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Meridional light path function and coma transfer for corrected holographic concave gratings

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The general form of the light path function for corrected holographic concave gratings in the meridional cut up to the third order in the transverse grating coordinate, using both aspheric wavefronts and aspheric grating supports for grating record is calculated. From this light path function one obtains a formula which describes the change of the third order part (comatic part) of the incident wavefront by the grating. Together with the focussing properties of the grating this gives the possibility for treating optical systems containing gratings.

1. Introduction

In optics the paraxial approximation is a useful first step for the treatment of a system. An interesting possibility for such a treatment is the formalism of the ABCD-matrix in the resonator theory [1]. In the framework of this theory the local transformation of the paraxial properties (curvature) of the wavefront by an optical surface is followed by the transformation of these properties in a homogeneous medium to the next optical surface and so on.

If the optical surface is a concave grating the transformation of the paraxial properties can be derived from the focussing formulae given for classical concave gratings by BEUTLER [2] and for holographic concave gratings in [3]. In the paper [6] the formulation in the framework of the ABCD-matrix was given.

Practical experiences in the correction of holographic gratings show that the paraxial properties of a grating are not sufficient for obtaining good correction, because at least comatic properties should be included. This is also valid for systems.

We follow the paraxial method "transformation by a surface-propagation in a medium - transformation by the next surface" and try the same method for the comatic part (third power of the wavefront expansion).

The third-order properties of a holographic grating are determined by the third-order properties of the wavefronts used for producing this grating and by the third-order properties of the grating support. Therefore, we add to the usual second-order terms (parabolic wavefronts which approximate spherical waves) the coefficients of the third power of a coordinate perpendicularly to the propagation direction of the central ray of the wavefront. Such wavefronts can be generated by oblique passage of optical elements (for instance, corrected holographic gratings).

The idea of our calculations is: A wavefront with paraxial and comatic properties is given. Then we look for the paraxial and comatic properties of this wavefront after diffraction by the grating. Finally, we need the propagation of such a wavefront in a medium ("free space").

Then we have the tool to construct systems up to the third order. A simple example of a system will be the classical Czerny-Turner-mounting.

The above explained inclusion of the third-order coefficients in the production of gratings is a further step for the extension of the conception of "deformed wavefronts" which up to the paraxial approximation was given in [6] and in a constructive way - in [5]. The new third-order coefficients in the light path function can be used for the correction of the coma of a grating or of a system.

The inclusion of the mentioned deformations has another aspect. The computer-controlled ruling engine of HARADA [4] has a great flexibility because it is able to generate the groove curvatures by a computer programme. Comparing this technique with the usual holographic production we have to take into account that the two point-sources of holographic production offer a system of interference patterns which consists of ellipsoids or hyperboloids. This optically possible system of interefernce patterns can be generalized by adding comatic parts to the wavefronts. We do not know to what extent a reasonable deformation of optical wavefronts can offer the results given in [4], but the point source is not the last possibility of optics.

As a first step we treat in this paper the meridional cut of a grating, because the meridional coma is the aberration of third order which is corrected first. The more complicated calculations including the sagittal coma are under investigation.

2. Formulation of the aberration expansion

The usual formulation of the light path function Δ is represented in Fig. 1. Two spherical waves, emitted by the point sources C and D, interfere on the grating support and yield the grating pattern. If we illuminate the recorded grating by point source A we obtain image B which is mostly disturbed by aberration. The usual light path function is written in the form

$$\Delta = \overline{AM} + \overline{BM} - \frac{k\lambda}{\lambda_0}(\overline{CM} - \overline{DM}). \qquad (1)$$

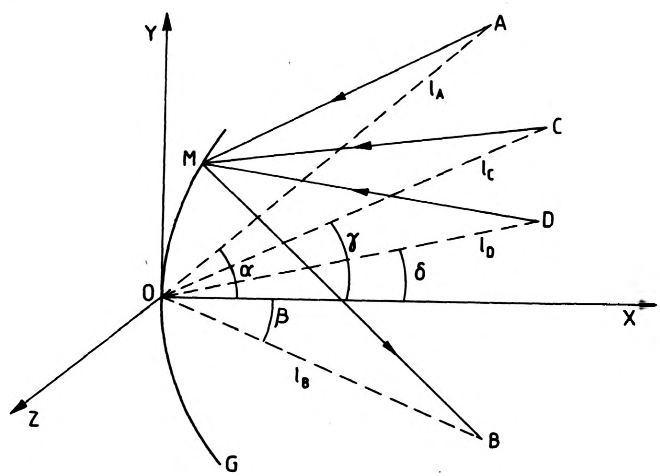

Fig. 1. Usual production and use of corrected concave gratings

Equation (1) means that the optical path from A via M to B is to be corrected by the phase change which must be taken into account if M varies from groove to groove. Point M on the grating surface is determined by the coordinates Y_M and Z_M and by the surface equation $X_M = X_M(Y_M, Z_M)$. If we look for the meridional part of the light path function we specify $Z_M = 0$. This specification does not mean that we cut the real spectral line because the sagittal coma also contributes to the line width. Our aim is to calculate meridional focussing and meridional coma.

