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LOGIC AND RISK AS QUALITATIVE AND QUANTITATIVE DIMENSIONS OF DECISION-MAKING PROCESS

Key problems in the field of decision-making have been considered. The authors' aim was to indicate the extremely important for management role of logic and risk in relation to decisions taken under conditions of uncertainty. In the course of the research, the following hypothesis was tested: the complexity of risk is determined by the diversity of reality. The result of this is that in science there is no current study developing a uniform methodology for the assessment of risk. It might even be doubtful whether it can be created. In a certain sense, this is indicated in the article by the discussion about the dimensions of logic and risk apparent in any decisions taken by a man. The paper presents the complexity and diversity of risk assessment on the basis of selected, but essential to the discussed issue, fields of knowledge. This is valid when the numerical or qualitative level of risk is substantial in the context of the analyzed problem.

Keywords: *logic, conditions of uncertainty, risk, decision-making management*

1. Introduction

The majority of human activities are associated with decision-making. Usually, the environment, as a set of conditions under which man makes decisions, has an undetermined character. As a result, this leads to situations in which decisions are taken in

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conditions involving risk (uncertainty). It also happens that decisions are sometimes taken under specified, in mathematical terms deterministic, conditions. However, deterministic does not necessarily mean simple in terms of computational complexity. Therefore, such decision-making may not be computationally easier in practice than decisions made under uncertainty⁴. However, the dimensions of risk and logic are characterized by a certain level of scientific knowledge. Science is created by scholars but the product of science, in the form of knowledge, is considered by science studies, the scientific study of science itself. Thus, knowledge is an essential object of interest to science. Science studies have been applied to obtain many interesting results of a general nature, which are sometimes a kind of invariant knowledge. The search for invariants with respect to any human activity is also considered by praxeology.

For some time, however, scientists' attention has been focused on the management of science, in particular decision-making processes⁵. In reality, though, it is impossible to manage knowledge, just as you cannot manage the weather, or anything that is not entirely dependent on us. There are specific fields of knowledge that can be used to manage organizations such as the economy, banks or even the state. In general, there are organizations which, in an orderly, organized manner provide knowledge for man, such that on the basis of data, it is possible to manage such organizations both in terms of content and quantitatively.

However, some invariant knowledge which has the character of a kind of general knowledge may be applied in practice, that is, in relation to knowledge management. Therefore, the management of knowledge and information has a remarkably pragmatic character and combines the dimensions of both logic and risk, because even uncertain environments have a structure, that is, a form of logic.

2. The process of institutionalizing knowledge

At present, knowledge seems to be generally available. As part of science studies, scientists tend to organize and order knowledge in the praxeological dimension. However, man was already gaining information and knowledge about the reality around him

⁴The problem of decision made in a deterministic environment does not constitute the primary object of analysis here.

⁵Science sometimes considers so-called self-organizing systems whose organization, in some sense, has an ontological character. The idea is that man is able to understand the behavior of such systems by interacting with them. To a large extent, the problem of systems management can be reduced to this dimension. We can understand the system and thus have knowledge about it. Therefore, we can also, to some extent, manage it. From a technical point of view, this problem was analyzed many years ago by W. Ross Ashby ([4], p. 85, [28, 29]). An example of a self-organizing system created by man is a stock exchange. Such a system has its own autonomy, that is, behavior (operations), and the effect of human impact on the stock exchange is limited.

much sooner than institutionalized science (universities) was established⁶. It can be assumed that knowledge and information are ontological properties of mankind. This is evidenced by the innate social ability of man to communicate and exchange information⁷. Often, however, scientific knowledge about the world was simply the domain of privileged and at the same time chosen, that is, narrow social classes. In addition, knowledge was kept secret, sometimes for a very long time⁸. Actually, the Renaissance, which began in Italy, was based on new ways of thinking and led to the opening of art, theater and theoretical sciences to every human being. During the Renaissance, science changed its character and cultivation, meaning that the acquirement of a wide knowledge was theoretically possible for all social classes. Due to this openness, science gained another dimension – it became a more complicated system in relation to its previous state. Complex systems also require more complicated methods of managing them and their products. In relation to science, knowledge is its product, and without a doubt, logic is an important element of its management. Admittedly, logic arose as early as in the times of Ancient Greece, but was a product of science, that is, logic was simply an element of science. Logic has, however, a very important asset – pragmatism. There is therefore a need to focus on the main elements of the pragmatism of logic, as they constitute, to some extent, the foundation of the majority of important decisions undertaken by man.

3. Decisions taken in the dimension of schemes, algorithms and invariants

The decisions undertaken by people are correct when they are based on knowledge and the laws of logic. The basis of knowledge from the point of view of logic (in fact as the truth) is a tautology. In practice, a tautology is meant to present something through the same thing, that is, colloquially speaking, a tautology is an object that represents itself ([23], p. 190).

In logic, the concept of a tautology has a different meaning, namely a logical expression which is always true from the point of view of the logical values of the variables contained in it. In propositional calculus, which forms the basis of logic, there are several well-known operationally effective methods of testing. It can be checked

⁶Compare this with, e.g. J. Piaget's ideas about children learning and understanding time, space and other concepts and objects of the reality surrounding them in an intuitive way [28].

⁷The linguist, C.F. Hockett said that man has been given a so called rush for communication ([13], p. 100–101, [19]).

