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# SYNTHETIC MEASUREMENT OF DEVIATION IN BUDGETARY CONTROL

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# SYNTETYCZNY POMIAR ZRÓŻNICOWANIA ODCHYLEŃ W KONTROLI BUDŻETOWEJ

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**Summary:** Budget control is an important element of budgeting, which is an important management accounting tool. Hence, it is necessary to recognize the need for its development. The article presents the authors' method of measuring the variation of budget deviations. A synthetic measure of budget differentiation can be used to assess deviation levels in the budgetary control process. The method described herein provides a deviation measure that is a weighted average of the relative and absolute deviations using a convex combination of Gini coefficients, which are used as a measure of variance. The research used data including monthly budgeted and actual costs from a thermal power plant in Poland for the years 2014 through 2016. The article is a methodological approach that fits into the current research on tools used in budgetary control. In addition, the described method is integrated into the development of work on control systems.

**Keywords:** budget control, Gini coefficient, convex combination.

**Streszczenie:** W artykule przedstawiono autorską metodę pomiaru zróżnicowania odchyłeń budżetowych. Opisana metoda pozwala na sprowadzenie do porównywalności miernika zróżnicowania obliczonego na podstawie dwóch rodzajów odchyłeń: względnych i bezwzględnych. Jako miernik zróżnicowania przyjęto współczynnik Giniego. Następnie współczynniki obliczone dla obu rodzajów odchyłeń zsumowano, wykorzystując kombinację wypukłą. Syntetyczna miara zróżnicowania wykonania budżetów może być stosowana do oceny poziomu odchyłeń w procesie kontroli budżetowej. Do badań wykorzystano dane jednostki produkcyjnej zawierające miesięczne koszty planowane oraz ich wykonanie w przekroju pozycji kosztów rodzajowych. Dane obejmują lata 2014, 2015 i 2016. Artykuł ma charakter metodyczny, wpisujący się w nurt badań nad narzędziami stosowanymi w kontroli budżetowej.

**Słowa kluczowe:** kontrola budżetowa, współczynnik Giniego, kombinacja wypukła.

## 1. Introduction

Budget control uses two types of data: prospective normative information set in the form of budgets and retrospective data in the form of actual expenditures. Based on this data, budgetary deviations can be calculated. The deviations may be the basis for evaluation of budgeted activity, budget adjustments or changes in controlled activities. One of the stages of budgetary control is deviation analysis. To a large extent the effectiveness of budgetary control depends on the quality of this analysis. The deviation analysis may include, among other things, factor analysis, analysis of deviation behavior over time, methods of dividing important and non-significant deviations and methods of level measurement and variation of deviations, etc.

The results of literature review in the field of management accounting in the context of the methods used in budgetary control provide the basis for the thesis that deviation analysis requires intense work and research to fill the cognitive gap. An analysis of the solutions used in companies using budgeting, as well as the assessment of the content of the theory, indicates a lack of control methods in the field.

Hence, the purpose of this article is to present one of the issues of deviation analysis – how to determine a practical measurement of the differentiation between budgeted values and actual expenditures. Because relative (percent) and absolute deviations can provide incongruent insights, a synthetic method combining both types of deviations was deemed most appropriate. The total estimated deviation is calculated as a weighted average of the relative and absolute deviations using a convex combination of Gini coefficients, which are used as a measure of variance.

For each cost type and month, relative and absolute deviations were calculated. These deviations were then used to create a weighted sum of relative and absolute Gini coefficients for each analyzed year. The results of the calculations will be presented to the company under study for their consideration and feedback.

## 2. Theoretical approach of budget control methodology

Numerous definitions of budgetary control can be found in the subject literature. A.S. Dunk defines budgetary control as a process of developing a spending plan and periodically comparing actual expenditures with the plan to determine whether the plan or expenditure need to be adjusted to achieve the target objectives. This process is necessary not only to control costs but also to manage expenses [Dunk 2009]. Budget control is also defined as a management control system in which the actual income and spending are compared with those planned. The results of this control is the basis for making the decisions regarding the execution of plans or the need to change them in order to make a profit [Epstein, McFarlan 2011].

