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MODELING THE PROCESS OF PURCHASE PAYMENT AS A CONSTITUENT OF INFORMATION SECURITY IN E-COMMERCE

A mathematical model of the process of payment for purchases in an online store has been presented. The model belongs to the class of semi-open queueing networks with four phases of exponential servers and Poisson arrivals. The authors describe in detail the derivation of the equations describing the system. Analytic expressions are derived on the basis of the proposed model for the average number of online store customers who have already paid for goods. Practical implementation of the model allows us to determine the number of clients who have added goods to their cart, but have not yet passed through the payment verification system, and thus determine the stream of real customers of the online store.

Keywords: *online store, payment cards, verification, information security, queuing network, Poisson stream.*

1. Introduction

On one hand, information security is an obvious and clear notion at the intuitive level, which we face every day using smartphones, tablets or laptops for creating, transmitting or receiving various types of information. On the other hand, one must clearly understand what it is. Currently, there are different definitions of information security put forward in various government and industry standards, as well as a number of methodologies which can be applied to e-commerce. In particular, *information security en-*

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sures that within the enterprise, information is protected against disclosure to unauthorized users (confidentiality), improper modification (integrity), and non-access when required (availability) [6].

Much research by scientists specialists, as well as various specialized sites and forums, etc., see for example [1, 2, 4, 5, 7], is devoted to various aspects of the information security of online stores. Summing up information from these and other sources, we can conclude that information security in e-commerce is a problem that is relevant for any online store and we can distinguish the following threats [10]: leakages of the store's database of customers, fraud regarding electronic payments, making malicious changes to the store's catalogue; interference with the store's process of functioning (for example, moving the store's home page to another page, advantageous for fraudsters); embedding malicious code into the pages of the online store (for example, to steal the details of payment cards); DDoS-attacks etc. To avoid these and other threats, it is necessary to take appropriate measures, which are expensive. Such measures are normally taken by large online stores but not always by small ones. This is, in particular, due to the fact that investing in advertising or widening a store's assortment will lead to a probable increase in sales (and, accordingly, profit) which is more significant for a small store than investing in security [4, 5]. In addition, large online stores appreciate their reputation and therefore have to invest in their own security and use the whole range of available countermeasures, while small stores may outsource the processes involved in online payment or opt out of accepting payment cards online in favor of receiving money from the buyer via a courier, who delivers the good [4]. However, this approach to payment hinders the development of small online stores. Therefore, it does not make sense to apply this approach for a long time.

Nowadays, customer profiles, together with information about credit cards, are stored by a large number of online stores. Therefore, Internet shops are able to accelerate the process of purchasing. As a result, by knowing only a customer's login and password, purchases can be made by a fraudster instead of a real buyer. In this case, in our view, the implementation of additional verification during the payment process is a must, in order to reduce the risk of fraud.

Likewise, it is quite common that incorrect payment information is entered many times. In this case, it is necessary to ascertain whether the customer has mistakenly entered incorrect information a number of times or malicious software has tried to make a purchase, by seeking valid data.

In traditional and electronic commerce, days when the number of buyers increases significantly are quite common. There are a variety of sales, e.g. Black Friday, as well as panic purchases of certain popular products, etc. For online shops, the start of such a sale is always accompanied by a significant increase in the computational load on the server. For instance, selling tickets for various concerts, movies or theatre events can cause peak load on the server when a lot of people are trying to buy tickets at one moment. Therefore, there is a need to take into account such situations when developing business processes (for e-shops, agencies selling tickets, etc.), because, in this case, the risk of fraud is high.

In order to mitigate threats to information security, the payment process in e-commerce may consist of the following phases [3]: 1) request information regarding the payment card, 2) obtain a signature, 3) confirm delivery, 4) match the ZIP codes for billing and shipping. Performing these steps allows us to improve the security of information held by the online store, as well as of the buyer.

2. The process of paying for goods in an online store as a queuing network

The sequence of steps taken by a buyer while paying for goods is illustrated in Fig. 1. This scheme is a queuing network with four phases. Each phase corresponds to one of the steps described above and includes one channel of service. Based on the scheme for verifying payment for goods presented in Fig. 1, it is clear that the transition of a client from one phase of payment to another depends only on satisfying the criteria regarding the current phase and not any previous phase. Thus, using the following generally accepted definition of a Markov process: "a set of random variables $\{X_n\}$ forms a Markov chain if the probability that the next value (state) is $\{X_{n+1}\}$ depends only on the current value (state) X_n and not upon any previous values" ([8], p. 21), it is possible to describe the process of verifying customer payment via a special class of Markov chains – birth-death process. Using the scheme shown in Fig. 1, in accordance with the most commonly used service discipline (First in, First out) and the construction of differential equations which describe a birth-death process ([8], p. 53), it is possible to form a homogeneous system of linear differential equations describing the process of paying for goods in an Internet-store.

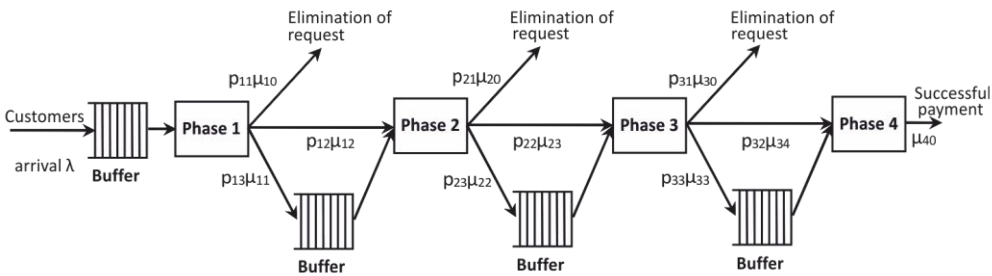

Fig. 1. The scheme of paying for an online purchase in an online store.
Source: proposed by the authors on the basis of [3]

Based on [8, 9, 12–14], we can hypothesize that a Poisson stream of requests arrives for processing at a rate of λ requests per minute (arrival rate) and the time for processing each request at each phase is exponentially distributed with parameter μ_{ij} (service rate).

