partial T = n_0 - \frac{\partial n}{\partial T} [T_0 - T(r)]. \tag{6}$$

Substituting $\big(T_0 - T(r)\big)$ from Eq. (5) we obtain

$$n(r) = n_0 - \frac{\partial n}{\partial T} \frac{\eta}{4\pi K l} \left(\frac{r}{R}\right)^2 M_{\mathrm{p}}^{3/2} \tag{7}$$

and

$$\Delta n_T(r) = \frac{\partial n}{\partial T} \frac{\eta}{4\pi K l} \left(\frac{r}{R}\right)^2 M_{\mathrm{p}}^{3/2}. \tag{8}$$

KOGELNIK [2] has presented the method of calculating parameters of the lens formed in a cylindrical medium with the varying n distribution given by the equation

$$n(r) = n_0\left(1 - 2\frac{r^2}{b^2}\right) \tag{9}$$

where n_0 is a constant value of n at the axis, r is a distance from the axis and b is a factor describing the variation of n. In this case the focal length f (Fig. 1) will be given by the expression

$$f = \frac{b}{2n_0 \sin 2l/b}, \tag{10}$$

and distance between the principal planes and the rod faces

$$h = \frac{b}{2n_0} \tan\frac{l}{b}. \tag{11}$$

Comparing Equations (7) and (9) we will obtain

$$b^2 = \frac{8\pi K n_0}{\eta \frac{\partial n}{\partial T}} \frac{R^2 l}{M_p^{3/2}} = \frac{8 K n_0}{\frac{\partial n}{\partial T} A}. \tag{12}$$

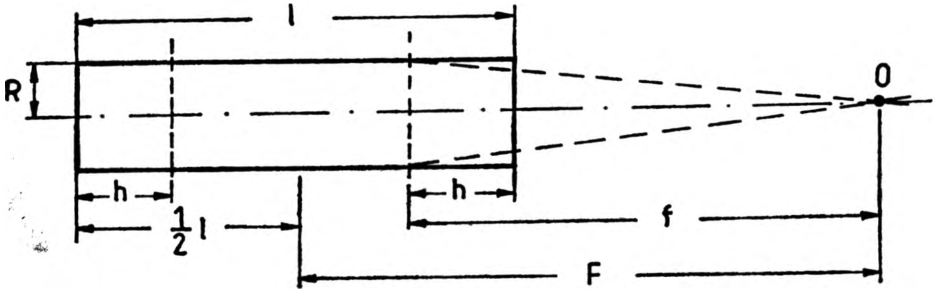

Fig. 1. Scheme of the thermal lans formed in the pumped laser rod. Symbols are given in the text

It is known from calculation of the numerical values that $l \ll b$ and hence Eqs. (10) and (11) may be substituted by simpler approximations

$$f = \frac{b^2}{4 n_0 l} \tag{13}$$

and

$$h = \frac{l}{2 n_0}. \tag{14}$$

Substituting b^2 from Eq. (12) into Eq. (13) we finally obtain a dependence of the thermal focal length upon the input power

$$f = \frac{2\pi K R^2}{\eta \frac{\partial n}{\partial T}} M_p^{-3/2} = a M_p^{-3/2}. \tag{15}$$

From the obtained equation it can be seen that the thermal lens focal length is directly proportional to the cross-section area of the rod R^2 and reversely proportional to the lamp input power M_p and that it depends upon the laser head construction and the lamp type (Kr pressure) η, as well as upon the rod-material constants K, and $\partial n/\partial T$.

Reversely proportional dependence of the focal length f upon the lamp power raised to the power of 3/2 attracts special attention. In papers [1] and [3] $f \propto M_p^{-1}$, and the experimental results presented therein do not agree with the given theoretical curves.

Equation (15) makes it possible to calculate the thermal focal length for a given lamp electrical input power and for known material constants and parameter η. In order to find η the focal length f should be determined experimentally for a given lamp power M_p and then

$$\eta = \frac{2\pi K R^2}{f \frac{\partial n}{\partial T}} M_p^{-3/2}. \tag{16}$$

3. Measurements of the thermal focal length as a function of the lamp electrical power

Experimental measurements have been performed for two Nd : YAG rods of dimensions: ø5 mm × 80 mm, placed in LCW–1/S laser head (Fig. 2). Dry single-elliptical pump cavity is covered with silver reflection coating. At one reflector focus the Nd : YAG rod is mounted, and at the other one a DNP–6/75 Kr arc lamp. The rod and the lamp are surrounded with pyrex tubes ø12 mm providing distilled water.