The indirect result of budgetary control are deviations, which are the differences between expected versus actual expenditures. In connection with the subject matter of this article, it should be noted that the determination of these differences can be

made in many ways. Z. Kes [2013] lists six forms of computational formulas, including Formulas (1) and (2) below. For the purposes of this paper, note that Formulas (1) and (2) are used to calculate incremental values, not cumulative ones.

$$OB = P_w - P_B, \quad (1)$$

$$OW = \frac{P_w - P_B}{P_B} \cdot 100\%, \quad (2)$$

where:  $OW$  – relative variance;  $OB$  – absolute variance;  $P$  – value for a budget item;  $P_B$  – budgeted value (denoted in the lower index);  $P_w$  – value reached in a budget period (denoted in the upper index).

Except for a small number of publications, the topic of variance analysis with the use of various methods has been present in the literature for a long time. The methods of partitioning variations into parts due to quantitative or price changes were presented by Kwang and Slavin [1962]. These methods were further developed for the study of indirect costs by Zannetos [1963] and Weber [1963]. The authors drew attention to the use of statistical methods in deviation studies. This trend also includes the work of Bierman, Fouraker and Robert [1961], Salman [2008] and others. The authors of this paper have also contributed to this matter. Kes points to the measurement of differentiation by various methods (cf. [Kes 2015a; Kes, Kuźmiński 2011a; 2011b]).

Common methodologies for measuring differentiation (variability, scattering, scattering, dispersion) use many ways to measure deviation. These include stretching, variance, standard deviation, interquartile spacing, average deviation, curtailed index, concentration factor, and indices calculated on the basis of the area under the Lorenz curve (cf. [Aczel 2011, p. 41; Wierziński 2006, p. 88; Zimny 2010, p. 28]). The Gini, Hirschman-Herfindahl, Theila and Isarda [Antczak, Żółtaszek 2009] coefficients can also be used to measure variance. Note that these studies are not in the field of accounting management.

The research presented in this paper focuses on assessing the level of differentiation using the Gini index. The index will be calculated for both the relative and absolute deviations generated by the cost control procedures used by the company under study.

### 3. Procedure for measuring the variation of budget deviations

Over the course of this study, a research problem was identified regarding the ambiguity of the assessment of budget deviations by means of either a relative or absolute formula. Deviations in absolute terms (usually in monetary terms) are straightforward and tangible, but lack context regarding how the deviation relates to the base value (most often it is a value from the budget). For example, a deviation

of 1,000 PLN is materially more significant for a budgeted cost of 1,500 PLN versus a budgeted cost of 1,500,000 PLN.

Similar conclusions can be drawn from evaluations received on the basis of relative deviation data. The relative deviation indicates the percentage of absolute deviation from the baseline. For example, a value of 10% means that budget execution was 10% lower (higher) than budgeted. In this case, 10% of PLN 1,500,000 is materially more significant than 10% of PLN 1,500. These examples demonstrate that both absolute and relative deviations should be considered, making it more difficult to interpret the results.

Analogical relationships exist when defining control limits for deviations. Limits based on percentage deviations that may be appropriate for budget items of a certain magnitude will be inappropriate for budget items in other orders of magnitude. For example, a materiality limit set at 5% may be considered too low for a base amount of PLN 1500, but too high for a base amount of PLN 1,500,000.

Therefore, data presented separately in absolute and relative terms do not have high cognitive value unless they are interpreted together. Taking into account both types of deviations allows for the proper assessment of the level of budget implementation. Introducing a measure that will take into account both relative and absolute deviations will simultaneously simplify control procedures and will facilitate the interpretation of control reports for managers and controllers.

In order to eliminate the above assessment deviations, a synthetic measure has been created to quantify the variation in deviations in both relative and absolute terms. The Gini coefficient was used as a “building block” to construct this measure. It relates the cumulative proportion of the population to the cumulative proportionality of a particular metric (e.g., total income, etc.). Graphically, the coefficient is calculated as the surface area between by the Lorenz curve and the egalitarian curve divided by the total area under the egalitarian curve. Since the area under the egalitarian curve is  $\frac{1}{2}$  and the maximum value between the two curves is  $\frac{1}{2}$ , the Gini coefficient assumes values in the interval  $[0; 1]$ . Populations that can be called egalitarian are characterized by a Gini coefficient close to zero, whereas in the case where only one element of the population is significant and the others are not, the coefficient is close to 1. Panek proposes [2007, p. 119] an analytical form of the Gini coefficient as shown in Formula (3).

$$GINI = 1 - \frac{1}{n^2 \bar{y}} \left( \sum_{i=1}^n (2(n-i) + 1) y_i \right), \quad (3)$$

where:  $n$  – sample size;  $i$  – position of observation in the sequence (in ascending order);  $y_i$  – value of  $i$ -th variable;  $\bar{y}$  – the average value of the variable in the sample.

