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AN EFFECTIVE SYSTEM OF SPORTS COMPETITION MANAGEMENT

An innovatory system of managing sports competitions has been presented. Its advantages with regard to other currently used systems are discussed. A theorem connected with such a system has been presented in the last section of the paper. Sports competitions aim to establish a ranking of the participating teams. This consists of sorting teams according to a quality which can be thought of as the ability to win matches. Direct measurement of this quality is not possible, since the ability to win matches depends on a great variety of factors being difficult to determine. Nevertheless, it is possible to compare any two teams if they play a match. These matches are played under normal rules. In turn, all the rules valid during sports competitions, outside the matches, make a system of sport competition. Sorting sports teams differs from typical problems of sorting. The result of a comparison of teams is sometimes misleading. It happens that a team with a greater ability to win matches loses a match to a team with a smaller ability to win matches. Thus, the problem of sorting teams is a probabilistic problem. Due to this reason, traditional sorting methods are ineffective in terms of managing sports competitions.

Keywords: *sports games, championships, system of sports competitions, probabilistic sorting, computational complexity*

1. Introduction

The aim of sports competitions is to rank teams or competitors taking part in sports games. Thus, the problem is how to sort teams by their ability to win matches. However, it is not possible to directly measure this characteristic because such ability depends on many factors that are difficult to define. It is possible, however, to compare two randomly chosen teams provided they have played a match. All the rules

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applied in various competitions are systems of managing sports competitions. Thus a system of sports competitions consists of the rules which decide who plays against whom, in what sequence and where, as well as the criteria that decide the teams places in the final ranking list.

Sorting sports teams differs from standard sorting problems, due to the fact that the result of comparing two teams is sometimes incorrect. It does happen that a team with a greater ability to win gets defeated by a team whose winning potential is undoubtedly lower. Thus the problem of sorting teams is a probabilistic one. This is a reason why traditional methods of sorting do not work in the case of the management of sports competitions.

2. Formulation of the problem

There are n objects $d_1, d_2, ..., d_n \in D$. Some numerical value called quality is assigned to each object. The object d_i has quality $x_i \in R$ ($i = 1, 2, ..., n$). It is not possible to measure the quality of any of the objects. However, comparison of the quality of any two objects is possible (e.g. by the result of a game). As a result of comparing two objects d_i and d_j, the relation established is $d_i \angle d_j$ (d_j wins against d_i) or $d_j \angle d_i$ (d_i wins against d_j) such that

$$\underset{i,j}{\forall} \quad x_i \le x_j \quad \Rightarrow \quad \left[\Pr(d_i \angle d_j) = p_{ij} \ge \Pr(d_j \angle d_i) = q_{ij} \right] \tag{1}$$

where: $p_{ij} \ge 1/2$ is the probability that the highest quality team wins, $q_{ij} \le 1/2$ is the probability that the lowest quality team wins, $p_{ij} + q_{ij} = 1$.

We assume that the probability p_{ij} is a non-decreasing function of the distance between the qualities x_i and x_j . Consequently, we have

$$p_{ij} = p_{ji} = f(|x_i - x_j|) \tag{2}$$

$$0 \le a \le b \quad \Rightarrow \quad f(a) \le f(b) \tag{3}$$

$$f(0) = \frac{1}{2} \tag{4}$$

Objects belonging to the set D are to be sorted with respect to quality according to the conditions stated above.

3. Discussion of the problem

A management system for sports competitions is a solution to the problem of probabilistic sorting. It consists of sorting objects under conditions where the only tool that we possess is probabilistic pan scales. Their properties are presented in conditions (1)–(4). The probabilistic character of the scales results in the fact that the relation established between the same pair of objects can differ according to different weightings.

Condition (1) guarantees that if two objects have different qualities, the probability p_{ij} that the relation between them will be consistent with the relation between the qualities is greater than the probability q_{ij} that the established relation will not be consistent with the relation between the qualities. This dependence contains also an assumption that the result of the comparison of two objects never indicates an equality relation between them ($p_{ij} + q_{ij} = 1$).

Condition (2) guarantees that the probability p_{ij} that the relation between two compared objects will be consistent with the relation between their qualities depends only on the extent of the difference between these qualities. Therefore, the order of the compared objects is not important, thus $p_{ij} = p_{ji}$.

