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GEOMETRICAL PERSPECTIVE ON ROTATION AND DATA STRUCTURE DIAGNOSIS IN FACTOR ANALYSIS

Abstract: Geometry has always contributed to a great extent and played a significant role in the development of many of the principles of the factor models. While factor-analytic principles and procedures have been generally developed by the heavy emphasis on matrix algebra, there is still a grave importance and need towards a geometrical approach and its application in the factor analysis. In this article the author provides, on selected issues, a description in reference to factor models from a geometric viewpoint with a discussion running through its advantages and disadvantages. Finally, at the end of the paper, conclusions in reference to good conditions of factors rotation are given. This article explains to what extent a geometrical approach brings specific value and offers an extra insight into factor analysis. As proved, geometry still provides an alternative framework which may be helpful for better understanding and data structure diagnosis.

Keywords: geometry, factor analysis, rotation methods.

1. Introduction

In the history of factor analysis, geometry has been used in the development of many principal factor models. The impact of geometry can be found in phrases such as "rotation of factors", and while factor-analytic principles and procedures are generally developed by matrix algebra, a brief overview of the factor models from a geometric viewpoint can greatly add to one's intuitive understanding. This is particularly helpful when we consider such problems as the number and placement of factors in the space [Gorsuch 1974; Skinner 1984]. In factor analysis the geometrical approach has a very special pedagogical value which offers an extra insight into some rather complex algebraic results. In the geometrical approach, where we have p random variables with finite variances, they will be represented as p vectors in p-dimensional Euclidean space, with the vectors' lengths being equal to the standard deviations of the variables and the cosines of the angles between the vectors being equal to their correlations.

2. The general idea, model and assumptions of factor analysis

In order to remind us what the idea and the general assumption hold in the factor analysis model, we need to start with the reconstruction of its well-known form:

$$\mathbf{X} = \mathbf{AF} + \mathbf{U}, \tag{1}$$

where **X** is matrix of observed variables, **F** is a matrix of common factors, and **U** is a matrix of unique factors defined on respective population. The matrix **A** is a *p* by *k* matrix of factor loadings. It is also assumed that the **F** and **U** have mean zero and remain uncorrelated, so the components **F** have variance one, and that the components of **U** are uncorrelated. More precisely [Jennrich 2007]:

$$\begin{aligned} E(F) &= 0, \\ E(U) &= 0, \\ \mathrm{Cov}(U_i, U_j) &= 0, \\ \mathrm{Cov}(F, U) &= 0. \end{aligned} \tag{2}$$

The covariance matrix **Σ** of **X** has the following structure:

$$\mathbf{\Sigma} = \mathbf{A\Phi A'} + \mathbf{\Psi}, \tag{3}$$

where **Σ** = cov**F**, **Ψ** = cov**U** and **Ψ** is diagonal.

If there are no constraints that affect **Σ** then we call it an *exploratory model*. On the other hand, if there are enough constraints to uniquely identify **A** and **Φ**, then we call it a *confirmatory model.* Models that are neither exploratory nor confirmatory do not seem to have a name. The two named models represent a major division in the empirical studies and application of factor analysis. Often an exploratory analysis is used to help formulate the next steps undertaken in the confirmatory analysis. In this article predominantly the issues in reference to exploratory analysis will be considered.

For exploratory analysis there are two important steps. The first depends on the estimation:

$$\mathbf{\Omega} = \mathbf{A\Phi A'} \tag{4}$$

and **Ψ** from a sample of **X** values. We call it an extraction. The second is the estimation of **A** and **Φ** from the estimate **Ω** and is therefore called rotation. The rotation problem is the major aspect of exploratory factor analysis.

Given now **Ω**, there are many **A** and **Φ** that satisfy the estimation (4), the ordinary approach to estimate **A** and **Φ** (that is also to the rotation problem purposes), is to find **A** that looks nice, or slightly more specifically has a simple structure. The main problem is: what does this vague statement mean? One case is clear. If each row of **A** has at most one nonzero element, **A** is said to have a *perfect simple structure*, an example of which is displayed in Table 1. The difficulty is that among all factorizations (4) which means there may not be **A** with a perfect simple structure and that is the usual case. Thurstone proposed a less demanding definition of simple structure. As we can observe, the second loading matrix in Table 1 has a *Thurstone simple structure.*

A Thurstone simple structure requires a fair number of zeros, but far fewer than a perfect simple structure. The complexity of a row of **A** is the number of nonzero elements in the row. A Thurstone simple structure allows for row complexities of 1, 2, 3, or more. As with a perfect simple structure, however, there may be no factorization (4) of **Ω** that has **A** with a Thurstone simple structure and this is the usual case. It may however be possible to find that **A** approximates a Thurstone simple structure or even a perfect simple structure.