Because of the properties of the Gini coefficient, it cannot be used for negative values. Consequently, zero-unity [Jarocka 2015] was included in the calculation of budget deviations.

The summation of Gini coefficients for relative and absolute deviations is based on a convex combination. The convex combination of points  $u_1, u_2, \dots, u_n$  is called a point which is defined by Formula (4).

$$u = \alpha_1 u_1 + \alpha_2 u_2 + \dots + \alpha_n u_n, \quad (4)$$

where:  $\alpha_i$  – scalars (weights) meeting the conditions  $\alpha_i \geq 0$  i  $\sum \alpha_i = 1$ .

If  $u_1$  is the value of the Gini coefficient for relative deviations and  $u_2$  is the value for absolute deviations, then the sum of the scalars (weights) will be 1. Ideally, the weights should reflect the magnitude of the variable being scaled. For a low cost budget item, the weight of the relative coefficient should be significantly higher than the weight of the absolute coefficient. Conversely, for a high cost budget item, the weight of the absolute coefficient should be higher than the weight of the relative coefficient.

Thus, a synthetic measure of variation in deviations will be a weighted sum of the Gini coefficients (Formula 5).

$$G = \alpha_1 G_1 + (1 - \alpha_1) G_2, \quad (5)$$

where:  $G$  – synthetic measure of variation of deviations;  $G_1$  – Gini coefficient for absolute variances;  $G_2$  – Gini coefficient for relative variances;  $\alpha_1$  – scalar (weight) determined on the basis of absolute variances.

The parameter in Formula (5) is relatively easy to calculate and interpret as demonstrated in the next section of this study.

#### 4. Characteristic of the source data

In order to test the methodology used to evaluate the differentiation of budget deviations, deviation calculations were performed using the budgeting data, including planned and actual expenditures, provided by the company under study. This company operates in the heat sector and manufactures and distributes heating services.

The overall budget approved by the management of the company is a static budget, prepared once a year. The implementation of the budget by individual cells within the company is reported monthly. This reporting includes deviations for budgeted costs.

The test sample consisted of fixed and variable costs segmented into generic cost groups for three calendar years. Figure 1 shows the share of total cost, as a percent of total annual cost, by cost type for each year.

The distribution of costs associated with this company is indicative of the nature of the business – plant operating costs, fuel costs, wage costs, and depreciation account for most of the expenditures. With respect to costs over time (2014-2016),