Let us describe in detail the process of the buyer filling in the required forms at each phase of the process of paying for goods. Thus, in the first phase, the buyer must "provide the name on the card, the billing address, card number, expiration date, and the CVV code on the back of the card" [3]. If the buyer provides any incorrect information (such as entering an invalid CVV code or card number), he will be denied confirmation of the order (purchase) of goods. That is to say that, in this phase, the system handles incorrectly entered information (request for service) with intensity μ_{10} requests per minute and as a result the buyer will leave the system (payment form). In this case, access to the second phase of purchasing will be denied. If the buyer has entered information correctly, such a request is processed with intensity (service rate) μ_{12} requests per minute and the buyer will proceed to the second phase of service. However, in practice it often happens that there are many users in the system and transition from one window of the online store site to another often requires additional waiting time. In this case (i.e., service of the second phase is occupied with handling other customer requests), a client who has successfully passed through the first phase has to wait for transition to the next phase (this request is held in the system's buffer memory). Therefore, if a client has successfully passed through the first phase of service, but the second phase is occupied, then with intensity μ_{11} requests per minute the client's request is moved to the buffer (queue). The probability of the client successfully passing through the first phase and entering the second phase is p_{12}, the probability of transition to the waiting block– p_{13}, and the probability of the client not entering the information correctly – p_{11}.

It is necessary to note that the above events form a partition. The probabilities p_{11}, p_{12}, p_{13} take values between 0 and 1, are selected either by an expert or by processing statistical information and their sum equals one. In addition, the intensities of processing requests in the first phase μ_{10}, μ_{11}, μ_{12} can be either the same or different and take values from 0 to some upper bound. In the case of the incorrect entry of information in the course of the payment process, the lines which are filled incorrectly are often highlighted, the processing of incorrect information μ_{10} may be greater than both μ_{12} and μ_{11}. If the server that processes requests is powerful enough, the waiting block can be ignored by setting μ_{11} to 0, and transition to the next phase should take place immediately. Hence, the proposed scheme is quite flexible and can be easily customized to specific technological features.

During the second phase of the service, the client is asked to sign an invoice, credit card authorization form, or a contract that authorizes the payment and outlines refund policies. The discipline of the service is the same as in the first phase (first in, first out). The intensity of processing clients' requests during the second phase in the case that the form is filled in correctly and then transition to the third phase occurs is μ_{23} requests per minute, the intensity of processing requests in the case of successfully completing the second phase, but then being transferred to the waiting block due to the third phase being occupied, is μ_{22}, the intensity of processing a client's request when the form is filled in incorrectly is μ_{20} requests per minute. The probability of the customer passing through the second phase successfully and being transferred to the third phase is p_{22}, the probability of successful completion, but then being transferred to the waiting block p_{23}, and the probability of failing to complete the second phase and logging off $-p_{21}$. These events are mutually exclusive and $p_{21} + p_{22} + p_{23} = 1$.

During the third phase of service, the customer should provide tracking information and a delivery receipt. Service rates are analogous to those in the previous two phases, i.e. with intensities μ_{30}, μ_{33}, μ_{34} requests per minute and the corresponding mutually exclusive transitions with probabilities p_{31}, p_{32}, p_{33} that form a partition.

During the fourth phase, the billing and shipping ZIP codes are compared. If these codes match, then the client receives confirmation of the order, which is passed on to the stock and packaging department. Such transitions occur with intensity μ_{40} per minute.

The intensity of the service of requests at each phase of the verification of payment is set according to the computational load of the hardware that the online store possesses. In other words, the intensity of the input flow of requests for goods cannot exceed the system's capacity. Thus, let ρ be the working capacity or utilization factor of the system. It is generally accepted that ρ equals the ratio of the intensity of the input flow to the intensity of service, where $\lambda < \mu_{ij}$. Therefore, the system is only stable if $0 \le \rho < 1$, otherwise the system is unstable and payment for purchases will become impossible, i.e. the processor goes down. In our case, according to ([8], p. 19), the working capacity of the system is given by $\rho = 1 - P_{\substack{0(0,0,0,0) \\ (0,0,0)}}$. Moreover, the capacity of a queue is set according to the number of requests that the processor is able to handle at the appropriate phases of processing. In the case when the waiting block for the next phase is full, incoming requests will not be serviced – they will be lost. In practice, such a situation can happen in the case of a DDoS-attack or holiday sales, when a large number of simultaneous requests typically results in a hardware failure and subsequent denial of service. In order to mitigate such threats, queues (buffers) are added to the system. Such queues form a buffer of requests which the system is not able to handle immediately because of the limited computational load of hardware (processor, memory, etc.). This reduces the number of requests which are lost.

Note that a request will be removed from the system when the verification of a payment gives an unsuccessful result (due to incorrect entry of information). The customer can try again to pay for a purchase, but he must start from the first phase, and in this case, he is considered by the system to be a new customer. For example, a product remains in the basket (reserved for this order) until the client removes it from the cart or a pre-determined period of time (15 minutes, one, two hours etc.) has passed. A customer can try to make a purchase as many times as he wants, but he must pass through all four phases of the payment process.

Hence, we take into account the possibility that the number of customers in the system can exceed the capacity of the server (such as in the case of a DDoS attack or the influx of customers during sales, etc.). In order to mitigate this situation and secure the whole process of verification, we propose that queues (buffers) are included in the verification system between the phases of service. Hence, requests which have completed one phase, will go to the buffer memory, if the next phase is occupied (full).