⁸For example, knowledge held by priests in Egypt.

whether the status of a decision-making process, the behavior of an object, the organizational structure of a system accords with the laws of logic, that is, whether it has a structure logically represented by a tautology or not. One of these methods of testing is the natural deduction method, otherwise called the propositional method. The second method is constituted by the so called resolution rule⁹. Correct decisions in terms of logic should be taken on the basis of the laws of logic (as has already been remarked) and therefore at this point the role and the significance (for decision-making processes) of the concept of a scheme (invariant) should also be emphasized. In terms of logic, a scheme corresponds to the laws of logic and abstractly reflects humans' fundamental thinking patterns. However, the concept of a scheme also appears, for example, in psychology, biology and physics¹⁰, which proves that this is an important scientific topic of a cognitive and ontological character¹¹. Patterns can have various characters. If well-defined, a particular form of a pattern constitutes the concept of algorithm in mathematics. In the context of this article, algorithms play, in a certain sense, the central role, because only in the mathematical definition of this term is order mentioned, that is, the hierarchy of the procedural rules used in a given pattern. Intuitively, the specified rules of a procedure form an algorithm. As a result, on the basis of various sciences, the concept of algorithm is not understood (defined) clearly. Many algorithms based on logic (e.g. to find a way out of a labyrinth) have been developed, as well as numerical (computational), genetic and many other types of algorithms. However, algorithms have certain universal properties, such as: determinism, mass-appeal, effectiveness, which should be fulfilled by any system of a practical nature that we call an algorithm¹². Whenever understood in this way, the concept of algorithm must always meet these three

⁹These methods are not only applied in logic, but have an important practical dimension and reflect processes occurring in the human environment well, that is, they reflect their nature in the logical dimension (especially simple processes). Due to the usefulness of decision-making processes, these methods effectively allow us to demonstrate, among other things, whether premises and a conclusion, taken together as an implication, constitute a law. For more details, see ([11], p. 35–58).

¹⁰We list here these four fields of knowledge (mathematics is also present together with logic), because they form the circle of fundamental sciences. Such an analysis can be seen in ([28], p. 121–126).

¹¹In psychology, the concept of a scheme consists of various principles of human behavior, but the fundamental role of a scheme (pattern) lies in the ability of transferring one psychological situation to another. In biology, the concept of a pattern refers to the examination of organisms' and species' organization. A pattern is understood primarily in the sense of an invariant. Although the system as a whole is subject to change, certain properties remain constant ([3], p. 111). In other aspects, the role of a scheme is understood as in psychology, that is, as a state in which nature uses a given scheme to transform a given situation, i.e. a biological process, and when biological diversity changes, then a new pattern is created – invariant [10]. In relation to physics, schemes tend to be understood as the laws of physics.

¹²The determinism of an algorithm is expressed in its strictly defined procedure of calculation, etc. This means the elimination of any free choice and the introduction of a clearly defined order in which rules are applied. The mass scale, in turn, refers to the fact that an algorithm must apply to different variants of

basic characteristics. However, as noted in various scientific fields, this concept is understood variously. In order to standardize this concept, the Russian mathematician Markov formulated, in a mathematical sense, a strict definition of algorithm¹³.

These scientific terms and the concept of an invariant as an algorithmic pattern, related to decision-making, have a common feature. This property is the concept of structure. In mathematics, there is a strict definition of structure, but from the point of view of the analysis of decision-making processes, it is more important to answer the question about the genesis, that is, the dynamics of structure formation. In decision-making, an important role is played by the human psyche, because the genesis of structure is worth forming as a concept in the psychology dimension. According to the view about the genesis of structure developed by Piaget – one of the greatest scientists of the twentieth century, any structure has some kind of genesis, and that any genesis is derived from a given structure and leads to another ([26], p. 146–149, [27], p. 9). Piaget argued that structure is neither innate, nor a set of solid characteristics but constitutes a system of transformations (dynamics) subject to laws specific to that given structure. He mentions three basic properties of such a structure¹⁴, namely: a structure of states and a total system, a structure as a system of transformations, transformations as a result of specific regulating processes¹⁵.

With regard to the problem of knowledge management, especially from the viewpoint of schemas, invariants, or algorithms, it is essential to analyze the role of the normal form of a logical expression. The normal form can represent any (pragmatically simple) process. It plays an important *operational-organizational and optimization* role in logic and in its practical application, i.e. decision-making [11]. Man operates with practical and theoretical knowledge. In particular, practical knowledge, for example, any instructions for handling devices, should be presented simply, clearly and unambiguously to their users. Such explicitness requires instructions to be provided in accordance with the construction (definition) of normal form. This logical construction has

a problem, that is, it should have a mass character (it cannot relate only to a single case, however complicated). Effectiveness is achieved when an algorithm involves a finite number of procedures. Hence, an unambiguous result should be obtained in a finite number of steps. Such an algorithm is used, for example, in classical propositional calculus to determine whether an expression is a tautology.

¹³According to Markov's definition, an algorithm means a finite set of assigned elements called an alphabet and a set of substitution rules (for replacing symbols) for converting the word generated from an assigned series of symbols in a given order. This can also be done on the basis of a set of generation rules (these rules do not need to be known, they are merely transformations of a given word). At each step of the procedure only one rule is applied, until all the possibilities of transforming a given word have been utilized [24].

¹⁴At the end of his life, Piaget adapted the mathematical theory of categories, and in particular, the concept of morphism, which generally meant the conversion and/or preservation of some kind of structure, to the research area of changes in structure and its regulation ([29], p. 191–192).

¹⁵In his work, Ashby presents a similar view about the understanding of structure from the point of view of variety understood very broadly [3].

many theoretical structures, which are used to solve practical problems. The normal form, in particular the conjunctive-alternative form, also plays a very important theoretical role in logic. With its aid, it has been proved that classical calculus of logic is finitely axiomatizable¹⁶. In the case of theoretical science, this means that fundamental human thought patterns may be reducible to a finite number of invariants, whose output can be other solutions, in some sense, secondary to the primary ones. In terms of the social sciences, such an idea is proclaimed by reductionism. The normal form plays an important role in practice, such as in the optimization of logical networks, that is, any structure of logical connections. It can be concluded that the normal form constitutes an important concept in relation to decision-making in both the theoretical and practical dimensions of science, e.g. in the theoretical dimension based on axiomatization and, in relation to the social sciences, on reductionism and in the practical dimension based on optimization, producing instructions, etc.

4. Rationalism in decision-making

The methods of logical inference indicated above have the attribute of representing knowledge in terms of certain defined concepts, such as schemes and classical logic in its very basic dimension. Furthermore, the role of these concepts, that is, for example, of patterns and algorithms was presented in relation to the fundamental fields of the knowledge-circle based on Piaget's ideas [28]. In reality, however, scientists investigate and obtain from their research results representing all fields of knowledge and life. However, in these various fields of knowledge and life, there exist different, in relation to each other, concepts, languages and methods. Logic combines them, but conceptual systems in science often differ in terms of their content. Each domain of investigating reality is valid in respect to the acquisition of new scientific knowledge, but the lack of precise definitions or of a logically compact and methodically conceptual system often leads to the formulation of equivocal conclusions in relation to the objects tested.

