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Upconversion fluorescence and laser emission in the visible under Ti:sapphire laser pumping at 800 nm in the ZBLAN:Er\textsuperscript{3+} optical fiber

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Infrared-to-visible upconversion in rare-earth doped optical fibers is a very interesting phenomenon, but its application in building a practically useful laser requires much more experimental work. In this paper, infrared-to-visible upconversion fluorescence observed perpendicularly to the fiber and laser emission in ZBLAN:Er\textsuperscript{3+} fiber under 800 nm excitation from a Ti:sapphire laser are described. Absorption at the pump wavelength, threshold power pumping density, and output power dependence vs. fiber length have been measured. The output power in the green (544 nm) amounts to 22 mW. Important technical details of the experiment are described.

**Keywords:** fiber laser, upconversion fluorescence.

1. Introduction

Infrared-to-visible upconversion fluorescence and laser emission in optical fibers have been intensively studied since 1989 [1], [2]. A review article of Joubert [3] presents a summary of the results obtained up to 1999. Later on, the number of papers dealing with lasers operating on the upconversion principle slowly decreased in time. To our knowledge, no lasers of that kind are now commercially available. There are several reasons for this. Let us mention only a few of them. Upconversion lasers are not very stable, the wall-plug efficiency is very low (only several percent). The main difficulty in constructing a Fabry–Perot (FP) cavity lies in the attachment of the fiber terminals to the dichroic mirrors. To overcome this problem one can evaporate these mirrors directly onto the fiber ends. However, to ensure cleanliness of the mirrors they have to be kept in vacuum! The point is that the core diameter (several \(\mu\)m) is of the order of a typical dust particle, commonly present in the air. Attaching the fiber ends directly to the mirrors (butt coupling) removes this problem, however, the optical contact must
be almost perfect and stable in time. We have applied this method by pushing slightly
the fiber terminal to the mirrors and then the fiber was mechanically clamped.

We have studied the infrared-to-visible fluorescence and laser emission at 544 nm
under 800 nm pumping.

2. Technical details of the experiment

The ZBLAN:Er$^{3+}$ optical fiber used had the following parameters: erbium doping –
580 ppm, core diameter – 10 µm. The fiber was multimode at both pump and laser
wavelength. Pumping source: Ti:sapphire laser, Tsunami-Spectra Physics, operating
at 800 nm with maximum output power of 600 mW. The pumping beam was focused
onto the fiber terminal using a Thorlabs AR coated lens of $f = 7.5$ mm. The launching
efficiency obtained was quite satisfactory (73%). The entrance AR coated dichroic
mirror transmits almost totally the pump beam, and reflects, also almost totally, the
green radiation at 544 nm. The output mirror reflects back to the fiber the pump power
that has not been absorbed, and transmits a fraction of the generated beam. In our case,
we have used output mirror with 25% transmission for the green beam.

3. Experimental results

The experimental arrangement used is shown in Fig. 1. The ZBLAN:Er$^{3+}$ fiber was
85 cm in length in this case.

The green fluorescence observed perpendicularly to the fiber, below and above the
threshold for laser emission is shown in Fig. 2. Its overall shape does not change with
increasing pumping power and when crossing the threshold for laser emission. A large

![Fig. 1. Infrared-to-visible upconversion fiber laser at 544 nm pumped by Tsunami Spectra Physics Laser at 800 nm.](image)
amount of green radiation is not being kept within the fiber core and it is purely spontaneous.

The parasitic fluorescence at 850 nm clearly seen when the fiber is pumped at 800 nm causes the emission at 544 nm to be strongly saturated. Lasers and amplifiers at this wavelength were studied by Whitley et al. [4] and Millar et al. [5]. Competition between generated beams at 544 nm and 850 nm was first observed by Allain et al. [6]. The spectrum of the pumping beam and fluorescence at 544 nm and 850 nm is shown in Fig. 3 (no output mirror in place). In the case of laser operation, the mirror transmits 25% of the green beam, and only a very small fraction of the 800 nm and 850 nm beams. Of course, this mirror does not prevent oscillation at 850 nm. According to Allain et al. [6], pumping at 970 nm removes the undesired 850 nm lasing. Unfortunately, our Tsunami laser cannot be tuned to that wavelength.

The decrease in pumping power along the fiber length, $P = P_0 \exp(-\alpha l)$, was calculated with measured $\alpha$ (at the pump wavelength) equal to 0.004 cm$^{-1}$. The output power of the laser strongly depends upon the fiber length (see Fig. 4). Absorption of the pump power within the fiber core does not explain totally this dependence, because the threshold for laser emission is kept up to the end of the fiber, when, of course, the input pumping power is strong enough (e.g., $P_{\text{input}} = 3P_{\text{threshold}}$). Output power in the green amounts to 22 mW when pumped at 800 nm with 408 mW input power. So, the efficiency is very low and amounts to 5.4%, however, we did not optimize the system. The drawback of green radiation and oscillation at 850 nm, as observed by Allain et al. [6] may be partially responsible for this length dependence. Careful examination of the optimum output mirror transmission and fiber length can yield better efficiency. In an earlier paper of the present author [7], we obtained maximum...
Fig. 3. Spectrum of the pumping beam at 800 nm, and fluorescence at 544 nm and 850 nm, measured at the fiber end, with no output mirror in place (dashed line). Laser emission at 544 nm with output mirror of 25% transmittance at that wavelength. The mirror reflects almost totally the beams at 800 nm and 850 nm (solid line).