The discipline of customer service is FIFO (first come – first served), but the number of requests in the investigated queuing network is limited by the capacities of queues as the quantity of goods is limited and/or an overwhelming number of requests for purchases can cause the site to crash. The number of customers in the system cannot exceed the number $N_1 = N + k_1 + k_2 + k_3 + 4$, where N is the number of customers in the queue waiting for entry into the system, $N = 0, 1, 2, 3, ...,$ k_1, k_2, k_3 – the capacity of the queues (buffer memory) for the first, second and third phases, respectively, $k_i = 0, 1, 2, 3, ..., i = \overline{1, 3}.$ Each query is processed one by one at each of the four phases of the investigated queuing network. In addition, there are restrictions on the capacity of the buffer memory caused by the limited capacity of the information processing system, and the number of customers waiting in line to pay for goods is limited according to the volume of goods available in the store.

The probabilistic model proposed for such a queuing network belongs to the class of semi-open queueing networks with four phases of exponential servers and Poisson arrivals [13, 14]. The construction of such a model based on birth-death equations is described in detail, for example, in [9, 14].

3. A stochastic model of the process of paying for purchases in an online store

A stochastic model of the process of paying for online purchases in an online store can be represented as a system of differential equations of the following form:

$$
\begin{aligned}
&P'_{0\substack{(0,0,0,0,0)\\(0,0,0)}}(t) = -\lambda P_{0\substack{(0,0,0,0,0)\\(0,0,0)}}(t) + \mu_{40} P_{0\substack{(0,0,0,0,1)\\(0,0,0)}}(t) \\
&P'_{0\substack{(1,0,0,0)\\(0,0,0)}}(t) = -\left(\lambda + p_{11}\mu_{10} + p_{12}\mu_{12}\right) P_{0\substack{(1,0,0,0)\\(0,0,0)}}(t) + \lambda P_{0\substack{(0,0,0,0)\\(0,0,0)}}(t) \\
&\qquad + p_{11}\mu_{10} P_{1\substack{(1,0,0,0)\\(0,0,0)}}(t) + \mu_{40} P_{0\substack{(1,0,0,1)\\(0,0,0)}}(t) + p_{21}\mu_{20} P_{0\substack{(1,1,0,0)\\(0,0,0)}}(t) \\
&\qquad + p_{31}\mu_{30} P_{0\substack{(1,0,1,0)\\(0,0,0)}}(t) \\
&\vdots \\
&P'_{N\substack{(1,0,0,0)\\(0,0,0)}}(t) = -\left(\lambda + p_{11}\mu_{10} + p_{12}\mu_{12}\right) P_{N\substack{(1,0,0,0)\\(0,0,0)}}(t) + \lambda P_{N-1\substack{(1,0,0,0)\\(0,0,0)}}(t) \\
&\qquad + p_{11}\mu_{10} P_{N+1\substack{(1,0,0,0)\\(0,0,0)}}(t) + \mu_{40} P_{N\substack{(1,0,0,1)\\(0,0,0)}}(t) + p_{21}\mu_{20} P_{N\substack{(1,1,0,0)\\(0,0,0)}}(t) \\
&\qquad + p_{31}\mu_{30} P_{N\substack{(1,0,1,0)\\(0,0,0)}}(t) \\
&\vdots \\
&P'_{0\substack{(0,1,0,0)\\(0,0,0)}}(t) - \left(\lambda + p_{21}\mu_{20} + p_{22}\mu_{23}\right) P_{0\substack{(0,1,0,0)\\(0,0,0)}}(t) + p_{12}\mu_{12} P_{0\substack{(1,0,0,0)\\(0,0,0)}}(t) \\
&\qquad + p_{11}\mu_{10} P_{0\substack{(1,1,0,0)\\(0,0,0)}}(t) + p_{21}\mu_{20} P_{0\substack{(0,1,0,0)\\(1,0,0)}}(t) + p_{31}\mu_{30} P_{0\substack{(0,1,1,0)\\(0,0,0)}}(t) \\
&\qquad + \mu_{40} P_{0\substack{(0,1,0,1)\\(0,0,0)}}(t) + p_{12}\mu_{12} P_{0\substack{(1,0,0,0)\\(0,0,0)}}(t) \\
&P'_{0\substack{(0,1,0,0)\\(1,0,0)}}(t) = -\left(\lambda + p_{21}\mu_{20} + p_{22}\mu_{23}\right) P_{0\substack{(0,1,0,0)\\(1,0,0)}}(t) + p_{11}\mu_{10} P_{0\substack{(1,1,0,0)\\(1,0,0)}}(t) \\
&\qquad + p_{13}\mu_{11} P_{0\substack{(1,1,0,0)\\(0,0,0)}}(t) + p_{21}\mu_{20} P_{0\substack{(0,1,0,0)\\(2,0,0)}}(t) + p_{31}\mu_{30} P_{0\substack{(0,1,1,0)\\(1,0,0)}}(t) \\
&\qquad + \mu_{40} P_{0\substack{(0,1,0,1)\\(0,0,0)}}(t) \\
&\vdots \\
&P'_{0\substack{(0,1,0,0)\\(k_1,0,0)}}(t) = -\left(\lambda + p_{21}\mu_{20} + p_{22}\mu_{23}\right) P_{0\substack{(0,1,0,0)\\(k_1,0,0)}}(t) + p_{11}\mu_{10} P_{0\substack{(1,1,0,0)\\(k_1,0,0)}}(t) \\
&\qquad + p_{13}\mu_{11} P_{0\substack{(1,1,0,0)\\(k_1-1,0,0)}}(t) + p_{21}\mu_{20} P_{0\substack{(0,1,0,0)\\(k_1+1,0,0)}}(t) \\
&\qquad + p_{31}\mu_{30} P_{0\substack{(0,1,1,0)\\(k_1,0,0)}}(t) + \mu_{40} P_{0\substack{(0,1,0,1)\\(k_1,0,0)}}(t) \\
&\vdots
\end{aligned}
\tag{1}
$$

$$P'_{\substack{N(1,1,1,0)\\(0,0,0)}}(t) = -\big(\lambda + p_{11}\mu_{10} + p_{13}\mu_{11} + p_{21}\mu_{20} + p_{23}\mu_{22} + p_{31}\mu_{30}$$