Science, or rather its product, that is knowledge, collects many facts from various sources. Therefore, René Descartes postulated the need for research on the basis of rationality which is reflected by the phrase: *cogito ergo sum*. *Cogito ergo sum*, this is the method of thinking based on mathematical reasoning patterns, recognized by Descartes as a universal and completely certain method¹⁷.

¹⁶See ([12], p. 101–105).

¹⁷Currently, mathematics brings a lot to the development and interpretation of experimental results in many fields of science. An example is the application of statistics, created on the fundamentals of the queen of sciences (mathematics), in many areas of experimental sciences. Mathematical formulas are the consequence of so-called mathematical thinking. Descartes lived in the seventeenth century, which is so distant in terms of the thinking and scientific view based on modern methodology. However, the fundamental and

The concept of Cartesian rationalism is in some sense connected with another scientific direction cultivated in the framework of philosophy and methodology subsequent in time with respect to rationalism, called reductionism, which has already been mentioned. The concept of reductionism generally lies in the fact that scientific knowledge (represented by laws, properties, processes and other objects such as schemas and algorithms), is divided into two dimensions. They are namely: important (or primary) knowledge, which is fundamental, and the less important secondary knowledge that could, ideally, be obtained from primary knowledge by logical inference. There are opponents of this concept. However, reductionism is, at least to some degree, in harmony with Cartesian rationalism, since reductionism claims the need to arrange knowledge¹⁸.

widely cited works of René Descartes: *Discourse on the Method* [8], *The rules of mind control. The search for truth by natural light* [9], also indicate to us, contemporary people, the need for mathematical thinking. In truth, does Descartes' view on the practicing of science through mathematical diagrams differ from a system paradigm? Systems are largely schemas and Brusilowsky proclaimed the need to also unify scientific theories through system paradigms into a methodologically compact system [6]. Moreover, as it is clear from numerous scientific studies, Descartes' thoughts about the need for mathematical thinking resulted from his dissatisfaction regarding the scientific results obtained from the concept of scientific research based on empiricism as proclaimed by Bacon.

Regardless of the passage of time, the results of mathematics do not age: for example, the Cartesian system created by Descartes is still valid. His ideas of the need to view knowledge through the prism of logical thinking and the need for reliable patterns that can be transferred from one field of knowledge or one research area to another are still applied today. Piaget conducted such reasoning several centuries after Descartes [26–29]. A similar way of viewing the reality that surrounds us was professed for example by Dräschner [10] and Ashby, who spoke about the patterns of diversity in Nature [3]. However Bogdanov, the Russian scientist of Polish origin, is regarded as having laid down the foundations of systems theory, and von Bertalanffy as its creator.

¹⁸The arrangement of knowledge is particularly needed for science in its present form, because it is, as it has already been noted, fragmented and much attention is paid to experiments. Methodologically compact tools constituting synthetic knowledge are required to elaborate research results. Today, such tools are provided by probability theory and mathematical statistics, which are extensively used in practice. However, despite the fact that they are exact sciences, they are not universal in terms of either quantity or quality, namely in terms of the content. Therefore, there is the need to create a system of sciences [7]. At present, there is no methodologically cohesive theory and because of this there is a pragmatic need for reductionism in the cultivation of science. Although it seems rather strange, in the context of scientific discourse on reductionism, an idea of a philosophical nature, has been implemented under some disciplines of mathematics and logic in the form of axiomatic systems. Moreover, this has not been yet done in any of the human, social, economic, or experimental sciences. It should additionally be stressed at this point that calculus has managed to achieve the mentioned postulate of reductionism in an ideal manner, because the axiomatics of this field of science fulfill all the methodological requirements for axiomatic systems [11, 12]. Moreover, it may be guessed from the theoretical results from logic described above that the laws of nature have logical relationships between themselves. Some are dependent on others, and in addition, some of them derive directly from others. There is an order of things in nature, and logic in some way can represent it.

5. Decision-making in the dimension of truth and evidence

In the field of logic, in addition to the previously described concepts, the two concepts of truth and evidence, which are very important for science, are also in operation. At this point, there is no need to give any formal expressions, or a wealth of other concepts, associated with their definition. It is enough to emphasize what they relate to and what they resolve in science, in practical life, and in relation to the management of accumulated knowledge. The concept of truth is considered in the field of logic called logical semantics and in the field of philosophy, actually in one of its branches, which is called the theory of knowledge or epistemology [12, 23].

In logical semantics, Alfred Tarski, a Polish mathematician and logician, formulated a commonly-used definition of truth¹⁹. This definition leads to statements completely consistent with intuition. For example, any statement is either true or false, the consequences of true sentences are also true sentences and the most important statement, that the theorems of logic are true in any field. This concept of the truth allows us to avoid paradoxes of the type known in logic as the liar antinomy, because the differentiation made between language and metalanguage is sufficient to prevent an expression stating something about itself²⁰.

Based on this definition of truth, one can introduce the concept of proof. Based on the axioms of logic, only the laws of logic can carry out the transition from truth to proof. The set of the laws of propositional calculus can be defined in a synthetic manner by formulating the theorem of decidability, stating that the operational method for checking whether a logical expression of propositional calculus is a law of logic or it is not. This is an effective method, because it requires a finite number of steps and in a sense constitutes a procedural algorithm. Typically, particularly in the formal sciences, as previously mentioned, the aim is to ensure that theories are created in an axiomatic manner²¹.

¹⁹According to which: the expression W is true in the range of $D = \langle X, p_1, \dots, p_n \rangle$ under the interpretation of the predicates P_1, \dots, P_n as the names of relations p_1, \dots, p_n if and only if, any sequence of objects in the collection X fulfils W in D with the above interpretation of the predicates.