In Figure 2 we demonstrate the general scheme for record and use of gratings which will be treated in the meridional cut up to the third order. Between A, B, C, D and G there are optical systems. G has an aspherical part in the X-Y plane. Obviously, the light path func-

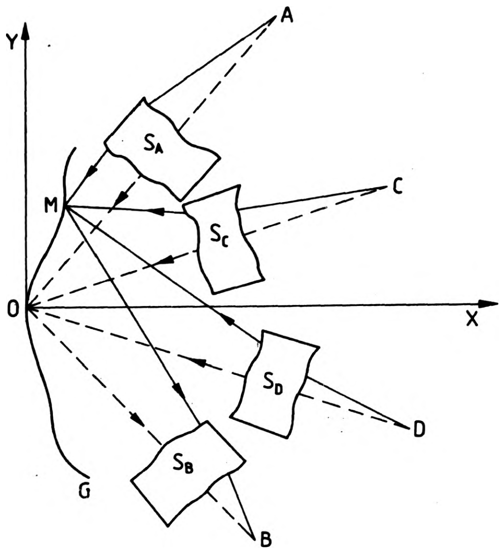

Fig. 2. Concave grating generated by aspherical wavefronts on an aspherical grating support

tion has again the form (1), where $\overline{AM}$ means the optical path length.

If M is given, the calculation of the optical path through the general optical system is difficult. Therefore, we reduce the calculation of $\overline{AM}$ to a local expansion of the wavefront at point O. The procedure is explained in Fig. 3. If the light starts in A it generates an aspherical wavefront W which contains point O. The optical path length from A to wavefront W has the constant values $\tilde{l}_A$ for all points A on W. This means

$$\overline{AM} = \overline{\tilde{A}M} + \tilde{l}_A. \tag{2}$$

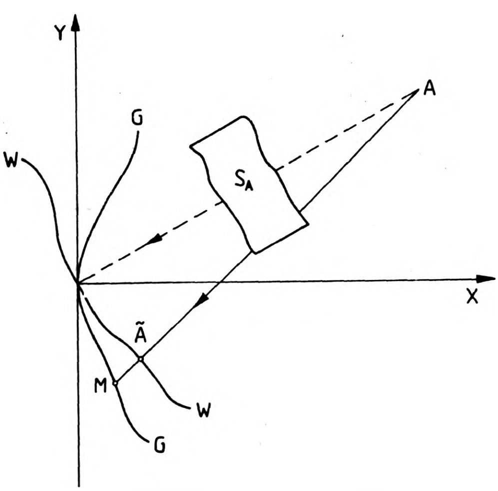

Fig. 3. Reduction of the optical path length $\overline{AM}$ to the local path length $\overline{\tilde{A}M}$

Combining (2) and the equivalent decompositions for $\overline{BM}$, $\overline{CM}$ and $\overline{DM}$ with (1), we can omit the constant terms $\tilde{l}_A$, $\tilde{l}_B$, $\tilde{l}_C$ and $\tilde{l}_D$, since they are constant during a variation of point M. Therefore, $\overline{\tilde{A}M}$ is the important term, i.e., a local formulation of the problem is possible.

Since we treat the problem to the third order only, we expand the wavefront perpendicularly to its principal ray. traversing point O,

$$\hat{X} = A_A \hat{Y}^2 + B_A \hat{Y}^3 \tag{3}$$

in the $\hat{X}-\hat{Y}$ coordinate system, the X-axis of which is directed from O to A. A_A is called the "parabolic part" ($= 1/l_{A\,eff}^2$, $l_{A\,eff}$ - effective radius of curvature of the wavefront), and B_A - the "comatic part". If we require eq. (3), we know that a ray starting from point A is located perpendicularly on the wavefront W and traverses the grating support at point M. We can construct the normal to W at point A. The intersection point M of this normal with the grating surface G can be calculated. Through the inversion of the series we can express the coordinates of A by X_M and Y_M. Hence, $\overline{AM}$ is available as function of X_M and Y_M. Taking into account $X_M = X_M(Y_M) = A_G Y_M^2 - B_G Y_M^3$, $A_G = 1/2R$, and R the radius of curvature of the grating support, $\overline{AM}$ is a function of Y_M only. Similar lengthy though simple calculations with the help of differential geometry provide equivalent formulae for $\overline{BM}$, $\overline{CM}$ and $\overline{DM}$, where (3) must be changed in a corresponding manner.