$$+p_{32}\mu_{34}\big)P_{\substack{N(1,1,1,0)\\(0,0,0)}}(t) + \lambda P_{\substack{N-1(0,1,1,0)\\(0,0,0)}}(t) + p_{11}\mu_{10}P_{\substack{N+1(1,1,1,0)\\(0,0,0)}}(t)$$

$$+p_{21}\mu_{20}P_{\substack{N\ (1,1,1,0)\\(1,0,0)}}(t) + p_{31}\mu_{30}P_{\substack{N\ (1,1,1,0)\\(0,1,0)}}(t) + \mu_{40}P_{\substack{N(1,1,1,1)\\(0,0,0)}}(t)$$

$$+p_{12}\mu_{12}P_{\substack{N+1(1,0,1,0)\\(0,0,0)}}(t) + p_{22}\mu_{23}P_{\substack{N(1,1,0,0)\\(1,0,0)}}(t)$$

$$\vdots$$

$$P'_{\substack{N\ (1,1,1,1)\\(k_1,k_2,k_3)}}(t) = -\big(p_{11}\mu_{10} + p_{13}\mu_{11} + p_{21}\mu_{20} + p_{22}\mu_{23} + \mu_{40}\big)P_{\substack{N\ (1,1,1,1)\\(k_1,k_2,k_3)}}(t)$$

$$+\lambda P_{\substack{N-1(1,1,1,1)\\(k_1,k_2,k_3)}}(t)$$

where $P_{\substack{N\ (i,j,m,l)\\(k_1,k_2,k_3)}}(t)$ is the probability that at time t there are N requests waiting in the queue to enter the system and i, j, m, l – the number of requests which are being processed at each of the four phases in turn, $N = 0, 1, 2, 3, \ldots$ $i = \overline{0,1}$, $j = \overline{0,1}$, $m = \overline{0,1}$, $l = \overline{0,1}$. Moreover, k_1 requests have passed through the first phase of service, but are still waiting for service in the queue for the second phase, $k_1 = 0, 1, 2, 3, \ldots$, k_2 requests have passed through the first and second phase of the service, but are waiting for service in the queue for the third phase, $k_2 = 0, 1, 2, 3, \ldots$, k_3 requests have passed through the first, second and third phase of the service, but are waiting for service in the queue for the fourth phase, $k_3 = 0, 1, 2, 3, \ldots$

Let us demonstrate how to lock the system when the total number of customers in the system reaches a given limit. Thus, when a maximum of nine customers are allowed in the system at one time, system (1) will have the following form:

$$P'_{\substack{0(0,0,0,0)\\(0,0,0)}}(t) = -\lambda P_{\substack{0(0,0,0,0)\\(0,0,0)}}(t) + \mu_{40}P_{\substack{0(0,0,0,1)\\(0,0,0)}}(t)$$

$$P'_{\substack{0(1,0,0,0)\\(0,0,0)}}(t) = -\big(\lambda + p_{11}\mu_{10} + p_{12}\mu_{12}\big)P_{\substack{0(1,0,0,0)\\(0,0,0)}}(t) + \lambda P_{\substack{0(0,0,0,0)\\(0,0,0)}}(t)$$

$$+p_{11}\mu_{10}P_{\substack{1(1,0,0,0)\\(0,0,0)}}(t) + \mu_{40}P_{\substack{0(1,0,0,1)\\(0,0,0)}}(t) + p_{21}\mu_{20}P_{\substack{0(1,1,0,0)\\(0,0,0)}}(t)$$

$$+p_{31}\mu_{30}P_{\substack{0(1,0,1,0)\\(0,0,0)}}(t)$$

$$P'_{\substack{1(1,0,0,0)\\(0,0,0)}}(t) = -\big(\lambda + p_{11}\mu_{10} + p_{12}\mu_{12}\big)P_{\substack{1(1,0,0,0)\\(0,0,0)}}(t) + \lambda P_{\substack{0(1,0,0,0)\\(0,0,0)}}(t)$$

$$+p_{11}\mu_{10}P_{\substack{2(1,0,0,0)\\(0,0,0)}}(t) + \mu_{40}P_{\substack{1(1,0,0,1)\\(0,0,0)}}(t) + p_{21}\mu_{20}P_{\substack{0(1,1,0,0)\\(0,0,0)}}(t)$$

$$+p_{31}\mu_{30}P_{\substack{0(1,0,1,0)\\(0,0,0)}}(t)$$

$$
\begin{aligned}
P'_{\substack{2(1,0,0,0)\\(0,0,0)}}(t) &= -\left(\lambda + p_{11}\mu_{10} + p_{12}\mu_{12}\right) P_{\substack{2(1,0,0,0)\\(0,0,0)}}(t) + \lambda P_{\substack{1(1,0,0,0)\\(0,0,0)}}(t) \\
&\quad + p_{11}\mu_{10} P_{\substack{3(1,0,0,0)\\(0,0,0)}}(t) + p_{21}\mu_{20} P_{\substack{2(1,1,0,0)\\(0,0,0)}}(t) + \mu_{40} P_{\substack{2(1,0,0,1)\\(0,0,0)}}(t) \\
&\quad + p_{31}\mu_{30} P_{\substack{0(1,0,1,0)\\(0,0,0)}}(t) \\
&\vdots \\
P'_{\substack{4(1,1,0,1)\\(1,0,0)}}(t) &= -\left(p_{11}\mu_{10} + p_{13}\mu_{11} + p_{21}\mu_{20} + p_{22}\mu_{23} + \mu_{40}\right) P_{\substack{4(1,1,0,1)\\(1,0,0)}}(t) \\
&\quad + \lambda P_{\substack{3(1,1,0,1)\\(1,0,0)}}(t) + p_{13}\mu_{11} P_{\substack{5(1,1,0,1)\\(0,0,0)}}(t) + p_{31}\mu_{30} P_{\substack{4(1,1,1,1)\\(1,0,0)}}(t) \\
&\quad + p_{32}\mu_{34} P_{\substack{4(1,1,1,0)\\(1,0,0)}}(t) \\
P'_{\substack{0(1,1,0,1)\\(5,0,0)}}(t) &= -\left(p_{11}\mu_{10} + p_{13}\mu_{11} + p_{21}\mu_{20} + p_{22}\mu_{23} + \mu_{40}\right) P_{\substack{0(1,1,0,1)\\(5,0,0)}}(t) \\
&\quad + p_{13}\mu_{11} P_{\substack{1(1,1,0,1)\\(4,0,0)}}(t) + p_{31}\mu_{30} P_{\substack{0(1,1,1,1)\\(5,0,0)}}(t) + p_{32}\mu_{34} P_{\substack{0(1,1,1,0)\\(5,0,0)}}(t) \\
P'_{\substack{1(1,1,0,1)\\(0,0,4)}}(t) &= -\left(p_{11}\mu_{10} + p_{13}\mu_{11} + p_{21}\mu_{20} + p_{22}\mu_{23} + \mu_{40}\right) P_{\substack{1(1,1,0,1)\\(0,0,4)}}(t) \\
&\quad + \lambda P_{\substack{0(1,1,0,1)\\(0,0,4)}}(t) + p_{31}\mu_{30} P_{\substack{1(1,1,1,1)\\(0,0,4)}}(t) + p_{33}\mu_{33} P_{\substack{1(1,1,1,1)\\(0,0,3)}}(t) \\
&\vdots
\end{aligned}
\tag{2}
$$