For investigation of the focussing effect He-Ne laser has been employed, placed at such a distance from LCW–1/S head as to make its beam fill the whole

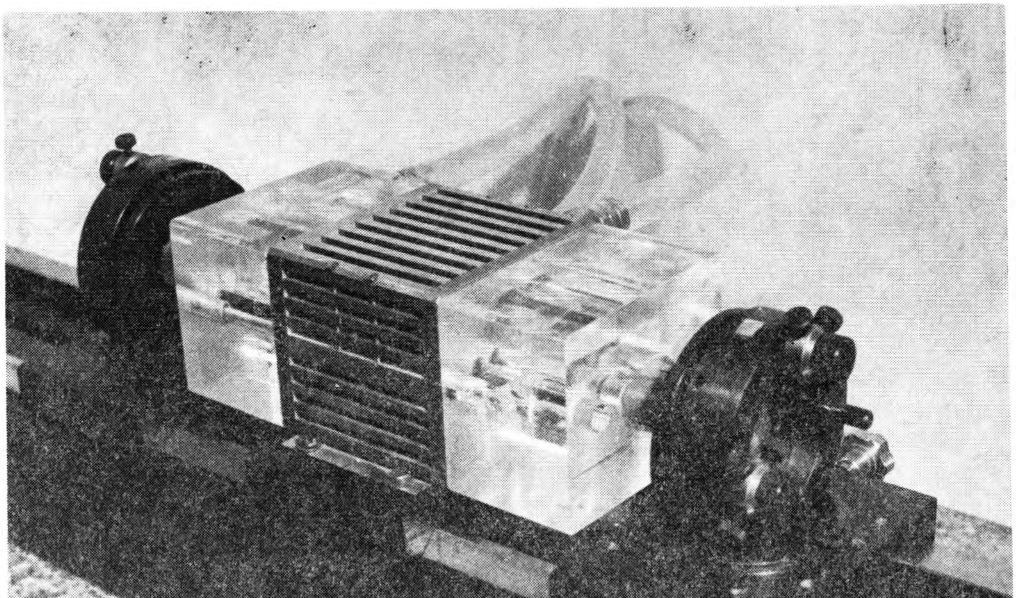

Fig. 2. Photograph of LCW–1/S laser which was used for the thermal focal length measurements

rod diameter and to make it parallel with its axis. During the measurements LCW-1/S laser mirrors have been removed.

The focus position has been determined by shifting a movable screen along the optical rail on which LCW-1/S head has also been situated. The correction for small He-Nd laser-beam divergence has not been taken into account in the focal-length measurements. These measurements have been carried out for lamp input powers ranging from 1.5 to 6 kW. For each rod the measurements have been performed thrice, the scatter of results being less than ±5%.