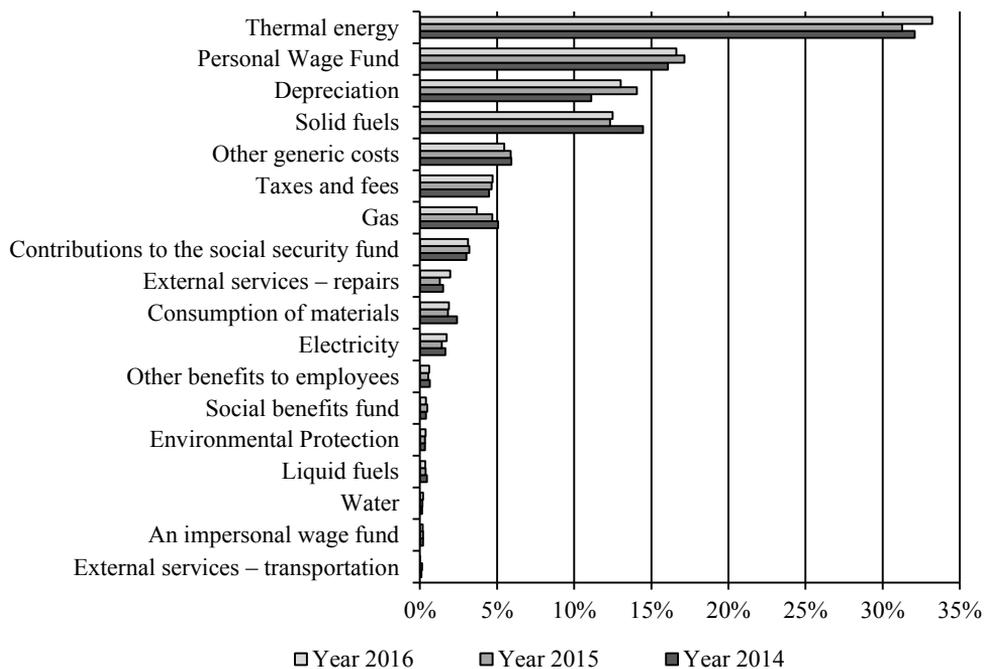


Fig. 1. Distribution of generic costs in the company under study from 2014-2016

Source: own elaboration.

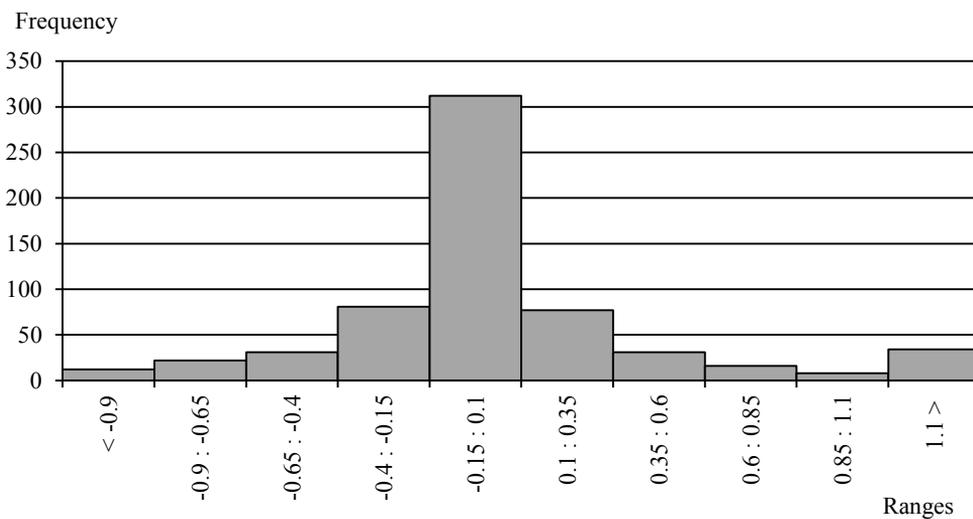


Fig. 2. Frequency of relative deviations in the company under study from 2014-2016

Source: own elaboration.

there was a relatively significant increase in depreciation cost and a relatively significant decrease in the cost of using solid fuels.

The relative and absolute deviations (calculated according to Formulas (1) and (2) were further analyzed. Due to the lack of data for solid fuels and environmental protection in the off-peak months, relative deviation values could not be calculated. Consequently, all data for these two cost items were removed from further study, leaving 576 values for each type of deviation.

A graph of the distribution of relative deviations was drawn to assess the nature of the data. For purposes of illustration, deviations for all cost types were combined to produce Figure 2.

Based on Figure 4, the distribution of deviations is orthogonal and leptokurtic. This involves overrepresentation of deviations in the range of 15% to 100% and a significant number of samples above 110%.

## 5. Measurement of deviation variation in a heat distribution company

Computations of the synthetic value of the variation deviation measure were made in two steps. The first step was to calculate the monthly arithmetic mean for three years for the individual costs on the basis of the “execution” series. Monthly arithmetic means were subjected to unitarisation. This resulted in values ranging from 0% to 100%. Based on the point ranges shown in Table 1, costs were assigned to one of nine groups. The selection of nine groups was arbitrary, as well as the selection of weights for each group.