Table 1. Example of perfect and Thurstone simple structure

Perfect			Thurstone		
1	0	0	1	0	0
1	0	0	0	1	0
1	0	0	0	0	1
0	1	0	.89	.45	0
0	1	0	.89	0	.45
0	0	1	0	.71	.71

Source: [Jennrich 2007].

Rather than attempting to define a simple structure, one might attempt to identify a simpler structure. That is, given two **A** decides on which is simpler. This might be done by just looking at them. Sometimes one is clearly simpler than the other. But also rather than just looking, one might consider the whole context of the problem. The trouble with this approach is that it does not tell us how to estimate **A** and **Φ**. This is usually done by choosing a rotation criterion, for instance, Q that assigns a numerical complexity $Q(A)$ to **A**. The **A** that satisfies (4) for some **Φ** and which minimizes Q, is the rotated value of **A** corresponding to Q. Unfortunately there are many choices for Q. A special version of the classical factor analysis model assumes the factors are uncorrelated. Hence, then **Φ** is an identity matrix and (4) becomes:

$$\mathbf{\Omega} = \mathbf{AA'}. \qquad (5)$$

This form is called the orthogonal factor analysis model. But when the factors are correlated, then it is called the oblique factor analysis model.

In the *oblique* approach, we have factors which are dependent on each other. In contrast, the *orthogonal* factors remain independent or otherwise uncorrelated with each other. In short, an orthogonal transformation depends on turning a matrix with factor loadings **A** into another **B**, which is made by the specific rotation of coordinate configuration in factorial space within the close range of starting coordinate point [Browne 2001].

In the practice of many market research projects, the most often applied technique is the *Varimax* rotation, which tends to purify the factors and to account for as much of the covariance in the data as possible, so as to make it easier to conceptualize the entire research domain. An algorithm to rotate the loadings was proposed by Kaiser [1958]. It is based on a criterion which maximizes squared variance in factor loadings for each factor, on a given number of factors and given communality:

$$V = \sum_{j=1}^{k} \frac{p\sum_{i=1}^{p} a_{ij}^4 - \left(\sum_{i=1}^{p} a_{ij}^2\right)^2}{p^2} \rightarrow \max \,, \tag{6}$$

where: k – the number of factors,
p – number of variables,
a_{ij} – factor loading i-th variable on j-th factor.

Another type of rotation, *Quartimax*, is quite similar to *Varimax*. However *Quartimax* is much more focused on simplifying the columns of a factor matrix. Generally it is considered to be less effective than *Varimax*. Technically speaking, *Q*-rotation maximizes squared variance in factor loadings for each variable (in contrast to *Varimax*), on a given number of factors and given communality that is leading to retain orthogonal factors [Gatnar 2003]:

$$Q = \sum_{i=1}^{p} \sum_{j=1}^{k} a_{ij}^4 \rightarrow \max \,. \tag{7}$$

The merits of the *Quartimax* and the *Varimax* solutions lie in the background where they simplify the rotational method by looking at it as a problem of maximization of a single criterion function. Their effects agree well with several empirical studies results which attained simple structure criteria through subjectively used methods. Moreover, the *Varimax* solution has generally been preferred to the *Quartimax* solution, as various studies reported [Kaiser 1958], however the former one satisfies the simple structure criteria better. But the *Varimax* solution, because it is based on the concept of factorial invariance (which is assumed to be the ultimate criterion), is not necessarily crucial for supporting the rotation solutions. For this reason *Varimax* may not satisfy all the requirements of an objective and analytic approach to the simple structure. It does not always satisfy the criteria of the positive manifold and the level contributions of all factors to the orthogonal rotation [Kashiwagi 1965].

If the restriction of orthogonality is relaxed, it will be impossible to apply directly the *Quartimax* or the *Varimax* criterion. This is because the interfactor relationships are not considered when the criteria are in this form, and when applied, all the factors will collapse into the same factor. However, Carroll's version of the *Quartimax* explicitly considers interfactor relationships and an oblique solution that is attainable. Assuredly, *Varimax* should not be used when there is a theoretical expectation of a general factor. Because *Varimax* serves to spread variance evenly among factors, it will distort any general factor in the data. That is why *Quartimax* is probably the orthogonal rotation procedure of choice when a general factor is expected [Carroll 1953].