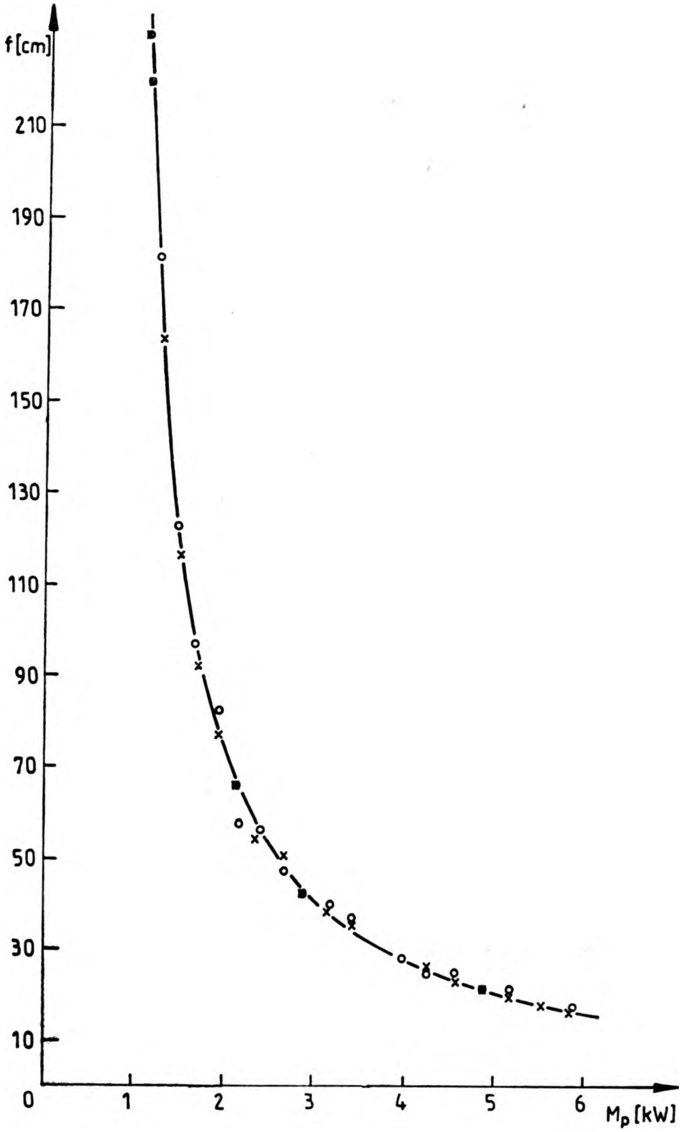

Fig. 3. Results of the thermal focal length measurements as a function of the electrical input into the lamp obtained for both the Nd : YAG rods. Experimental points: ○ – Nd : YAG ø 5 × 80, No. 7; X – Nd : YAG ø 5 × 80, No. 13

The thermal focal length vs. the lamp input power for both the rods is plotted in Fig. 3. The shape of this dependence for both the rods is, within the measurement accuracy, identical.

An approximation of the experimental results with exponential function of the aM^x type has given the following results: $a = 222.78$ and $x = -1.4636$, these factors being obtained with the linear-regression method. The shape of the function found in this way is the same as the theoretical one in the whole range of the input lamp powers applied. The functional form agrees with the previously presented results of the theoretical analysis. Very close results of the measurements were obtained in paper [3] in which the relation of $f \propto M_p^{-1.47}$ was given.

Both in paper [1] and in [3] theoretical analysis of the problem led to the dependence of $f \propto M_p^{-1}$, which caused that the theoretical curves agreed with the measurements results only in a narrow range of the lamp input powers.

Substituting the found value of coefficient a into Eq. (15) one obtains the dependence of η upon a, R and the crystal material constants

$$\eta = \frac{2\pi K R^2}{a\frac{\partial n}{\partial T}}. \tag{17}$$

For the following numerical values [1]: $K = 0.11$ Wcm^{-1} K, $R = 0.25$ cm, $a = 222.8$ cmkW$^{3/2}$ $= 7045.2$ cmW$^{3/2}$, $\partial n/\partial T = 7.3 \times 10^{-6}$ K^{-1} one obtains

$$\eta = 8.5 \times 10^{-4}\ [\mathrm{W}^{-1/2}]. \tag{18}$$

Knowing the exact value of η the total heat power generated in the Nd : YAG rod can be calculated from Eq. (3). The results obtained for the pumping powers ranging from 1.5 to 6 kW are shown in the Table. As can be seen from it, when pumping the Nd : YAG crystal with the continuous Kr lamp, 3.5%–6.5% of the input power is generated in the form of heat which should be carried away from the active material.

An influence of the thermal focussing effect on cw Nd : YAG laser operation is illustrated in Fig. 4. The power characteristics of the cw Nd : YAG laser, obtained for different resonator lengths L_R have been presented there. It can be seen that the focussing influence becomes more critical with increasing L_R. For lower and lower powers the resonator ceases to be stable and a break of generation may follow.

$\eta = 8.5 \times 10^{-4}\ [\mathrm{W}^{-1/2}]$

M_p [W]	P [W]	(P/M_p) 100%
1500	49	3.3
2000	75	3.8
3000	139	4.6
4000	215	5.4
5000	300	6.0
6000	394	6.5

The resonator theory [2] says that the system with given mirror curvatures ceases to be stable when the focal length of any inner lens approaches 1/2 L_R. This is confirmed by the shape of the power characteristics of LCW–1/S laser obtained for the smallest length of the resonator, i.e., $L_R = 30$ cm and shown in Figs. 4 and 5. As can be seen from them, the resonator with this length retains stability up to 6 kW of the input power, which corresponds to $f = 16$ cm. A saturation of the characteristics occurs for powers as high as 6–6.25 kW, when the thermal lens focal length approaches 15 cm and the resonator goes out of the stability range.