Condition (3) guarantees that the probability p_{ij} that the relation between two compared objects will be consistent with the relation between their qualities is increasing in the difference between these qualities, i.e. the result of weighing is more often true when two objects differ greatly than when two objects are more similar.

Condition (4) guarantees that the result of the comparison of two objects of the same quality does not favour either of the objects.

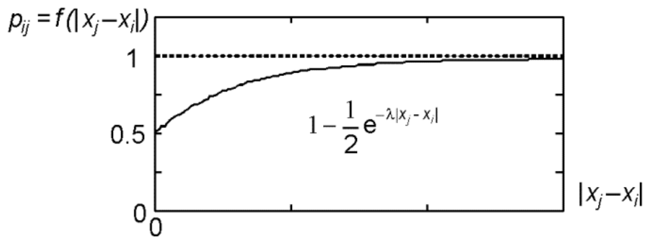

Fig. 1. An example of the function p_{ij}

An example function p_{ij} is presented in Fig. 1. It is obvious that under probabilistic sorting no method guarantees obtaining exactly the same ordering as the ordering of the qualities of the sorted objects. Appropriate methods may merely guarantee a high probability of obtaining such an order.

4. Solutions applied in sport

Two common systems of managing sports games are presently used all over the world. The former is the round-robin system (the peer-to-peer system) known as the English system. The other one is the knockout system. Also, various variants combining these two systems are applied.

In the round-robin system all possible pairs of participating teams are compared. Each pair of teams is compared a number of times set in advance. One of the disadvantages of the system is the need for making numerous comparisons. The number of comparisons is a multiple of $n(n - 1)/2$, where n is the number of teams. Thus, the computational complexity of this method is polynomial. Another disadvantage of this method is the fact that one cannot freely arrange the number of all planned comparisons during the process of sorting. This method is also ineffective because teams that differ considerably are compared unnecessarily often. Another drawback concerns the impossibility of a precise, that is to say, frequent comparison of teams that are similar. It is the result of the comparison of such teams that is often incorrect and needs checking several times.

The knockout system appears in several variants. In the basic variant, a team that has lost a match is eliminated from games. Thus, after each sequence of matches, only half of the teams remain. The team that does not lose any match becomes the winner. The number of comparisons made equals n–1. Various combinations of the round-robin system and the knockout system are used. One disadvantage of a simple knockout system is the fact that it does not allow arranging all the teams yet merely determines a winner. Besides, this system assumes that the probability p_{ij} is constant and equals 1.

In chess competitions, the Swiss system is used. The drawback of this system for ranking chess players concerns the fact that it does not guarantee the possibility of more precise, that is, more frequent comparisons of players that have similar qualities, since two players can be compared only once.

5. Algorithm for probabilistic sorting

The proposed algorithm of probabilistic sorting has the following properties.

1. Sorting is done in stages. At each stage objects are placed in a sequence. Neighbouring objects are compared, i.e. the first one with the second one, the third one with the fourth one. The first arrangement of objects, the so-called starting arrangement, may be made at random.

2. After each comparison, a new arrangement of objects in the sequence is made. It is established in such a way that if $d_i \angle d_j$, then the object d_j stays in its place or moves up the sequence, whereas the object d_i stays in its place or moves down the sequence.

3. Every object has its own level. Thanks to this fact, objects can obtain a differing number of points according to the results of a comparison at this level. At the beginning, all the objects are placed on the same level, which is the lowest one.

4. After each comparison, objects move to new levels. They move in such a way that if $d_i \angle d_j$, then object d_j moves to a level not lower than its current one, whereas the object d_i moves to a level not higher than its current one (according to this procedure, objects will change levels at various stages of the algorithm).

5. After each comparison, objects gain points. If $d_i \angle d_j$, then d_j gains more points than d_i. Additionally, more points are gained by objects that are at higher levels than those at lower levels.

Many variants of this algorithm can be constructed in accordance with the above rules. If the number of objects is odd, the sequence should be completed with a virtual object of a quality that is smaller than any of the qualities of the teams.

A formal definition of the algorithm for an even number of objects is given below.

The present sequence of objects is put into pairs which are numbered 1, 2, …, $n/2$. Let a_i^k denote the number of the pair containing object d_i in the k-th sequence.