Some other well-known rotation approaches are *Equimax* and *Biquartimax* . But the *Equimax* compromises either on *Varimax* or *Quartimax*:

$$\alpha Q + \beta V \rightarrow \max, \tag{8}$$

where: α and β – weights both criterions.

The simplified *Equimax* criterion can be stated as follows:

$$E = \sum_{j=1}^{k} \left[\left(\sum_{i=1}^{p} a_{ij}^2 \right) - \frac{\gamma \left(\sum_{i=1}^{p} a_{ij}^2 \right)^2}{p} \right] \rightarrow \max, \tag{9}$$

where: $\gamma = \dfrac{\beta}{\alpha + \beta}$.

And if by any chance, $\gamma = 0.5$, then we obtain the *Biquartimax* rotation.

In the oblique rotation techniques, these factors (as mentioned above) remain uncorrelated. Their objective is to identify factors correlated with other observed variables at the highest level, and on the other hand, correlated (with other variables) at the lowest level. In the *Oblimin* rotation type, the sum of squared correlation coefficient for variables is minimized (with axes perpendicular to the hyperplanes designated by axes fitted the variables – according to simple structure of factors) [Browne 2001; Aranowska 2005]:

$$O = \sum_{j<r=1}^{k} \left(p \sum_{i=1}^{p} a_{ij}^2 a_{ir}^2 - \gamma \sum_{i=1}^{p} a_{ij}^2 \sum_{i=1}^{p} a_{ir}^2 \right) \rightarrow \min, \tag{10}$$

where: a_{ij}^2 – loading for factor j,
a_{ir}^2 – loading for factor r.

The Oblimin procedure is based on: 1). *primary axes configuration* represented by oblique factors, fitted to variables, and 2). *reference axes*.

In the case of the *Quartimin* type rotation, the function will be minimized without γ.

$$Qu = \sum_{j<r=1}^{k} \left(p \sum_{i=1}^{p} a_{ij}^2 a_{ir}^2 - \sum_{i=1}^{p} a_{ij}^2 \sum_{i=1}^{p} a_{ir}^2 \right) \rightarrow \min. \tag{11}$$

Quartimin criteria have been used for both indirect and direct rotation. In the direct case, Jennrich [1979] has proved that for every $\gamma > 0$ there is an initial loading matrix **A** such that the *Oblimin* criterion with parameter γ is unbounded below over oblique rotations of **A** and as a consequence **Φ** (that equals *F*-factors) approaches singularity. The matrix of factor loadings **A** approaches infinity when attempting to minimize the criterion. Although there always exists such an initial loading matrix, this need not happen for a specific initial loading matrix. There is also known as the *Promax* rotation, which maximizes a simple structure while allowing the factors to become correlated [Gorsuch 1970].

The literature, though filled with factor analytic reports, provides few examples of oblique rotations. The orthogonal rotation techniques dominate, despite the strong likelihood that correlated factors and hierarchical factor solutions are intuitively attractive and theoretically justified in many scientific applications. The careful researcher should almost invariably perform both either an orthogonal or oblique rotation, particularly in exploratory works. These solutions can be compared to identify the simpler structure and to determine whether the oblique rotation produces a marked increase in the hyperplane count. Oblique solutions have been found particularly useful in the theory building of other disciplines (e.g. psychology, sociology), and are likely to play a significant role in the development of any theory, such as customer behavior or attitudes measurement [Stewart 1981].

3. Principles of factor-score indeterminancy

The most obvious consequence of indeterminancy is the prevention from the development of unique factor measures. As a result, factor analysis does not seem very useful in describing the individual subject. No doubt Guttman [1955] had this potential usage of factor analysis in mind when he remarked "that unless indeterminancy of factor scores was resolved, factor analysis should be abandoned as a scientific instrument". However, a description of the individual subject is not necessarily a major objective of factor analysis. Thurstone [1947] noted that "the individual subjects are examined, not for the purpose of learning something about them individually, but rather for the purpose of discovering the underlying factors".