²⁰The protection against using semantic antinomies resulting from such a definition is possible only when the language used is clearly determined by listing all of its output symbols and all the rules for constructing complex expressions. This is not possible in the case of a natural language, unless an appropriate metalanguage has been defined, to which the definition of truth given above is applied. The philosophical problem of the definition of truth constitutes one of the fundamental questions of philosophy and is connected with the definition of truth in the sense of its philosophical understanding and with the question about the criteria for truth. The answers are contained in various theories of truth, but this thread is not going to be developed here. For more details, see ([23], p. 153–154).

²¹The basis of theorem proving is the thought according to which the basic properties of the objects considered are categorized, that is, the philosophical concept of reductionism. In general, the most obvious properties of the objects considered are called certainties, assumptions or most often axioms. Axioms are

This approach to logical inference contains the concept of proof, which is important in logic and mathematics, as well as in everyday life, and in turn contains two rules of evidence: *the rule of detachment* and *the rule of substitution* [12].

However, attention will be focused at first on the presentation of the essential concept of proof, and then its formal character will be introduced²². The very concept of proof requires a reference, i.e. relativization with respect to the set of assumptions and rules of evidence, which have the nature of deductive rules. It can therefore be concluded that evidence is a string of sentences or logical formulas, any proof begins with the expressions adopted as a foundation, then it contains the expressions obtained from the assumptions as a result of the transformations allowed by the deductive rules and, possibly further, expressions derived from earlier expressions, etc. The last link in the series of transformations is the sentence to be proved²³.

6. The pragmatism of the concepts of theory, truth and proof

After defining and then appropriately understanding the concepts of truth, evidence and theory, one might raise the question as to what the definitions of truth, proof and theory indicated above bring to the practice of inference, i.e. obtaining knowledge in the context of decision-making? Above all, the answer to this question should be related to mathematics. In mathematics, the majority of deductive work, that is in fact evidential, is based on the derivation of new theorems, i.e. new knowledge from already proven theorems. Such mathematical proofs are logically concise, that is, from both a methodological and logical perspective, mathematical proofs have a reductionist nature. This is due to the fact that the essence of the methodology of mathematics is based on reducing some domain of knowledge (a T theory), into such a form where each derived theorem of T theory is either an axiom, or is obtained from the system of axioms by using inference rules a certain number of times. This is consistent with the given definition of proof. In addition, such a procedure is called the formalization of T theory. In practice,

the presuppositions of a field of study. All its other theorems are derived from the axioms by logical inference.

²²The natural deduction method mentions evidence as a primary concept.

²³If the rules of a given system are the rule of detachment and substitution, then the formal description of proof in the framework of propositional calculus takes the following form²³. D is a proof of sentence B based on a set of formulas X adopted as assumptions, if and only if D is a finite sequence of formulas $D = \{D_1, D_2, \dots, D_n\}$, such that the last formula of this sequence is identical to the sentence B : $D_n = B$ and each formula D_k of the sequence D ($1 \leq k \leq n$), either (i) belongs to the set X , or (ii) arises from an earlier formula D_j , i.e. ($j < k$) in this sequence by the appropriate substitution, or (iii) arises from two earlier formulas in the sequence $A : D_j, D_i$ i.e. ($j < k, i < k$) by detachment: $D_j = (D_i \rightarrow D_k)$. It should be added that there is also a slightly different definition of proof, but this refers to mathematical theories.

the evidence used for reasoning should not be formalized. However, in every formalized field, that is, for which a T theory has been created, it is theoretically possible to generate all its truths²⁴. Thanks to such idealization, on one hand, man becomes independent from randomness in science (randomness in the sense of probability calculus) and the psyche of its creators. On the other hand, this psyche exerts a significant influence on the development of science, because in the definitions of truth and proof presented earlier, information is included only about what these concepts are and how the proof of a given theorem should proceed in logical terms. Generally, in science there is no algorithm for deriving proofs in relation to any selected process, or even in respect to any formalized field of knowledge. Therefore, the majority of potential theorems, even within mathematics itself as the most formalized science, will never be derived. What does this mean for cognition, and in fact for decision-making: chaos or order? On one hand, it is possible to deduce everything, but looking at this problem from the perspective of time, there is no answer to the question of when this is going to be achieved. Thus, the human psyche is not to be underestimated in the decision-making process or its analysis. Since this dimension of thinking is studied by psychology, therefore a lot of this article is devoted to this aspect.

Dealing with issues of truth and proof requires consideration of the scientific concept of a model, which is very important methodologically. However, this is not just about a definition of this concept in the context of logic. It is more important to understand when it is possible to use the concept of model in its scientific sense, especially in the analysis or synthesis of knowledge. In everyday life and the practice of science, people often use the term model. There are some differences between the understandings of this term in these two fields. Constructing models is a methodological approach used in science. Its aim is to simplify and reduce the complexity of a problem in any given field of knowledge, while retaining its essence. This methodological approach increases the chances of solving problems analyzed scientifically in this way. The scientific concept of a model is helpful in a given field of knowledge when solving research problems, with the assumption that models are used in a given branch of science. A model understood in this way has a methodological dimension and is associated primarily with the ability to perform certain calculations and operations related to the relationships existing between concepts. That is to say, it provides operational tools with respect to a specific problem for solving this particular problem²⁵. In science, the term model is used with such a meaning which is called the semantic model. In general, semantics is the science studying the way in which diverse languages express and represent concepts. Semantics is an important and wide-rang-

²⁴In practice, in the form of theorems, properties and descriptions of certain processes and other phenomena.

²⁵More specifically, this problem will be considered in the section concerning the classification of terms.

ing discipline of linguistics. Scientists in the field of semantics investigate the relationship between language and the reality to which linguistic expressions are related. Because reality can be real and abstract, knowledge also has these two dimensions. In the area of the methodology of mathematics (metamathematics), the relationship between language and reality are recognized as the relationship that occurs between a mathematical theory and the field described by this theory²⁶.

The theory of semantic models also deals with the relation between mathematical theories (language) and their models²⁷ (realizations of a theory), i.e. theories that satisfy the axioms of these theories. The model of a theory accepted at a given time does not always fulfill these requirements. In addition, the total set of our knowledge is not constant over time²⁸.