It should be noted that the outgoing stream from the first queuing system (QS) is also the Poisson input flow into the second QS. Similarly, the output stream from the second QS is the Poisson input flow into the third QS, etc. The output streams from the first, second and third phases are all divided into three substreams each with the appropriate probability of occurrence.

Let us consider the solution of the system (1). In vector-matrix form, the stationary distribution of the system (1) satisfies the following equation:

$$A\vec{P} = 0 \tag{3}$$

where A – square matrix of coefficients, whose elements, a_{ij}, $i \neq j$, correspond to the transition rates between states according to the system parameters λ, μ_{ij}, p_{ij}, $\vec{P}$ – vector of probabilities of the system states $P_{\substack{0(0,0,0,0)\\(0,0,0)}}$, $P_{\substack{0(0,0,0,1)\\(0,0,0)}}$, …, $P_{\substack{N\ (1,1,1,1)\\(k_1,k_2,k_3)}}$. The values of the elements a_{ii}, i.e. along the leading diagonal of the matrix, are negative and their absolute values equal the total rate at which the system leaves the state corresponding to row i. The remaining elements of this matrix, a_{ij}, may equal μ_{ij}, p_{ij} (or product of two such terms), λ or zero. The majority of entries in each row are equal to zero.

Let us add a condition normalizing the system (3), i.e. ensuring that the terms in the probability vector sum to one. Thus, the following system of linear equations is obtained:

$$\begin{cases} A\cdot\vec{P}=0 \\ P_{0\ \substack{(0,0,0,0)\\(0,0,0)}}+P_{0\ \substack{(1,0,0,0)\\(0,0,0)}}+\sum_{k_1=0}^{N_1}P_{0\ \substack{(0,1,0,0)\\(k_1,0,0)}}+\sum_{k_2=0}^{N_1}P_{0\ \substack{(0,0,1,0)\\(0,k_2,0)}}+\sum_{k_3=0}^{N_1}P_{0\ \substack{(0,0,0,1)\\(0,0,k_3)}} \\ +\sum_{k_3=0}^{N_1}P_{0\ \substack{(1,0,0,1)\\(0,0,k_3)}}+\sum_{k_1=0}^{N_1}P_{0\ \substack{(1,1,0,0)\\(k_1,0,0)}}+\sum_{k_2=0}^{N_1}P_{0\ \substack{(1,0,1,0)\\(0,k_2,0)}}+\sum_{k_2=0}^{N_1}\sum_{k_3=0}^{N_1}P_{0\ \substack{(1,0,1,1)\\(0,k_2,k_3)}}+\sum_{k_1=0}^{N_1}\sum_{k_3=0}^{N_1}P_{0\ \substack{(1,1,0,1)\\(k_1,0,k_3)}} \\ +\sum_{k_1=0}^{N_1}\sum_{k_2=0}^{N_1}P_{0\ \substack{(1,1,1,0)\\(k_1,k_2,0)}}+\sum_{k_1=0}^{N_1}\sum_{k_2=0}^{N_1}\sum_{k_3=0}^{N_1}P_{0\ \substack{(0,1,1,1)\\(k_1,k_2,k_3)}}+\sum_{k_1=0}^{N_1}\sum_{k_2=0}^{N_1}\sum_{k_3=0}^{N_1}P_{N\ \substack{(1,1,1,1)\\(k_1,k_2,k_3)}}=1 \end{cases} \quad (4)$$

Let us now solve the system (4) by substituting in the following expression for $P_{0\ \substack{(0,0,0,0)\\(0,0,0)}}$ which results from the last equation above:

$$\begin{aligned} P_{0\ \substack{(0,0,0,0)\\(0,0,0)}}&=1-P_{0\ \substack{(1,0,0,0)\\(0,0,0)}}-\sum_{k_1=0}^{N_1}P_{0\ \substack{(0,1,0,0)\\(k_1,0,0)}}-\sum_{k_2=0}^{N_1}P_{0\ \substack{(0,0,1,0)\\(0,k_2,0)}}-\sum_{k_3=0}^{N_1}P_{0\ \substack{(0,0,0,1)\\(0,0,k_3)}} \\ &-\sum_{k_3=0}^{N_1}P_{0\ \substack{(1,0,0,1)\\(0,0,k_3)}}-\sum_{k_1=0}^{N_1}P_{0\ \substack{(1,1,0,0)\\(k_1,0,0)}}-\sum_{k_2=0}^{N_1}P_{0\ \substack{(1,0,1,0)\\(0,k_2,0)}}-\sum_{k_2=0}^{N_1}\sum_{k_3=0}^{N_1}P_{0\ \substack{(1,0,1,1)\\(0,k_2,k_3)}} \\ &-\sum_{k_1=0}^{N_1}\sum_{k_3=0}^{N_1}P_{0\ \substack{(1,1,0,1)\\(k_1,0,k_3)}}-\sum_{k_1=0}^{N_1}\sum_{k_2=0}^{N_1}P_{0\ \substack{(1,1,1,0)\\(k_1,k_2,0)}}-\sum_{k_1=0}^{N_1}\sum_{k_2=0}^{N_1}\sum_{k_3=0}^{N_1}P_{0\ \substack{(0,1,1,1)\\(k_1,k_2,k_3)}}-\sum_{k_1=0}^{N_1}\sum_{k_2=0}^{N_1}\sum_{k_3=0}^{N_1}P_{N\ \substack{(1,1,1,1)\\(k_1,k_2,k_3)}} \end{aligned} \quad (5)$$