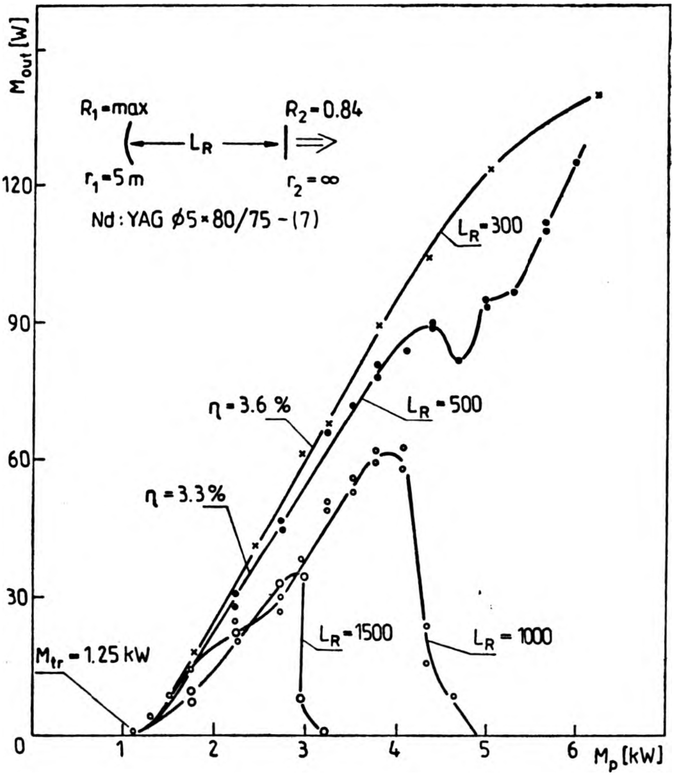

Fig. 4. Output power of LCW–1/S multi-mode cw Na : YAG laser beam as a function of the input power for different resonator lengths L_R

4. Conclusions

The presented analysis of the thermal focusing effect in Nd : YAG laser rod takes account of the change of the lamp spectral characteristics with increasing current density flowing through the lamp.

The assumption of $P = \eta M_p^{3/2}$ leads to the relation $f \propto M_p^{-3/2}$ which has been confirmed by the experimental results. The latter agrees with the results given in paper [3].

The experimentally determined value of the parameter η has made it possible to calculate the heat generation rate in the Nd : YAG rod for a given lamp input power. The obtained results agree, within a measuring error, with those of the power balance of the optical pump in LCW–1/S laser head. The power

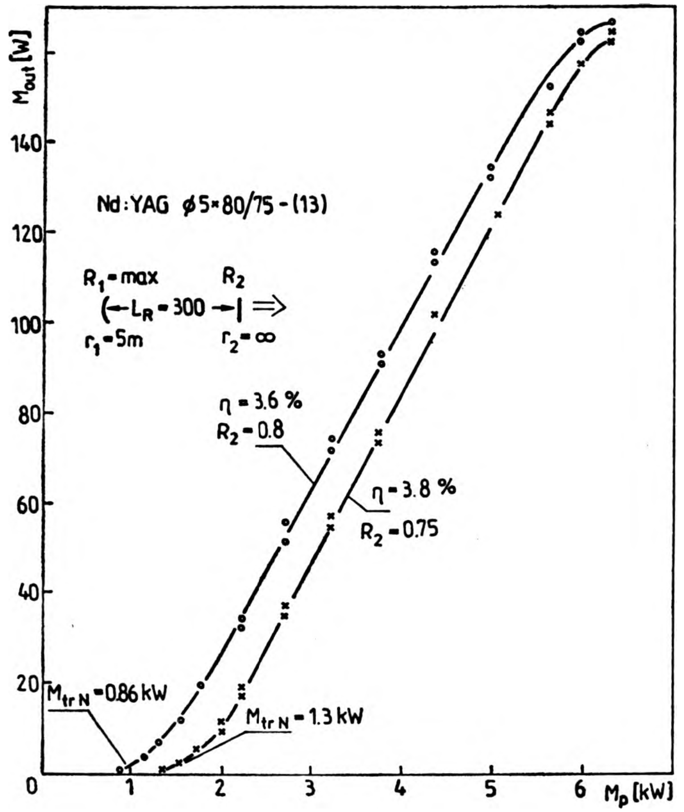

Fig. 5. Output power of LCW-1/S multi-mode cw Nd : YAG laser beam as a function of the input power for different reflectivity R_2 of the output mirror

balance calculated on a basis of calorimetric measurements has been presented in paper [5].

The exactly determined dependence of focal length of the thermal lens formed in the active material upon the input power will constitute the ground for elaboration of the focusing-effect compensation method and for optimalization of single- and multi-mode generation for high lamp input powers.

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Исследование термической фокусировки излучения Nd:YAG лазера непрерывного действия

В настоящем сообщении приведен теоретический расчет теплового режима активного элемента аналогично, как это показано в работе [1]. Показана необходимость учета изменения спектра излучения лампы накачки от плотности ее тока. Результаты измерений совпадают с теоретическими предпосылками.