7. Classification of decisions

When describing the progress of human knowledge, it is necessary to consider the problem of classifying concepts. The significance of this issue results from the fact that human knowledge depends greatly on the way in which ideas are defined and classified, resulting in their later understanding in science and everyday life. In practice, this also transfers into decision-making²⁹.

The problem of defining concepts and their classifications will only be discussed in outline, because generally its methodology has still not been fully explored. In this context,

²⁶This relates to logical semantics and the semantics of natural language, that is, to the reality surrounding us. However, logical considerations sometimes also constitute reality in relation to the problems considered therein, which have to have a carrier, i.e. a logical form. One might ask, how can logical forms in syntactic character, i.e. content carriers, be distinguished from forms regarding contents, i.e. semantic forms representing the content of the object analyzed. A good example from the field of logic might be predicate calculus: an expression of the form $R(x, y)$. This represents a compound (relationship) in any field occurring between the variables x and y . It is a syntactic representation in this language, because it does not state anything about the content of this relationship. However, if we are explicitly talking about such a relationship in a particular field, this will now be a semantic representation (e.g. take the set of natural numbers and the expression $\exists x \forall z (x \leq z)$, which says that in the set of natural numbers, there exists a number which is smallest and this is a semantic property clearly defined in this field of knowledge).

²⁷The domain $\mathfrak{R} = \langle X, p_1, \dots, p_n \rangle$ is called a model for the series Z of sentences with constants P_1, \dots, P_n if and only if X is not empty and $Z \subset E(\mathfrak{R})$, that is, all the sentences from the series Z are true in the domain \mathfrak{R} with the terms interpretation of time, scientific model P_1, \dots, P_n as names of the relations p_1, \dots, p_n .

²⁸The model of stimulus and reaction is an example of a model that was accepted for a certain time, but which in the end did not meet the desired level of effectiveness in describing reality. See, e.g. ([23], p.129).

²⁹To some extent, the problem of understanding concepts, even as fundamental as a model, truth or proof in science discussed there indicates this.

consider the following example: the definition of a sign and a language sign – issues that form an essential basis for scientific terminology. The French linguist F. de Saussure was the first to raise the problem the arbitrariness of signs in the twenties of the last century. Many years later, another French scholar, this time the mathematician René Thom (founder of catastrophe theory), postulated that each sign has some form of motivation, a reason for its occurrence or appearance. This issue was also considered, among others, by the Polish semiotician, J. Pelc, who thought that there was no so-called pure use of a sign³⁰.

Very little empirical work has been done on the variety of ways in which people understand, use and define signs (the authors only know of one survey). Moreover, this indicates that the logical precision in defining concepts is subject to the arbitrariness of people's understanding of them, that is, their interpretation of a sign. This constitutes the intersection between the understanding of a sign and its nature.

In every field of science, some problems are seen to be very important and some seen to have secondary importance³¹. This reasoning also refers to the concepts used in a given field of knowledge. With respect to knowledge analyzed from the point of view of methodology (more generally, from philosophy) and logic, in practice the issue of concepts can be expressed in two dimensions. One of them is composed of the concepts necessary for calculations, such as accounts, which are not necessarily simple. The second category includes important relational connections, which form between themselves different but essential concepts, and even fields of knowledge. An example is the definition of proof and model quoted above. Axiomatic systems of binary-valued logic, also previously mentioned, are evidence that every theorem of logic (except axioms) is directly associated with a number of other theorems, which can be derived from them by logical inference (deduction). In other words, they are linked together by logical relationships. For the pragmatics of knowledge, this means that man basically uses numbers and words, and so generally the quantitative dimension can be understood as measure and the quality dimension as content. This first concept has mathematically formalized metrics such as: Euclidean, Manhattan, Chebyshev, or Minkowski and dimensions: topological, Hausdorff, Kolmogorov, and recently the fractal dimension defined by Mandelbrot [22]. Thanks to the concept of metrics and dimensions, many processes occurring in the reality surrounding humans can be better understood than previously. However, content is generally immeasurable. Content is considered by such important fields of knowledge as: linguistics, psychology, sociology and many other disciplines. Hence, the following question emerges at this point: which of these two dimensions of science is more important with respect to issues, fields of science and

³⁰All these semiotic threads are discussed in more detail in [19].

³¹This problem was indicated during the discussion about the role of Cartesian rationalism in the cultivation of knowledge and reductionism in science.

concepts in regard to the acquisition of knowledge? The answer is simple. Both dimensions are components of knowledge, the constituents of the human mind, and only together can they form a whole, that is, a unity³².

Conceptual science is therefore recognized to have a measurable dimension and one that is difficult to measure, but without which the practice of science would be impossible, because everything that surrounds us needs to be somehow understood. There have been attempts to define measures for concepts that do not belong to the measurable class. In this case, it is maybe worth considering the creation of qualitative forms (measures) corresponding to the nature of the conceptual category that cannot be represented by the quantitative (numerical) dimension. At present, the dimension of concepts, terms and immeasurable issues is generally best represented (measured) simply by natural language. On the other hand, mathematical measures have also been applied (e.g. formal grammars, a topological approach to language or the statistical analysis of language) but these most often refer to the study of natural language, that is, to linguistics.

The aspect analyzed concerns the role of logic in decision-making. Although science knows many types of logic, in practice man makes decisions based on Boolean logic (yes or no alternatives). This form of logic does not resolve operationally whether the event analyzed has a deterministic or random character, and therefore whether it is worthwhile to examine the second dimension of decision-making. This is connected with risk, especially with the assessment of its level. Moreover, this area already covers the content of the decision problem analyzed.

8. On the issue of risk assessment

Risk is an important colloquial concept and scientific term and is inseparable from decision-making. In practice, we distinguish between decision-making under risk and making decisions under conditions of uncertainty³³. In relation to the first case, it is assumed that the decision-maker knows the probabilities of the possible states of the external world (Nature). In the second case, which is a state of uncertainty, the decision

³²The anatomical construction of the human brain is asymmetric, because one hemisphere is more responsible for discrete processes – measurable, and the second for substantial – semantic, and so continuous. However, a man has only one brain, and thus only one reality.