Then the first equation of the system (4) will be as follows:

$$\begin{aligned} &-\lambda(1-P_{0\ \substack{(1,0,0,0)\\(0,0,0)}}-\sum_{k_1=0}^{N_1}P_{0\ \substack{(0,1,0,0)\\(k_1,0,0)}}-\sum_{k_2=0}^{N_1}P_{0\ \substack{(0,0,1,0)\\(0,k_2,0)}}-\sum_{k_3=0}^{N_1}P_{0\ \substack{(0,0,0,1)\\(0,0,k_3)}}-\sum_{k_3=0}^{N_1}P_{0\ \substack{(1,0,0,1)\\(0,0,k_3)}} \\ &-\sum_{k_1=0}^{N_1}P_{0\ \substack{(1,1,0,0)\\(k_1,0,0)}}-\sum_{k_2=0}^{N_1}P_{0\ \substack{(1,0,1,0)\\(0,k_2,0)}}-\sum_{k_2=0}^{N_1}\sum_{k_3=0}^{N_1}P_{0\ \substack{(1,0,1,1)\\(0,k_2,k_3)}}-\sum_{k_1=0}^{N_1}\sum_{k_3=0}^{N_1}P_{0\ \substack{(1,1,0,1)\\(k_1,0,k_3)}} \\ &-\sum_{k_1=0}^{N_1}\sum_{k_2=0}^{N_1}P_{0\ \substack{(1,1,1,0)\\(k_1,k_2,0)}}-\sum_{k_1=0}^{N_1}\sum_{k_2=0}^{N_1}\sum_{k_3=0}^{N_1}P_{0\ \substack{(0,1,1,1)\\(k_1,k_2,k_3)}}-\sum_{k_1=0}^{N_1}\sum_{k_2=0}^{N_1}\sum_{k_3=0}^{N_1}P_{N\ \substack{(1,1,1,1)\\(k_1,k_2,k_3)}}) \\ &+\mu_{40}P_{0\substack{(0,0,0,1)\\(0,0,0)}}(t)=0 \end{aligned} \quad (6)$$

Thus we obtain a system of the form $A^{*}\vec{P}=\vec{B},$ where $\vec{B}$ is a vector of the form

$$\vec{B}=\begin{pmatrix}\lambda\\0\\0\\0\\\vdots\\0\end{pmatrix}$$

matrix A^{*} is obtained from matrix A by replacing the coefficients in the first row of Eq. (4) with the coefficients in the first row of Eq. (6). Having solved this system, we obtain the probability of each state of the system according to the stationary distribution $\vec{P}.$

Obviously, the system (1) can be solved to find the vector $\vec{P}$ using algebraic transformations. However, based on the complexity of the system's structure, in our opinion, the method described above is a reasonable method of solution, and the problem of finding analytical expressions for all the states of the system in this manner is a topic for further research.

Having the stationary solutions of the system (1), it is easy to obtain the basic operating characteristics of the process considered. In the case of our problem, one of the most interesting measures is the average number of customers in the system. The commonly used formula ([8], p. 96) for finding the average number of requests in the system (mathematical expectation) is applied to the problem of finding the average number of customers at each phase of the payment process. Thus:

a) the average number of customers presently in the first phase (including the queue) who will fill in the initial form incorrectly, and thus leave the system, is given by:

$$M_{10}=\sum_{N=0}^{N_1}(N+1)p_{11}P_{N\binom{1,0,0,0}{0,0,0}}+\sum_{N=0}^{N_1}(N+1)p_{11}P_{N\binom{1,0,0,1}{0,0,k_3}}+\sum_{N=0}^{N_1}(N+1)p_{11}P_{N\binom{1,1,0,0}{k_1,0,0}}$$
$$+\sum_{N=0}^{N_1}(N+1)p_{11}P_{N\binom{1,1,1,0}{k_1,k_2,0}}+\sum_{N=0}^{N_1}(N+1)p_{11}P_{N\binom{1,0,1,0}{0,k_2,0}}+\sum_{N=0}^{N_1}(N+1)p_{11}P_{N\binom{1,1,1,1}{k_1,k_2,k_3}}$$
$$+\sum_{N=0}^{N_1}(N+1)p_{11}P_{N\binom{1,0,1,1}{0,k_2,k_3}} \qquad (7)$$

b) the average number of customers who will successfully fill in the initial form on their next attempt and move on to the second phase is given by:

$$M_{12} = \sum_{N=0}^{N_1} (N+1) p_{12} P_{N\left(\substack{1,0,0,0 \\ 0,0,0}\right)} + \sum_{N=0}^{N_1} (N+1) p_{12} P_{N\left(\substack{1,0,1,0 \\ 0,k_2,0}\right)} + \sum_{N=0}^{N_1} (N+1) p_{12} P_{N\left(\substack{1,0,1,1 \\ 0,k_2,k_3}\right)} + \sum_{N=0}^{N_1} (N+1) p_{12} P_{N\left(\substack{1,0,1,0 \\ 0,k_2,0}\right)} \tag{8}$$

c) the average number of clients who will successfully pass through the first phase on their next attempt and move on to the queue waiting for the second phase:

$$M_{11} = \sum_{N=0}^{N_1} (N+1) p_{13} P_{N\left(\substack{1,1,0,0 \\ k_1,0,0}\right)} + \sum_{N=0}^{N_1} (N+1) p_{13} P_{N\left(\substack{1,1,1,0 \\ k_1,k_2,0}\right)} + \sum_{N=0}^{N_1} (N+1) p_{13} P_{N\left(\substack{1,1,1,1 \\ k_1,k_2,k_3}\right)} + \sum_{N=0}^{N_1} (N+1) p_{13} P_{N\left(\substack{1,1,0,0 \\ k_1,0,0}\right)} \tag{9}$$

d) the average number of customers who have successfully passed through the payment procedure for the online purchase in an online store will be:

$$\begin{aligned} M = {} & \sum_{k_3=0}^{N_1} \sum_{k_2=0}^{N_1} \sum_{k_1=0}^{N_1} \sum_{N=0}^{N_1} (N+k_1+k_2+k_3+4) P_{N\left(\substack{1,1,1,1 \\ k_1,k_2,k_3}\right)} \\ & + \sum_{k_3=0}^{N_1} \sum_{k_2=0}^{N_1} \sum_{k_1=0}^{N_1} (k_1+k_2+k_3+3) P_{0\left(\substack{0,1,1,1 \\ k_1,k_2,k_3}\right)} + \sum_{k_3=0}^{N_1} \sum_{k_2=0}^{N_1} \sum_{N=0}^{N_1} (N+k_2+k_3+3) P_{N\left(\substack{1,0,1,1 \\ 0,k_2,k_3}\right)} \\ & + \sum_{k_3=0}^{N_1} \sum_{k_1=0}^{N_1} \sum_{N=0}^{N_1} (N+k_1+k_3+3) P_{N\left(\substack{1,1,0,1 \\ k_1,0,k_3}\right)} + \sum_{k_2=0}^{N_1} \sum_{k_1=0}^{N_1} \sum_{N=0}^{N_1} (N+k_1+k_2+3) P_{N\left(\substack{1,1,1,0 \\ k_1,k_2,0}\right)} \\ & + \sum_{k_2=0}^{N_1} \sum_{k_1=0}^{N_1} (k_1+k_2+2) P_{0\left(\substack{0,1,1,0 \\ k_1,k_2,0}\right)} + \sum_{k_3=0}^{N_1} \sum_{N=0}^{N_1} (N+k_3+2) P_{N\left(\substack{1,0,0,1 \\ 0,0,k_3}\right)} \\ & + \sum_{k_1=0}^{N_1} \sum_{N=0}^{N_1} (N+k_1+2) P_{N\left(\substack{1,1,0,0 \\ k_1,0,0}\right)} + \sum_{k_3=0}^{N_1} \sum_{k_2=0}^{N_1} (k_2+k_3+3) P_{0\left(\substack{0,0,1,1 \\ 0,k_2,k_3}\right)} \\ & + \sum_{k_3=0}^{N_1} \sum_{k_1=0}^{N_1} (k_1+k_3+2) P_{0\left(\substack{0,1,0,1 \\ k_1,0,k_3}\right)} + \sum_{k_2=0}^{N_1} \sum_{N=0}^{N_1} (N+k_2+2) P_{N\left(\substack{1,0,1,0 \\ 0,k_2,0}\right)} \\ & + \sum_{N=0}^{N_1} (N+1) P_{N\left(\substack{1,0,0,0 \\ 0,0,0}\right)} + \sum_{k_1=0}^{N_1} (k_1+1) P_{0\left(\substack{0,1,0,0 \\ k_1,0,0}\right)} \\ & + \sum_{k_2=0}^{N_1} (k_2+1) P_{0\left(\substack{0,0,1,0 \\ 0,k_2,0}\right)} + \sum_{k_3=0}^{N_1} (k_3+1) P_{0\left(\substack{0,0,0,1 \\ 0,0,k_3}\right)} \end{aligned} \tag{10}$$

Based on the scheme proposed in Fig. 1, it is easy to define a mathematical model for a process of online payment with any number of phases.

This research allows us to explore various issues regarding the stationary distribution of the system (1). The analytical expressions (7)–(10) allow us to solve a variety of interesting applied problems, like determining the average number of requests that can be serviced by the system, the average number of requests which leave the system at each phase of service, the average number of requests waiting in a queue (in the case of a DDoS attack or influx of customers), the average time spent waiting in a queue, the server's capacity at each phase of service and the optimal power of the whole system, in terms of the rate at which requests are satisfied etc.

In addition, in further research, the use of differential equations and a solution to the system of Eqs. (1) in dynamic rather than stationary, form will help to determine the time required for a system to approach the stationary distribution after a force majeure.

4. A numerical example

Let the intensity of processing queries in the first phase when the form is filled in incorrectly be μ_{10} = 0 (in the first phase, the client does not leave the system), and the rate of processing forms filled in correctly is 35 queries per hour or μ_{12} = 35/60 = 0.58 queries per minute. Similarly, the intensity of queries entering the waiting block before the second phase is μ_{11} = 35/60 = 0.58 queries per minute. In the second phase, the authorization of signatures is performed with processing intensities μ_{20} = 3.7/60 = 0.0617, μ_{22} = 3.7/60 = 0.0617, μ_{23} = 3.7/60 = 0.0617 queries per minute. Let us note that the time required to process a query is a positive real number and present the mean number of processed queries per hour, per minute, per second. For instance, let us similarly define the intensities of processing at the third phase: μ_{30} = 3.7/60 = 0.0617, μ_{34} = 3.7/60 = 0.0617, μ_{33} = 3.7/60 = 0.0617 queries per minute. The intensity of processing queries in the fourth phase and thus transferring customers' requests to the stockroom is carried out with intensity μ_{40} = 10.8/60 = 0.18. Then, determining the stationary distribution according to model (2) with, for example, λ = 9/60 = 0.15 queries per minute and p_{ij} = 1/3 at each phase, we obtain the distribution of the states of the system for online purchasing, which is represented in Table 1. Let us note that in order to solve the system (2), an algorithm was written in the *R* software environment [11].