The underlying factors are fewer in number than the observable variables and go beyond measures, in the sense that they do not tie in the variables space. Thurstone evidently conceived factor analysis as a tool for analyzing variables and its measures into more fundamental components, much as he analyzed box measurements into length, width, and height. The underlying factors suggest to the investigator the type and number of new measures which need to be reconstructed in order to depict what is common to a group of variables. Fortunately, an interpretation of underlying factors is untroubled by the indeterminancy since both of these processes are dependent only upon the factor matrix. Nevertheless, ambiguities are created by indeterminancy if one attempts to relate the underlying factors to variable measures not included in the original factor analysis. These outside measures occupy a unique position in *n*-space which is determined by their normalized score matrix, but factor vectors may occupy a wide variety of positions in *n*-space [Heermann 1969].

There is no doubt that the generality and scientific utility of the factor models would be enhanced if some meaningful method could be found to render factor scores determinate. It should be emphasized, however, that it is not advocated here to use the determinate factor model, i.e. a factor model which employs unities instead of communalities in the diagonals. Determinate factor solutions permit the unique calculation of factor scores, but the factors are always contained in the variables space and hence cannot be expected to represent anything which goes beyond the original measures. If we think of factor analysis as a tool for generating new measures which are more fundamental than our original measures, we have little choice except to retain the indeterminate factor model. However, to avoid the ambiguities of this model, it does seem wise to seek some rational criterion which would allow the investigator to select a particular set of factor scores from the infinite set of factor scores which correspond to a given factor solution. The criterion for resolving this indeterminancy would be more complex than the simple structure criterion for resolving factor matrix indeterminancy because it must remove indeterminancy of total factor space by specifying *r* – additional measures to supplement the test-score matrix, and it must remove rotational indeterminancy by specifying values[1].

4. Geometrical perspective on factor analysis

We may geometrically take a look at the factor models which will be represented by the space including variables and their factors as vectors. Geometry helps us to describe the relationship and provides significant information on how the factors (rotated or unrotated) are made. A geometrical solution to the basic problems associated with factor analysis has therefore two major premises. It explains why the graphical perspective of multivariate factor analysis makes up a significant element of procedure which assists and leads the whole analysis to the end. Secondly, any problems (appearing in the factor analysis pertaining to the relationship analysis) are detected and resolved with some graphical data visualization, provided we choose a preferable configuration on data [Yule 1897]. However the choice is always made by humans, which means the geometrical perspective of looking and finding the relationship is still more biased by our subjectivity.

In its technical understanding of the problem, the presentation of the correlation coefficients between observed variables and factors usually take place in

[1] While the determinate factor analysis is seen as less desirable than indeterminate factor analysis, the results of research conducted by Heermann [1969] suggest that the two types of factor solutions may not be so distinct as once thought. Since a determinate factor solution for *n* measures would at most contain *F* factors, it is clear that we cannot transform these factors into a set of linearly independent communality factors. However, a determinate factor solution for measures can be transformed into a communality factor solution for *n* of these measures [Ledermann 1938].

n-dimensional space (assuming that space dimension is defined by number of observations in matrix R). On the other hand, subspace can be reduced to k-dimensional ($k < p$) space (where p denotes collection of vectors in n-dimensional space containing vector-variables X_i (i = 1, 2, …, p). Observable variables, being correlated with each other, should be set on a vector, that is, a sort of segment of the straight line with – denoting ab. Its absolute value length is termed with |ab| symbol. We denote length of vector as h and i as a variable being presented on the vector according to its length h_i and another variable on h_k (length of vector for i-th variable). The length of the vector is computed from the Pythagorean theorem, where the square of the length of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides [Mendelson, Vershynin 2004]. As a result, factors are identified as the axes and the variables as lines, or vectors, drawn in what is called a Cartesian coordinate space. When the plot shows the variable to be physically close to a factor, then they are strongly related. The direction and length for the line representing the variable is determined by the factor pattern of the variable and the factor pattern gives the coordinates of each line's end point [Gorsuch 1974].

Then, plotting variables and factors in joint space can proceed in two ways. On the one hand, the variables can be plotted as a function of the factors. This procedure can be used to represent the multivariate linear model in which variables, factors and weights are all known. On the other hand, the variables can be plotted alone. Factors and weights are then determined as additional steps. If one decides to present variables as vectors themselves, the correlation between them equals a product scalar of two vectors, which is the product of absolute values in either vectors by the cosinus angle between vectors. To put it simply, the correlation coefficient between two variables equals the total (accumulated) length of two vectors multiplied by cosinus angle length between them [Thurstone 1947]. Otherwise it can be explained respectively as: r_{ik} = hihk cos Φ_{ik}, provided both vectors are made up of the same length, $h_i = h_k = 1$. Otherwise it can be also stated as follows:

$$r_{12} = h_1 h_2 \cos a_{12}, \tag{12}$$

where: r_{12} – correlation coefficient between variables 1 and 2,
h_1 – length of vector representing variable 1,
h_2 – length of vector representing variable 2,
a_{12} – angle between vectors in variables 1 and 2.