³³It should be emphasized at this point that many economists do not recognize the distinction mentioned above, or even consider it. This is because they consider that whenever the probabilities are not known, then they can be determined by an appropriately selected statistical sample (statistical game) or treated as subjective probabilities (based on the knowledge of the decision maker appropriate to the problem considered, or on knowledge obtained from competent experts, e.g. according to the Delphi method). This view constitutes a pragmatic and, in fact, correct point of view in relation to decision-making and the way of understanding the concept of risk [32].

maker does not know these probabilities, and sometimes, they are very difficult or even impossible to determine, basically everything depends on the complexity of the problem analyzed³⁴. This form of risk has probably always occurred and still occurs in every area of human life, but has gained a special meaning at the end of the twentieth century on the plane of economics and finance³⁵.

In science, the term models of decision-making under uncertainty and risk is used. Also, strategic models are considered according to which a decision-maker does not know the plan of his opponent until the end. According to strategic models, there are two parties who are antagonistic to each other. A good mathematical representation of strategic models is given by game theory, in particular zero-sum two-player games in which each player uses a probability distribution over his set of actions. This distribution is not known to the other player³⁶.

9. Risk and management

The term: risk management is used both in modern science and in the field of organizations operating in the market. Risk is understood in this case as the danger of making a wrong decision or incorrect behavior that will result in failing to achieve a goal ([14], p. 109). The goals of organizations operating in the market depend on the type of tasks carried out and so the primary goal of a profit-making organization is to make a profit in the financial dimension, and in the case of a non-profit organization – the maximum satisfaction of social needs. Decision-making is burdened with risk, because it concerns the future. The policy concerning risk applied at the level of an organization is called risk management and relates to both minimization of the incurred loss, as well as maximization of profit or satisfaction.

³⁴This distinction was introduced to science by Knight in 1921 ([18], p. 15 and others, [32]). Other definitions of risk have been given, but in reality there is only one category of uncertainty and one type of probability (in terms of axiomatic probability theory).

³⁵The acquisition of the same bundle of goods, cheaply or relatively dearly, or receiving a low or a high return on invested sums of money in conditions of uncertainty, is an important consequence (effect) for the decision-maker in respect of the investment decision taken.

³⁶It should be noted, however, that if only the second player acts rationally, the first player, on the basis of his payoff matrix, is able to deduce (calculate) his optimal behavior (frequency with which each action is taken) in terms of the probability distribution of the actions of his opponent, that is, the second player. This eliminates the element of uncertainty from the decision-making process [15–17]. Such behavior may prove to be misguided from the viewpoint of logic (rational decision-making). Science and reality do not always match each other, that is, fulfill the symmetry relationship. The problem of modern science is to answer the question as to whether cognition (science) corresponds to reality (intuitive human behavior) and in many cases it turns out that this is not so.

Defining the concept of risk management as a logical, orderly set of rules and principles, applied in a uniform and constant manner in relation to the activities of the whole organization ([14], p. 60), there is a desire to maximize the reduction in the level of risk and protect against its consequences (compare [14], p. 59).

In principle, however, risk cannot be managed, just as man cannot manage the weather, climate³⁷ and other such areas of life, on which man usually does not have a significant impact and cannot predict the future state of Nature accurately. Therefore, only one-dimensional measures of risk, the numerical value of risk, are used in management, which only partially account for the mathematical complexity of such problems.

A real problem associated with the concept of risk emerges here, the problem of defining categories of risk, its nature, as well as tools and methods for measuring the level of risk. We also need to use the appropriate substantive analysis (of content), that is, the field of knowledge (sphere of life) in which risk is present.

In the twentieth century, the linear paradigm is dominant in science, particularly in economics³⁸. The supporters of this paradigm accept that using linear equations³⁹ is a possible approach, or at least approximation, to modelling many measurable processes (which are not necessarily based on linear dependences) connected primarily with economics in its most general dimension⁴⁰. However, achievements in the area of mathematics, especially at the end of the twentieth century, have clearly demonstrated that Nature, that is, the Reality surrounding humans (in the mathematical dimension), is in fact constituted by dynamic processes of a non-linear character and, moreover, they are mostly so-called asymmetric processes [25, 30].

It should be asked here as to whether these dynamic processes are completely random, somehow ordered (do they have a trend, is there correlation between the past and the present), or are they even deterministic?

Randomness is an important determinant of life. However, the essence of randomness, from a scientific point of view, is recognized in an appropriate manner only by probability calculus. Probability calculus is one of many branches of mathematics. It has a conceptual system, which is ordered and most importantly enclosed in an axiomatic form⁴¹. Randomness is precisely that notional category which constitutes the basis

³⁷Although in general we know that in a certain climate zone there are four seasons, in another one, etc.

³⁸Linear regression model (multiple) at one time fascinated representatives of such science as psychology. On the basis of empirical data the linear models of such relations were attempted to be built: the adequate reaction corresponds with a given action.

³⁹Bertrand Russell also confessed idea that everything in nature can be expressed by the layout of linear differential equations. Later he withdrew from that idea [31].

⁴⁰Linear formulations are impressive because of their simplicity, but not always effective in terms of results.

⁴¹The axiomatic approach of a given branch of knowledge usually facilitates its development and the strict formulation of theorems and connections with other sciences – as in the case of probability calculus. The axiomatics of probability calculus caused it to be specifically bound together with the theory of measure

of uncertainty and is associated with the axioms of probability calculus. The term random event is adopted in this area of knowledge as a primary concept, that is, indefinable, accepted as something familiar, predetermined as obviousness or understood intuitively. In science, processes are generally divided into: deterministic or probabilistic – statistical or strategic⁴². Processes are also analyzed according to their behavior (model)⁴³ as static or dynamic, linear or non-linear, continuous or discrete, finite or infinite dimensional. This requires the use of many mathematical methods of testing.

10. Chaos and complexity theory as a new paradigm of science

In recent times, chaos theory has become a popular issue related to randomness and risk. Considerations on the relationship between chaos theory and risk should begin by citing a description of risk assessment. Risk assessment consists of a forward pass of an inductive calculation procedure and, as it has already been indicated in the article, is partly devoted to the logical aspect of decision-making. This algorithm must have an unambiguous result (this is a fundamental requirement). Having assessed the level of risk with regard to the issue analyzed, one can then look at the decision being taken or already undertaken. Such a numerical assessment of risk adds credibility to a decision.