Then we obtain the light path function

$$\begin{aligned}
\Delta &= \overline{AM} + \overline{BM} - \frac{k\lambda}{\lambda_0}(\overline{CM} - \overline{DM}) \\
&= Y_M\left[-\sin\alpha - \sin\beta - \frac{k\lambda}{\lambda_0}\langle -\sin\gamma + \sin\delta\rangle\right] \\
&+ Y_M^2\left[\frac{\cos^2\alpha}{2l_A} - A_G\cos\alpha + \frac{\cos^2\beta}{2l_B} - A_G\cos\beta\right. \\
&\left.- \frac{k\lambda}{\lambda_0}\left\langle\frac{\cos^2\gamma}{2l_C} - A_G\cos\gamma - \frac{\cos^2\delta}{2l_D} + A_G\cos\delta\right\rangle\right] \\
&+ Y_M^3\left[-\frac{A_G}{l_A}\cos\alpha\sin\alpha + \frac{\sin\alpha\cos^2\alpha}{2l_A^2} - B_G\cos\alpha + B_A\cos^3\alpha\right. \\
&- \frac{A_G}{l_B}\cos\beta\sin\beta + \frac{\sin\beta\cos^2\beta}{2l_B^2} - B_G\cos\beta + B_B\cos^3\beta \\
&- \frac{k\lambda}{\lambda_0}\left\langle -\frac{A_G}{l_C}\cos\gamma\sin\gamma + \frac{\sin\gamma\cos^2\gamma}{2l_C^2} - B_G\cos\gamma + B_C\cos^3\gamma\right. \\
&\left.\left.+ \frac{A_G}{l_D}\cos\delta\sin\delta - \frac{\sin\delta\cos^2\delta}{2l_D^2} + B_G\cos\delta - B_D\cos^3\delta\right\rangle\right].
\end{aligned} \tag{4}$$

Except for the well known aberration terms, here there are new contributions with the coefficients B_G, B_A, B_B, B_C and B_D. Their sources are the comatic parts produced by the optical systems used and by the grating support. In (4) new parameters are available for the correction of gratings. The light path function for curved transmission gratings and for gratings produced by a "backside technique" can be easily derived from (4) by multiplying the terms of eq. (1) by the corresponding index of refraction and by changing some signs.

3. Transfer of the meridional coma

The transfer (transformation) of the first order of Y_M of eq. (4) is given by a change in the direction of the principal ray which passes point O in accordance with the grating equation. The transfer of the second order in Y_M is given by the meridional focussing distance l_{B2} which results from demanding that the corresponding bracket in (4) be equal to zero. This transfer can be formulated by the ABCD-matrix [6].

A transfer of the comatic part (third order of Y_M) would be very useful for implementing gratings in multielement optical systems. If we know the "incident comatic part" B_A we look for the comatic part B_B of the wave leaving the grating. This comatic part is equal to that comatic part which had to be compensated by the system S_B in Fig. 2 in order to obtain an image with vanishing coma in B. Therefore, we demand that the bracket at Y_M^3 in (4) be equal to zero. This yields B_B.

The procedure of the transfer of the third-order aberration is equivalent to the transfer in the first and second orders, where β and l_B were calculated also by equating factors of powers of Y_M (in brackets) to zero. A control possibility for the calculation of B_B is a ray tracing connected with the calculation of the new wavefront from the optical path along the rays.

The treatment of multielement systems requires the transfer of the comatic part in a homogeneous medium without optical elements.

In Figure 4 we show a wavefront which propagates from U to V with the centre of curvature in P. At U the wavefront has the shape

$$X' = A_U Y^2 + B_U Y^3 \tag{5}$$

and at V the shape

$$X = A_V Y^2 + B_V Y^3 \tag{6}$$

omitting a constant. From A_U (= 1/2R, R – radius of curvature) there follows $A_V = 1/2(R - X_1) = (R/(R - X_1))A_U$. This is transfer of the second order. The calculation of B_V requires to start with the wave-

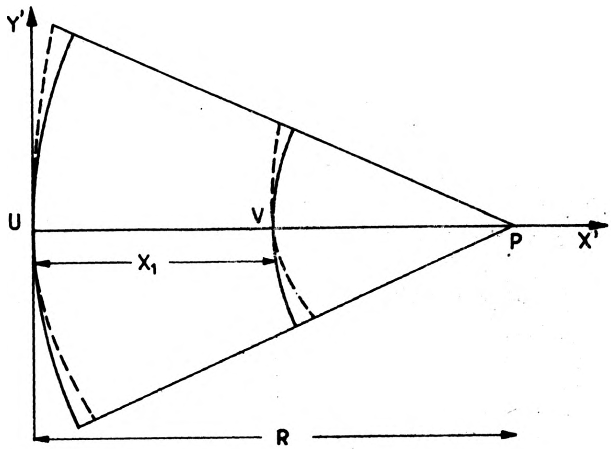

Fig. 4. Transfer of the coma

front at U and to proceed along distance X_1 to V. The new wavefront at V can be derived by means of differential geometry (calculation of the normal direction). The expansion of the new wavefront yields B_V. After an extended calculation we get the equation