$a_i^k \in \{1, 2, 3, ..., n/2\}$ is the number of the pair in the sequence containing object d_i,

$k \in \{1, 2, 3, ..., K\}$ is the number of the sequence,

$i \in \{1, 2, 3, ..., n\}$ is the number of the object.

For each k, $(a_1^k, a_2^k, ..., a_n^k)$ is a permutation of the elements of the set {1, 1, 2, 2, 3, 3, …, $n/2$, $n/2$}.. If in the k-th series of comparisons, the two objects d_i and d_j are compared (i.e. $a_i^k = a_j^k$) and it was established that $d_i \angle d_j$, then the position of these objects in the next sequence satisfies the condition

$$1 \le a_j^{k+1} \le a_j^k = a_i^k \le a_i^{k+1} \le n/2.$$

One of M levels is associated with each pair of objects. Let b_i^k denote the level of the pair containing object d_i in the k-th sequence:

$b_i^k \in \{1, 2, 3, ..., M\}$ is the level of the pair containing object d_i in the k-th sequence.

All the pairs in the first sequence have the lowest level (level 1). In all the following sequences the levels are assigned to the pairs in such a way that if, in the k-th sequence, it turns out that $d_i \angle\ d_j$ for two comparable objects, then

$$1 \le b_i^{k+1} \le b_i^k = b_j^k \le b_j^{k+1} \le M.$$

Beginning from a random sequence, after each set of comparisons of pairs of objects, the objects obtain points. The number of points obtained depends on the pair's level and the result of the comparison conducted in the way described above.

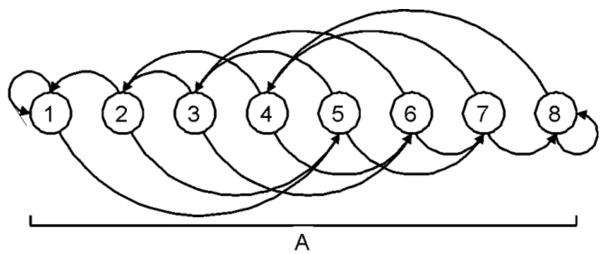

Fig. 2. The sorting algorithm – the first round

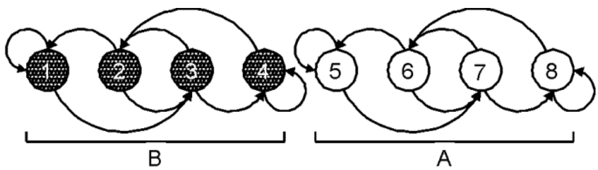

Fig. 3. The sorting algorithm – the second round

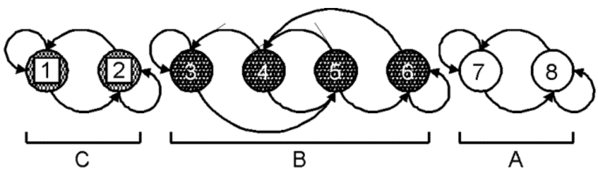

Fig. 4. The sorting algorithm – the third round

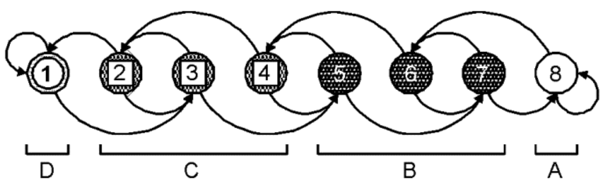

Fig. 5. The sorting algorithm – from the fourth round onwards

Figures 2–5 illustrate an example algorithm for 16 objects. In each sequence, the objects form eight pairs – a basis of arranging matches. Each pair is marked by a circle. The number of a circle stands for its place in the sequence and not for a particular pair of objects.

First, the starting arrangement of the objects is made, as well as comparisons of pairs in this first sequence. After eight comparisons based on this first sequence, eight objects that won and eight objects defeated are obtained. Then all the objects are arranged in the next sequence as shown in the figures. The upper curves indicate the movement of the objects which won in the previous round, while the lower curves indicate the movement of the objects which lost in the previous round. As can be seen in the figures, the winning objects move up the sequence, whereas the objects that lost move down the sequence. Thanks to this, objects of similar characteristics are more likely to be compared in the following rounds. It was originally assumed that a draw is out of the question. That is why, if a match ends in a draw, its result is decided by extra time or randomly.