Very different research methods are used in classical models of decision-making. The most important methods applied are: differential calculus, probability calculus, linear programming, nonlinear programming, mathematical statistics, game theory, dynamic programming, topology and category theory, chaos theory and fractal theory. The methods based on differential calculus, to a greater or smaller degree, benefit from the tools of differential and integral calculus. These tools are applicable to so-called mathematically smooth objects. Fractal theory considers objects that are not mathematically smooth, that is, cannot be differentiated, but have a specific structure. This seemingly rough structure can also be expressed mathematically⁴⁴. In addition, fractals exhibit many other interesting features that exist in the reality surrounding humans and are not

and mathematical analysis. Axiomatics are also an almost ideal realization of the idea of rationalism preached by Descartes, that is, reducing a given field of knowledge to a finite number of its most important concepts. From this, one can derive others and on their basis draw important rules for that area of knowledge modeled on the mathematical formulations proposed by Descartes.

⁴²Some strategic processes can be classified as deterministic or probabilistic in terms of their method of derivation. For example, the solution of a two-person, zero-sum game is deterministic, since it can be reduced to a linear program that is solved in a deterministic manner (i.e. not involving probability theory).

⁴³The concept of model is presented here in a more colloquial than logical, mathematical sense.

⁴⁴One of the mathematical features of fractals is generativity. However, proponents of methods based on differentiation, when fractals have smooth structures, at least locally, and are characterized by generativity, do not include them in the set of mathematical fractals. For example, see ([21], p. 61).

disposable, but on the contrary, are able to repeat their shape in the time and space dimensions. This is associated with the concept of generativity that is well-known in mathematics and represented by generative grammars, and more generally, in combinatorial systems [5].

A paradigm is an abstract model – a way of looking at things. In recent years in science, especially in economics and financial economics, the linear paradigm has been dominant. The linear paradigm is not only an assumption but also a thought pattern stating that every stimulus produces a reaction proportional to its strength. Meanwhile, many natural processes, and these include markets, are rarely organized in a linear manner, and it often happens that the response to a cause has, for example, an exponential form. This means that there are important economic, social and psychological processes that react to various stimuli in a nonlinear way. The following question may be posed: what constitutes the scientific strength of what might be called, the linear philosophy of science? The answer to this question is as follows: linear models are simple mathematical structures, easy to solve, and provide many useful benefits in practical terms, because what is not linear can quite often be well approximated by a linear model, with no major losses in the accuracy of the analysis.

11. The social dimension of decisions taken

Man, regardless of the achievements of science, has long been implicitly trying to give structure and arrangement to life. Such an attempt to understand the environment has an important practical purpose⁴⁵. In order to realize this purpose, mankind has developed legal systems, introduced (often to its own misfortune) bureaucracy as a system of control, calendars to order time, or rather divide it into units designating successive important events, as well as many other systems for ordering reality and societies. However, regardless of the precision of legislation or the level of development of organizational schemas, when we deal with natural systems, there are still problems with understanding the forces associated with their behavior and functioning. As a result, man needs various institutions such as courts to interpret the law, consultants and experts in various areas of life in order to understand, e.g. the group dynamics of the functioning of enterprises and the behavior of social groups, as well as scientists investigating nature. The outcome of such activities is that different societies have developed different

⁴⁵From a methodological and logical point of view, the use of the word structure automatically implies the concept of order. However, the concept of structure is superior to the notion of order. This means that if we are dealing with a structure, then its order is specified in an implicit way. Moreover, in the field of psychology, the outstanding French scientist, J. Piaget, stated in his research that every man strives to achieve a state of equilibrium in relation with Nature. This means that he wants to understand the environment, because it enables him to achieve a state of equilibrium, e.g. a lack of fear [28, 29].

systems of values. It should be emphasized here that all organizations must first be created by someone, that is, a certain philosophy of thinking must be adopted and then the view of society regarding the structure and construction of the surrounding reality evolves along with the system that has been developed. In addition, if the mental picture adopted is not consistent with reality, then the conclusions formulated by such science are not worth much, do not mean anything, or even harm the reality analyzed by that science. At this point, the following question, important from a methodological point of view, arises: what appropriate methods of analyzing our surrounding reality should be taken? In his book which was published many years ago but still remains actual, Edgar E. Peters makes a judgment that: *The world is not ordered* [25]. In nature, there is no order, nor does it exist in structures created by man and called institutions. Man established, for example, capital markets, which after a while became so developed that their functioning and behavior has ceased to be understandable to their creators. The complexity of capital markets meant that they became self-organizing systems. The same has happened with the functioning of social organizations, the behavior of political parties and even – the behavior of individual citizens in society

When Econometrics was developed more than half a century ago, it was expected that it would allow us to predict the economic future of man, but unfortunately, this field did not satisfy these expectations. This happened because the concept of equilibrium was applied, i.e. it was assumed that when there were no outside influences (disturbances), then the system was at rest⁴⁶. However, this does not take into account the fact that society is constantly developing. This means that the system is in a state of remoteness from equilibrium [25, 30].

In physics and the engineering sciences, linear differential equations were and are still often used, as they have a unique solution. For a long time, according to the pragmatic dimension of science non-linear equations were not useful in practice, because they have many independent solutions. It turned out, however, that the most complex natural systems can be modeled using non-linear differential equations. Life is characterized by diversity and there are many possible responses to the same set of stimuli – i.e. alternative behaviors. Therefore, we need equations with many solutions⁴⁷. A characteristic feature of nonlinear systems is self-similarity (as previously mentioned in the

⁴⁶This is the definition of equilibrium used by economists. A system likes order and after any disturbance tries to return to the previous state, and responds to any stimulus in a linear manner. Hence, the principle that supply is equal to demand, and many other principles of equilibrium are dominant. Meanwhile, in nature, if a system or species is to survive, it must evolve – according to Prigogine's view – and thus never be in equilibrium [30].