Now if we, for instance, assume that both h_1 and $h_2 > 0$ at $r_{12} = 0$, then $a_{12} = 90°$ and respectively for negative $r_{12} = –0.60$, gives $a_{12} > 90°$ or even $a = 180°$. Analogously for positive $r_{12} = +0.60$ the cosines angle between vectors can be $\alpha_{12} \geq 0^{\circ}$ and $\leq 90^{\circ}$. Both vectors will be expressed in unit length (which means they represent total variance), therefore $h_1 = 1$ and $h_2 = 1$. This implies product scalar in either vectors to be equal cosines angle between them: $r_{12} = \cos a_{12}$. For example, if $a_{12} = 45°$ then we obtain $r_{12} = 0.707$.

The process of placement points in geometrical configuration according to their specific vectors arranged between two axes begins, as mentioned, with the correlation estimation on hypothetical variables (e.g. two variables) and then the transformation correlation scores into vectors. In fact, the more variables come into data analysis, the more vectors come out onto geometrical space of vectors and the more complex geometrical presentation such data becomes [Rusnak 1999].

5. Some problems related to the factors rotation

In orthogonal models, the factors are placed in space, i.e. each factor will be represented in hyperspace by a unit length vector. The correlation matrix should represent the cosines between the vectors directly. In as much as they do not, the model fails to account for the data. In virtually most cases of factor analyses, unit length vectors are not preserved. But an attempt is made to represent the variables in a sufficient number of dimensions so that the reduction in vector length is minor. Determining the exact number of dimensions which will produce this “next to best” situation is the problem of determining the number of factors. When the variables have been plotted, the factors can then be added as deemed best as long as they are all orthogonal. Usually, a factor is placed near the center of a cluster of variables so that it can be identified by the conceptual central thrust of the variables. But once several factors have been added, they need not remain in the same position. Instead, they can be shifted or rotated to any position for which the investigator can present an appropriate rationale [Cattell 1944].

In oblique rotation (representing correlated factors), the general Cartesian coordinate system is used instead of the rectangular Cartesian coordinate system. The former allows the axes (or factors) to form an oblique angle and thus they are correlated. The latter allows only orthogonal factors which are therefore uncorrelated. In the general Cartesian system, the correlation between two factors plotted in hyperspace is still defined by the assumption (12). Since both factors are defined as unit length:

$$r_{12} = \cos a_{12}, \tag{13}$$

Let now the cosine of the angle be equal to the correlation between two factors. Since the angle between factors is oblique, they are referred to as *Oblique* factors. It should be noted here that the plotting of the vectors which represent the variables is identical to that in the uncorrelated factor model. Each correlation coefficient is converted to a cosine and, if two dimensions are adequate, the variable vectors are plotted in a plane. The use of correlated coordinates does not alter the configuration of the variables.

The problems with obtaining the best solutions in factor analysis depend heavily on determining the transformation matrix and finding an initial orthogonal solution. Two restrictions are placed on the transformation matrix. The first (and this is essential) is that the transformation matrix must be nonsingular. If not, the common factors space will collapse. Second (and this is for convenience only), the transformation matrix must be scaled such that the derived factors have unit variance. For derived uncorrelated factors, the restriction is that the transformation matrix must be orthonormal, which guarantees the imposition of the first two restrictions. When viewed in this way, the problem of developing derived oblique factors (which often proved to be quite troublesome, primarily because the transformation matrix is allowed to be nonorthogonal) is usually difficult to keep them from becoming singular [Harris, Kaiser 1964].

Finally, the more preferable entry of reference axes to vectors configuration, the better factor-solution, is obtained. Hence, depending on the coordinate system position (which is sometimes set on an arbitrary basis by the researcher) one can obtain a good or bad representation of factor loadings being the extension of variables vectors' projections on factor-axes. Sometimes if vectors create approximately one cohesive group in a space, then across such a group, a single factor will be drawn [Carroll 1953; Kaiser 1958].