Let us select the intensities of the arrival of queries in a way that the system is loaded on a 0–100% scale in increments of 10%. We determine, using Eq. (10), the average number of real potential customers making a purchase within the entire flow of customers. For this example, we obtain the results shown in Table 2. Let us define, using Eq. (10), how many real customers on average will make a purchase among all the customers. For this example, we obtain the results presented in Table 2.

Table 1. A probabilistic forecast of the states of the payment process according to the customers in the online store

State	Probability	Characteristic of the process state
$P_{0\binom{0,0,0,0}{0,0,0}}$	0.0170	probability of system downtime; there are 0 customers in the payment system
$P_{0\binom{0,0,0,1}{0,0,0}}$	0.0035	probability that the system is processing one query in the fourth phase and there are no queries in a queue
$P_{0\binom{0,0,0,1}{0,0,1}}$	0.0007	probability that the system is processing one query in the fourth phase, and one query is waiting to be processed in the queue to this phase
$P_{0\binom{0,0,0,1}{0,0,2}}$	0.0001	probability that the system is processing one query in the fourth phase, and two queries are waiting in the queue to this phase
⋮	⋮	⋮
$P_{5\binom{1,1,0,0}{0,0,0}}$	0.000006	probability that the system is processing one query in the first phase and one query in the second phase; there are five queries waiting in the queue to the first phase, but no queries in any other queue
$P_{5\binom{1,1,0,1}{0,0,0}}$	0.000001	probability that the system is processing one query in the first phase, one query in the second phase and one query in the fourth phase; there are five queries in the queue to the first phase, but no queries in any other queue
$P_{5\binom{1,1,1,0}{0,0,0}}$	0.000003	probability that the system is processing one query in the first phase, one query in the second phase and one query in the third phase. There are five queries in the queue to the first phase, but no queries in any other queue
$P_{5\binom{1,1,1,1}{0,0,0}}$	0.0000005	probability that the system is processing one query in the first phase, one query in the second phase, one query in the third phase, and one query in the fourth phase; there are five queries in the queue to the first phase, but no queries in any other queue

Source: calculated by the authors based on system (2), the transition matrix describing the system. Solved using an algorithm written in the *R* package [11].

This example is based on the assumption that there are no more than $N_1 = 9$ customers in the system (the system (2) is built exactly for $N_1 = 9$). It should be noted that as the allowable size of the system $N_1 = N + k_1 + k_2 + k_3 + 4$ increases, the number of differential equations in the system (2) increases significantly. Therefore, to construct the system of equations (2), the authors suggest that $N_1 = N + k_1 + k_2 + k_3 + 4 \leq 9$, in order to illustrate the payment process based on numerical calculations. In other words, the capacity of the waiting blocks k_1, k_2, k_3 are designed for not more than 9 customers. For example, if the system is in the state $P_{\substack{1(1,1,1,1)\\(0,0,3)}}$, which means $N = 1, k_3 = 3$, only one more client can enter the system (assuming that no query is fully processed beforehand), the next client to arrive will be lost.

Table 2. Forecasts of the average number of customers who pay for purchases at an online store at equilibrium

The intensity of the entry of queries into the system [persons/hour]	System load ρ	Average number of customers who paid for the purchase
4	0.1	1.06
8	0.2	2.2
12	0.3	3.46
18	0.4	5.57
24	0.5	8.04
30	0.6	10.96
38	0.7	15.68
45	0.8	20.62
60	0.9	32.83
70	0.95	40.86
80	0.97	47.78
90	1	53.32

Source: calculated by the authors based on system (2) using *R* [11].

The numerical example is solved for various loads on the system, ρ. The goals are to determine which intensity of arrivals will load the system by 10%, 20%, 30%, etc. and which proportion of customers will pass through the system of payment verification without any problems. The results of these calculations are presented in Table 2.

Based on the results obtained from Eqs. (7)–(10), we can conclude that some of the customers who put products into their cart, for whatever reasons, did not pay for their purchases, having left at the second or third phase of the payment process. According to the assumptions made in this example, we found that the maximum capacity of the payment verification system is 90 people. Moreover, if 90 customers attempt to make purchases, only 53 of them will successfully pay for a purchase at their first attempt, and the other 37 will have to try again, because the system does not accept their cards or the shipping and billing addresses do not match, etc. The payment system will be fully loaded when it receives 90 requests per hour. 53 of these requests will be successfully served through all four phases (i.e. 53 buyers will successfully make payments for goods) and 37 requests will not be successfully served for various reasons, and are eliminated at the first, second or third phase of the payment process. We can assume that these 37 requests (or 41% of requests) are problematic and they, in some way, threaten the information security of the online store. It is not known in advance whether a person accidentally or intentionally entered incorrect data, or whether a crook enters valid information or not, etc.

Thus, according to the data given in Table 2, the system is almost fully loaded when it receives from 60 to 90 customers (requests) per hour, while a little over half of them will carry out the payment without any problems.

5. Conclusions

One of the fastest growing spheres of the economy is e-commerce, in which a significant position is occupied by online stores. However, the rapid development of online trading leads to the emergence of a number of issues, including information security. Among the most common threats to the operation of online stores is the threat of fraud with payment cards when making online payments. This article has considered a payment process as a four-queue network, in which each phase consists of one service channel. The proposed queuing network has three queues which contain queries that have passed through the first, second or third phase of the service and cannot move onto the next phase, which is currently busy serving preceding queries.

Based on the results from this modelling, the advantage of using mathematical models of queuing networks to simulate the flow of real customers in an online store is obvious. As part of the scheme proposed in this paper (Fig. 1), model (1) can be modified according to the requirements for information security of each particular online store, by varying the key parameters μ_{ij}.

The results obtained may be the basis for further research into modelling, not only payment processes, but also other (similar) business processes realized by online stores.

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