Each object is on one of the levels: A, B, C or D. The lowest level is A and the highest one is D. At first, all the objects are on level A. This is shown in Fig. 2, which represents the first sequence. After the comparisons based on the first sequence, the objects which won move to the higher level B, while the objects which lost stay on level A. New levels are shown in Fig. 3, which represents the second sequence. Now, objects which won again move to a higher level, i.e. from A to B, or from B to C. The objects which lost remain on their levels. The new levels are shown in Fig. 4, which represents the third sequence. In the case of sequence three, the procedure is the same.

From the fourth sequence onwards, the procedure changes. Now only the level an object occupies will change. This occurs in the way shown in Fig. 5. For instance, if one looks at the objects in pair 1, they are on level D. After the objects are compared, the object that won stays in pair 1, whereas the one that lost moves to pair 3. Thus, a winning object stays on the same level or advances to a higher one. On the other hand, an object that lost remains on the same level or falls down to a lower level. Following rounds are held in the same manner as the fourth round. A “stop” condition is established at the beginning of the procedure.

After each round, objects get points for the results achieved. The number of points obtained depends on the level at which a given object is placed. An example point system is presented in Table 1. Objects placed on a higher level obtain more points than objects from lower levels. To obtain the final ranking, the points obtained by each object in all the rounds are added. The object that gets the most points is given first place.

Table 1. An example point scheme

The level of an object	Points for an object	
	Win	Lose
The highest – D	4	3
C	3	2
B	2	1
The lowest – A	1	0

In the conducted simulations, very good results were obtained when the objects were ordered after the fourth round. In the first three rounds, the objects competed simply to obtain a higher level. The number of comparisons required under the proposed algorithm for probabilistic sorting equals $n(t + \log_2 n–1)/2$, where t is the number of repetitions of the procedure shown in Fig. 5. It seems that it is sufficient that t is of order $\log_2 n$, thus the complexity of the algorithm is $O(n\log_2 n)$, which is the same as the mean complexity of the quicksort algorithm.

6. Theorem of neat systems

Let X and Y be pairs in a sequence. The notation X > Y will be adopted when the pair X is located higher than the pair Y, i.e. following the notation used in the example above the number of pair X is lower than that of pair Y.

6.1. Definition of a neat system

A neat system is a system where, for each X and Y, if X >Y, then in the next sequence the winner from pair X cannot be in a lower pair than the winner from pair Y. Similarly, the loser from pair X cannot be in a lower pair than the loser from pair Y. ■

The notation used is as follows:

S_1 – set of systems where, in each round, winners (from the previous round) can only compete with winners and losers (from the previous round) only with losers.

S_2 – set of systems where, for each two pair numbers X and Y, if the winners from the two pairs play each other in the next round, then the losers from these pairs also play each other in the next round.

6.2. Theorem on neat systems (winners – losers)

Premise: $x \in S_1$
Thesis: $x \in S_2$ ■

6.3. Proof (indirect)

Figure 6 presents three different fragments of the system where moves from any selected round (pairs X, Y, M) to the next round (pairs A, B, N) are marked. In the first round (enclosed within a broken line), the pairs situated to the left are higher than those to the right. According to the figure, the winners from pairs X and Y meet in the next round, but the losers from these pairs do not. Suppose the loser from pair X next plays the loser from any other pair M. The examples presented below take into account the three possible locations of M with regard to bases X and Y. It can be seen that each x such that $x \notin S_2$ and $x \in S_1$ presented as a system of graphs has to include at least one of the following three subgraphs.

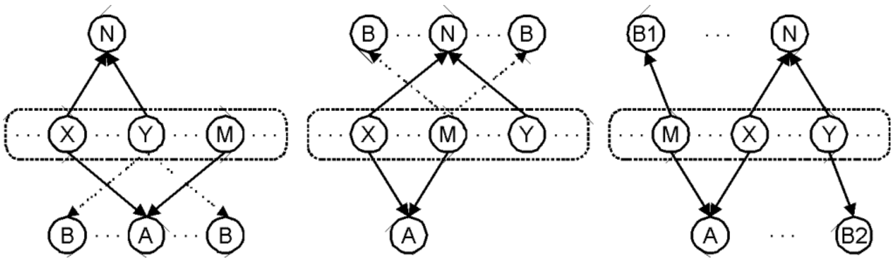

Fig. 6. Three possible subgraphs describing a system x, such that $x \in S_1 \wedge x \notin S_2$

It is enough to prove that a neat system $x \in S_1$ cannot contain any of the three presented fragments, i.e. $x \in S_2$.