6. Conclusions and specific rules in determining satisfactory levels of rotations

At the conclusion of this article it is worth mentioning five core principles of good rotational approach to factor analysis. Most of them are equally applicable to either orthogonal or oblique axes solutions:

1. Rotation to agree with factors from past factors analyses

This procedure, which has been widely resorted to, especially in the final stages of a rotation, consists in rotating until as many as possible of the factors agree with the factors previously established in independent researches. The factors of earlier researches have sometimes been established as single general factors, by concentrated research in one particular field (e.g. intelligence).

2. Rotation to put axes through the center of clusters

This may be done either by picking out the outstanding correlation clusters in the original correlation matrix, or by considering the clusters which exist in the projection on a single plane when the number of factors is known and plotted. In general, if there are two factors operating fairly evenly in a certain matrix, the noticeable correlation clusters are likely to occur in the regions of overlap of the two factors. In these regions the shared variance (communality) is higher. Such comparatively even distribution of loadings is likely to occur when the total variance is accounted for by a considerable number of factors. Clearly in such circumstances, a cluster is more likely to represent a region of overlap of several factors than the region of strong influence of one factor. On the other hand, with one or two factors, the high points (clusters) of the matrix may well be the variables best defining the factors. For example, in a matrix satisfying the two-factor theory we put the axis through the center of the most highly inter-correlating bunch of variables. However, since both possibilities exist, there is no guarantee that a salient cluster is anything more stable than a province of overlap of two or more real, functional factors.

3. The principle of orthogonal additions: rotation to agree with successively established factors

In an *n*-dimensional orthogonal system, if the position of $n - 1$ axes is known from previous sources of evidence, the position of the *n*-th axis is automatically established. One can begin therefore with variables which, apart from specific factors, measure only the known factors, or even only a single known factor. By trial and error, guided by the researcher's insight, one then attempts to add variables to the matrix which will introduce, apart from specifics, only one new factor. When the new factor is determined, a further set of variables can be added introducing another new factor, the position of which, in turn becomes fixed by the earlier factors. In this way, starting with one factor of known position, it should be possible, theoretically, by successive additions to fix the rotation of a most complex multi-dimensional factorization. Indeed, in a relatively inexplicit and unplanned fashion, this principle has been employed in practical research problems, as the history of the establishment of the factors during the past twenty years shows.

4. The principle of expected profiles: rotation to produce loading profiles congruent with general researcher's expectations

It is possible that on general psychological grounds, one could validly conclude that certain kinds of sources, e.g. traits, should manifest certain general forms of factor loading pattern in certain batteries of variables. One would then rotate so that the maximum number of factors would give loading profiles, i.e. factor patterns of the kind required. According to this principle, therefore, one would rotate to get profiles of loadings having a relationship to the nature of the source traits as shown by the nature of the trait elements in which the factor tends to appear most persistently.

5. The principle of parallel proportional profiles

This begins with the same general scientific "principle of parsimony" which forms the premise for Thurstone's simple structure, but arrives at a different formulation of the meaning of the principle in the field of factor analysis. The principle of parsimony, it seems, should not demand "which is the simplest set of factors for reproducing this particular correlation matrix?" but rather "which set of factors will be most parsimonious at once with respect to this and other matrices considered together?" This parsimony must show itself especially when the correlations emanate from many diverse fields of observation. The criterion is then no longer that the rotation shall offer the fewest factor loadings for any one matrix; but that it shall offer the fewest dissimilar (and therefore the fewest total) loadings in all the matrices together.

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UJĘCIE GEOMETRYCZNE W ANALIZIE CZYNNIKOWEJ – METODY ROTACJI I DIAGNOZA STRUKTURALNA DANYCH

Streszczenie: W niniejszym artykule autor opisuje zagadnienia związane z analizą czynnikową z perspektywy geometrycznego ujęcia wyników badań w modelach czynnikowych. Brane są pod uwagę zalety i wady stosowania poszczególnych metod rotacji. W pracy wyjaśniono także, w jakim stopniu podejście geometryczne może zmieniać ostatecznie wyniki prowadzonej analizy, a tym samym wygenerowaną wartość z badań. Geometryczne ujęcie modeli czynnikowych zapewnia badaczowi alternatywne podejście w ramach lepszego zrozumienia i docelowej diagnozy struktury danych.

Słowa kluczowe: geometria, analiza czynnikowa, metody rotacji.