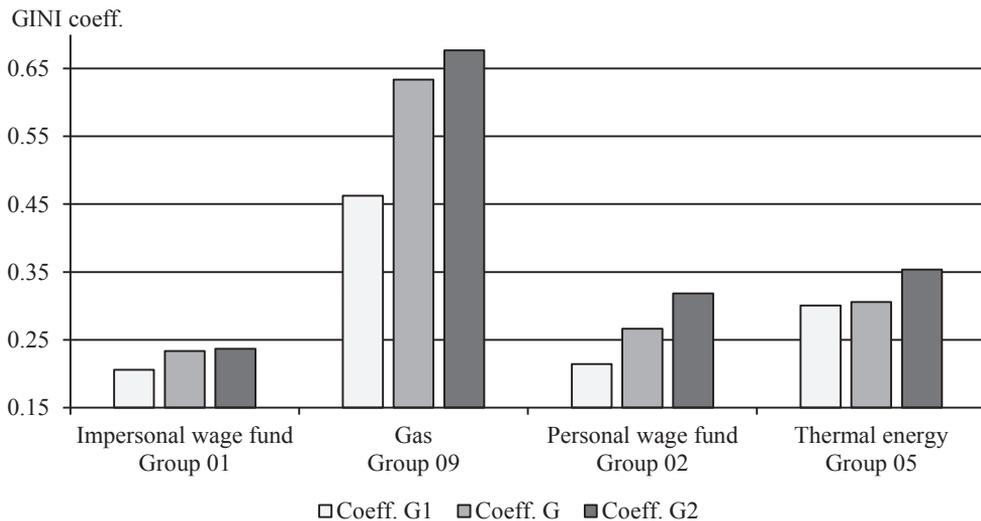
The data in Table 1 shows that the cost with the highest average over three years was classified as Group 09 with weights for the absolute and relative Gini coefficients set to 90% and 10%, respectively.

**Table 1.** Point ranges for the unitarisation means of the arithmetical mean

Point range	Group of costs	Absolute Gini Coeff. Weight $\alpha_1$	Relative Gini Coeff. Weight $\alpha_2$
[0% : 11%]	Group 01	10%	90%
(11% : 22%]	Group 02	20%	80%
(22% : 33%]	Group 03	30%	70%
(33% : 44%]	Group 04	40%	60%
(44% : 56%]	Group 05	50%	50%
(56% : 67%]	Group 06	60%	40%
(67% : 78%]	Group 07	70%	30%
(78% : 89%]	Group 08	80%	20%
(89% : 100%]	Group 09	90%	10%

Source: own elaboration.

The second step was to determine the differentiation coefficients for each of the costs (16 items) for a given year (3 items). In this way, 48 pairs of weighted Gini coefficients were generated from Table 1 using Formula (5). Figure 3 shows the Gini coefficients for four selected cost items for 2014.



**Fig. 3.** Gini coefficient for chosen costs in 2014

Source: own elaboration.

Figure 3 shows that the variation for absolute deviations is greater than the variation for relative deviations ( $G2 > G1$ ). Over the entire sample tested, this was the case 36 times out of 48. Of course the synthetic measure  $G$ , which is a weighted sum, always assumes a value between the Gini coefficients for absolute ( $G1$ ) and relative ( $G2$ ) deviations. Figure 3 also shows fourth individual costs. For relatively lower costs in the enterprise, the synthetic measure gravitates toward the variance established for relative deviations, while for relatively higher costs, the measure gravitates toward the variance established on the basis of absolute deviations.

## 6. Conclusions

The aim of this paper was to present the possibility of a synthetic measure of deviation based on relative and absolute differences between budgeted values and actual expenditures. For this measurement, Gini coefficients were calculated for both relative and absolute variances, using judgmentally selected, cost-dependent weights.

For costs representing a low share of total cost, the weight used for absolute deviations was lower than that for relative deviations. Conversely, for costs

representing a high share of total cost, the weight used for absolute deviations was higher than that for the relative deviations.

The application of the approach presented here is intended to simplify the analysis and interpretation of indicators for deviations in budgetary control. This is very important in situations in which the difference calculated using percentages is materially different than the difference calculated using absolute values. It is also important for the creation of control limits to determine the significance of deviations. Limits defined by percentages cannot be directly used to evaluate all budget items due to significant relativism. The use of synthetic measures in this case allows for the adoption of a uniform model for determining materiality limits irrespective of the level of baseline values.

The presented calculation procedure has some shortcomings. One of them is that it is impossible to calculate the Gini index for negative values. It is possible to replace this measure with others such as standard deviation or variation coefficient. However, the value of these measures is not standardized, which makes it difficult to track over time and to compare between budget units and cost or revenue items.

The next discussion will include issues related to the possibility of using not only different measures of differentiation, but also the level of deviations.

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