For the first subgraph of the system from Fig. 6, pair Y is higher than pair M (Y > M), so, with regard to the neatness of the system, pair B should be placed higher than pair A (B > A). At the same time, as pair X is higher than pair Y (X > Y), pair A should be higher than pair B (A > B). We thus obtain a contradiction. This means that the neat system $x \in S_1$ cannot contain the first of the three subgraphs. Similarly, one can prove that the neat system $x \in S_1$ cannot contain the second of the three subgraphs.

It is also necessary to prove that an $x \in S_1$ neat system with the third subgraph is not possible. Even if the subgraph is extended by adding extra pairs, it cannot contain one that is isomorphic to either the first or the second subgraph (it must look like a "zigzag"). Thus, the number of pairs in a previous round must always be smaller than in a later one. This is impossible, as the number of pairs in each round is constant.

Hence, the losers of pairs X and Y have to meet in the next round, i.e. $x \in S_2$. ■

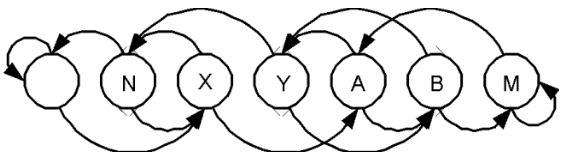

Fig. 7. An example of a neat system y such that $y \notin S_1 = S_2$

Thus, $S_1 \subset S_2$. Since one can prove that $S_2 \subset S_1$, then $S_1 = S_2$.

There arises the question of whether there are any neat y systems such that $y \notin S_1 = S_2$. If pair A can be joined by a winner from pair M and not a loser, as follows from the assumption of the theorem, then it is possible to create a system, e.g. the one shown in Fig. 7. In this system, a winner meets a loser in pair A, so it is an example of a neat system y such that $y \notin S_1 = S_2$. In the example above, each of the pairs: N, X, Y, A, B, M is different, but, in general, the pairs X, Y, M may coincide with some of the pairs A, B, N.

7. Conclusion

The paper presents an effective system for managing sports competitions. Such problems can be formulated as one of probabilistic sorting. The proposed method of organizing competitions is novel and has assets that other known and currently applied methods of sports competition management do not possess:

1. A competition can consist of a practically unlimited number of teams, so there is no need to play in leagues.
2. The number of sequences played in a season can be established in advance.
3. Each team plays the same number of matches in a season, as none of them are eliminated from the competition.
4. Each team takes a specific, individual place in the ranking, owing to the fact that pairs become more uniform after each round is completed. There are no 'groups of death', which are unfair.
5. Teams compete with each other on various levels. On these levels, constant competition concerning promotion to a higher level and defense against a fall to a lower level is in progress. Thus, the games are a source of entertainment.
6. Strong teams play more frequently with other strong teams, whereas weak teams compete more frequently with other weak teams. Therefore, there are less unnecessary matches to play. Moreover, each team competes with teams of a similar quality. Therefore, fewer matches that are unnecessary or uncompetitive are played in general.
7. There are several "big clashes" in each round. Such matches are played between top teams.
8. There is a chance that teams that have already played against each other will meet again. This is possible if their quality and number of points obtained are similar. In this way, only similar teams are compared on a relatively large number of occasions.
9. It makes corruption difficult, as matches are played between teams that are classified as being of a similar level. Furthermore, the result of each match is likely to be of great importance to the final ranking.

There are still numerous open questions. How is the probability of establishing the true ranking affected by the following:

- the starting arrangement,
- the variant of the algorithm presented,
- the number of games to be played,
- the form of the function from Fig. 1,
- the number of levels on which comparisons take place,
- the number of points obtained by each team.

The probability of establishing the true ranking regarding each of the above questions can be treated as the probability that the ordering of the teams made according to the sorting algorithm coincides with the ordering of their qualities (a global order) or as the probability that a specific team will be found in the appropriate position in the final ranking (a local order).

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