$$B_V = (R/(R - X_1))^3 B_U. \tag{7}$$

4. Example: the Czerny-Turner-mounting

For example we derive the formula of Fastie, Reader, Shafer et al.[7] for the Czerny-Turner-mounting (Fig. 5). The light starting from a slit A propagates via concave mirror B, plane grating C and concave mirror D to the receiver E, where we require the meridional coma to be equal to zero for a selected wavelength of the light, determined by the angles α_g and β_g at the grating. All the elements used are special cases of corrected concave gratings (concave mirror: order k = 0, plane grating: all focussing distances and R $\longrightarrow \infty$). From A to B no coma results. At B the concave mirror generates the comatic part

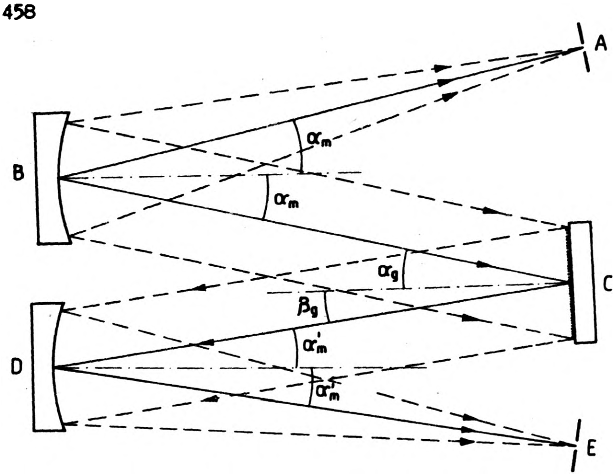

Fig. 5. Classical Czerny-Turner -mounting

$$B'_B = \frac{\sin\alpha_m}{2l_A \cos\alpha_m} \left[\frac{1}{R_1 \cos\alpha_m} - \frac{1}{l_A} \right] \tag{8}$$

(specification of the grating generated coma) or taking into account the collimation of the leaving bundle

$$B'_B = -\frac{\sin\alpha_m}{R_1^2 \cos^3\alpha_m}. \tag{9}$$

From B to C there is no change in the value of the coma, because from eq. (7), with $R \rightarrow \infty$, we get the transfer factor 1. Specifying the generated coma for a plane grating we obtain at C

$$B''_B = -\frac{\cos^3\alpha_g}{\cos^3\beta_g} B'_B. \tag{10}$$

From C to D there is no change in the coma again. The coma coefficient generated at D

$$B'''_B = -B''_B + \frac{\sin\alpha'_m}{R_2^2 \cos^3\alpha'_m} \tag{11}$$

should be equal to zero for grating no coma at E. If we combine the equations (9), (10) and (11) with $B'''_B = 0$ we obtain

$$\frac{\sin\alpha_m'}{\sin\alpha_m} = \frac{R_2^2}{R_1^2}\left(\frac{\cos\alpha_m' \cos\alpha_g}{\cos\alpha_m' \cos\beta_g}\right)^2, \tag{12}$$

wherein R_1 and R_2 are the radii of curvature of both concave mirrors. This formula is well known from literature. Here it was very simply derived by specializing our general formulae.

5. Discussion

The derived expansion of the meridional part of the light path function to the third order is the most general formula which describes the use of aspherical grating supports and aspherical wavefronts. By means of a practical example the usefulness of the formalism was demonstrated for the precalculation of optical systems to the third order containing gratings. The formalism also includes the generation of deformed wavefronts, which we need for the production of such gratings, by mirror, gratings and other optical surfaces.

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МЕРИДИОНАЛЬНЫЕ ФУНКЦИИ ОПТИЧЕСКОЙ ДЛИНЫ ПУТИ, А ТАКЖЕ ПЕРЕНОС КОМЫ В СЛУЧАЕ ПРОКОРРЕКТИРОВАННЫХ ВОГНУТЫХ РЕШЕТОК ГОЛОГРАММЫ

Определен общий вид функции оптической длины пути для прокорректированных вогнутых голографических решеток в меридиональном сечении вплоть до третьего порядка при использовании поперечных координат сетки. Расчеты выполнены для случая, когда как волновые фронты, так и основание решетки были сферическими. Исходя из полученных таким образом функций оптической длины пути была выведена формула, определяющая изменение члена третьего порядка (член комы), происходящее на решетке для падающего волнового фронта. Это дает, наряду с фокусирующими свойствами решетки, также возможность рассмотрения оптических систем, содержащих решетки.