⁴⁷Using the tools of logical deduction from Aristotelian logic, it can be shown that a single set of premises can generate many different conclusions which can be interpreted as the response of a system to this stimuli set.

case of fractals). This property only appears in systems which are far from equilibrium and sensitive to changes in the initial conditions.

12. The logical dimension of risk category

In the first part of this article, the logical dimension of concepts associated with decision-making was analyzed. In this section, attention is focused on the issue of risk diversity. Thus, the following question appears: can logic also be diversified? There exist well-known variants of binary-valued logic, as well as multi-valued logic, modal logic, fuzzy logic and temporal (time-dependent) logic. These systems of logic were created in response to humans' understanding of the reality surrounding them. However, many of these systems of logic have a purely abstract character, while others bring new insights to our interpretation of reality. For example, in the framework of Raichenbach's multi-valued logic, it is possible to interpret certain quantum phenomena formulated by physics⁴⁸. The intensive development of fuzzy logic has also brought measurable benefits to our interpretations of events and risks ([20], p. 1093–1098). However, in the end, man always makes dummy decisions. Even many systems of multi-valued logic can be reduced to dummy logic. Therefore, this analysis is limited to Aristotle's system of logic. What connection does it have with the concept of risk? Deduction does not have, in principle, a quantitative character, because it is embedded in the semantic thinking of man. However in logic, this form of thinking has been, formalized to some extent, and so-called logic diagrams have been built for some of its forms. On the basis of this, we can judge whether our thinking is logically correct or may prove to be wrong. Thus, man can minimize risk from making decisions within the logical dimension, sometimes even bringing it to a value of zero, without numerically calculating this risk, because it is a qualitative dimension of risk assessment. Therefore, a very important role in decision-making, from the point of view of risk minimization, is played by the correctness of the decision taken in qualitative terms. A formal evaluation of the qualitative correctness of such a decision is based on operational logic, which can be reduced to the verification of the decision taken in terms of logical correctness. Referring to the previously mentioned concept of logical schema, each logic diagram (law of logic) contains premises and a conclusion. Checking whether a correct conclusion (a correct possible decision) was generated from the given premises can be carried out in an algorithmic way. There are several ways to verify this correctness. One of them is the previously mentioned rule of resolution. The principle of resolution allows us to minimize the risk of the decision taken in the sense of content. Risk minimization in the sense of content

⁴⁸For examples, see: ([33], p. 57 and other).

means that this approach reveals all the possible variants, that is, the possible conclusions in relation to the known set of premises. However, based on this principle, it cannot be guaranteed that there exists an unequivocal answer to the question as to which of the possible variants (conclusions) is realized in a given situation. The choice may be up to the decision-maker, but Nature decides what the conclusion is. All the possible variants of events can be predicted (logically forecasted) in the case of simple situations, which is dictated by the fact that simple situations generate only a small number of possible outcomes.

13. On the issue of a universal algorithm for risk assessment

From the foregoing considerations, it has been concluded that each of the discussed fields of knowledge includes the issue of risk, but only in a few of them have attempts been made to assess risk in a practical sense. In this context, practical means that the results obtained are concrete, consistent with reality, and above all assessment takes the form of a calculation procedure. Large or small risks may be indicated, different qualitative categories of risk considered, but risk assessment is carried out in an algorithmic way in few of these fields. At this point, an even more significant problem emerges, because it is of a methodological nature: does the category of risk share a common feature with each of the areas in which risk occurs? In other words, can various risks be summarized, at least from the point of view of measurable risks, using a common denominator? With respect to bundles of goods, as considered in mathematical economics, such a universal equivalent is money. However, another such equivalent might be utility, and utility theory indicates that two bundles which give the same utility might have different prices⁴⁹. In addition, paradoxically, the utility gained by obtaining a given sum of money depends on how much money we already have [1]. Hence, both reality and the concept of risk are multi-dimensional. Therefore, it is difficult to create a unified theory related to the assessment of risk.

14. Conclusions

The issues presented in this article concerning decision-making processes and to some extent management, cover the approach to such problems only from the point of view of logic (the philosophy of its nature) and risk. However, according to the authors, logic is a fundamental dimension in the decision-making process, because it is the basis of human thinking and philosophy, and in turn, directs this thinking to different visions

⁴⁹The utility of a sum of money paradoxically depends on how much of money is possessed in total [32].

of understanding the World. Since processes often include some random elements, then many decisions are subject to risk, in the sense of a possible loss of utility. Logic's contribution manifests itself in two concepts, important for science and humanity: truth and proof. It can be philosophically said that if a man has knowledge, then at the same time he has truth. Formally, however, unambiguous results require a strict definition of the terms used within logic and mathematics, e.g. a definition of proof. In a more general statement, it is assumed that if a precise concept of proof exists, then truth also exists. In addition, the article draws attention to a concept from logic which is important in knowledge management from an axiomatic viewpoint. In relation to the social sciences, this concept is analogous to the concept of reductionism in science. The paper also presents issues concerning the Cartesian concept of rationalism in knowledge management, reductionism, the concept of truth and model, as well as the classification of concepts in terms of logic. All of these problems and concepts are understood from the position of the most important carrier of information for humankind, which is natural language. Hence, our understanding of formalism is intertwined with colloquial concepts and this means that scientific texts contain many threads that can be understood by the reader in different ways. All this together influences the process of making decisions.

In addition, the arbitrariness of signs and at the same time the need to strictly define terms, imply ambiguity in our understanding of the contents presented. Hence, the possibility of incomprehension or misunderstanding appears. Since science is valuable only when it has a utilitarian character, emphasis in the article was also put on the very concept of risk in its scientific and colloquial dimensions. Risk has been described from the point of view of content and the need for its quantization. In turn, from a methodological point of view, risk, as a category of conceptual science, is one of the fundamental problems for scientific studies in terms of its complexity.

In conclusion, we should not expect the development of a uniform methodology for the scientific assessment of risk in the near future [2, 14, 25], because any numerical assessment of risk only represents what it refers to, that is, only one aspect of risk. Logically speaking, the existence of such a methodology would mean that we fully understand the mechanisms of the reality surrounding us.

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