Fig. 4. Output power in the green as a function of the launched pumping power (squares) and the fiber length (triangles). A decrease of the pumping power along the fiber length due to absorption (circles).
laser power with no output mirror in place. The ZBLAN:Er$^{3+}$ optical fiber (1000 ppm of Er$^{3+}$ in weight) had a core diameter of 1.9 µm in this case. In another paper of Kaczmarek et al. [8] we had studied the kinetics of the fluorescence and laser emission, and found that the excited state absorption (ESA) is fully responsible for the process involved. Further experiments with pumping at 970 nm, and with dichroic mirrors evaporated directly onto the fiber terminals are now in progress.

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References


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Femtosecond laser-induced damage in dielectrics

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We present a new method to investigate the ablation phenomenon by a 100 fs, 1053 nm Gaussian laser pulse in fused silica and describe the different mechanisms of ablation in long pulse and ultrashort pulse lasers. A modified rate equation is used to numerically calculate damage in dielectrics. In addition, we examine the respective role of ionization and avalanche ionization in femtosecond laser-induced damage. We find that present results are in quantitative agreement with those of earlier study.

Keywords: femtosecond laser, ablation mechanism, damage threshold, electron number density.

1. Introduction

Laser-induced breakdown and damage in transparent dielectrics have been investigated extensively since the advent of high-power pulsed-lasers [1]–[5]. These lasers are uniquely characterized by their ultrashort pulse duration (from 10 fs to 1 ps) and extremely high intensity (10^{12}–10^{15} W/cm^2) [6]. Optical breakdown by ultrashort pulse laser in dielectrics presents an efficient method to deposit laser energy into materials that otherwise exhibit minimal absorption at low laser intensity. Compared with their relatively long-pulse counterparts, ultrashort-pulse lasers have many advantages for laser material processing, including negligible heat diffusion effects, absence of the liquid phase during material removal, minimal plasma absorption, and smaller laser fluences for processing, which makes them capable of producing high-quality features with high spatial resolution [7]. Additionally, in recent years, the structural alternations produced in transparent materials by ultrashort lasers have been used for micromachining, thin-film formation, and bioengineering applications. Understanding the mechanisms for optical damage may allow high-damage-threshold optics to be constructed for ultrashort laser systems.

Laser-induced breakdown results in damage to dielectrics in terms of three major processes: i) the excitation of electrons in the conduction band by impact and multiphoton ionization, ii) heating of the conduction band (henceforth free) electrons by the radiation, and iii) transfer of the plasma energy to the lattice [1], [4], [8], [9].
In a transparent dielectric, there is no linear absorption of the incident laser light, two nonlinear mechanisms are responsible for optical breakdown and material damage: photoionization and avalanche ionization [10]. These are two different mechanisms that play a role in this absorption; by these two nonlinear mechanisms, laser energy is deposited into the material by promoting electrons from the valence band to the conduction band. If the laser intensity is strong, a very high free electron density, \(i.e.,\) a plasma, of the order of \(10^{21} \text{ cm}^{-3}\), is produced. This high density plasma strongly absorbs laser energy by free-carrier absorption, the reflectivity of the plasma at the critical density is only a few percent, the shock-like energy deposition leads to ablation.

Optical breakdown threshold is associated with a threshold electron number density \(n_{th}\) and can be predicted by solving the electron number density rate equation. Research on electron number evolution induced by ultrashort laser pulses has been reported [1], [5]. In order to describe the ablation phenomenon accurately, we decided to present a new model which includes a reduction in the number of free electrons due to diffusion and recombination to determine the evolution of the free electron number density in dielectrics medium exposed to laser pulses of 100 fs. Results based on electron production via photoionization, avalanche ionization and loss via diffusion and recombination are in quantitative agreement with earlier studies [1], [5], [11], demonstrating that we present an effective method to determine the time-resolved electron number density and predict damage threshold.

2. Electron number density evolution

The present work modifies the rate equation model based on Stuart et al. [1], [5] by taking electron diffusion and recombination into account and use the expressions which are different from formulas of Stuart et al. [1], [5] to describe evolution of the conduction band electron number density.

2.1. Nonlinear photoionization

Photoionization refers to direct excitation of the electron by laser field. Because a single photon of visible light does not have enough energy to excite an electron in a transparent material from the valence band to the conduction band, so nonlinear ionization occurs due to the simultaneous absorption of several photons by an electron, the ionization rate describes the probability for multiphoton absorption [10]. During multiphoton ionization, the number of absorbed photons simultaneously by an electron depends on the ionization potential of the material \(\delta\) and the photon energy \(\omega\), the smallest number of the absorbed photons \(k\) satisfying \(k\omega \geq \delta\). In this regime, ionization rate depends strongly on laser intensity. Researchers used multiphoton ionization rate which was described as follows [1], [4], [5]:

\[
W(I)_{\text{MPI}} = \sigma_k I^k
\]
where $\sigma_k$ is the multiphoton absorption coefficient for absorption of $k$ photons, $I$ is the laser intensity. This is one case of photoionization which is at high laser frequencies limit (but still below that required for single photon absorption). While in the opposite limiting case, i.e., low frequency and strong fields, photoionization describes the tunnel effect [8]. In the tunneling ionization regime, the electric field of laser suppresses the Coulomb well that binds a valence electron to its parent atom. If the electric field is very strong, the Coulomb well can be suppressed enough for the bound electron to tunnel through the short barrier and become free [10]. The tunneling rate scales more weakly with the laser intensity than the multiphoton ionization rate. Researchers [1], [4], [5] also point out that the multiphoton ionization term should be replaced by the tunnel ionization expression in a field stronger than $\sim$100 MV/cm. Nevertheless, calculations including tunnel ionization have not been presented.

For a 1053 nm Gaussian laser pulse, the temporal pulse shape has the form:

$$I(t) = I_0 \exp \left(-\frac{4 \ln 2 \tau^2}{\tau^2} \right)$$  \hspace{1cm} (2)

where $\tau$ is the full width at half-maximum (FWHM) pulse duration. In this case, eight-photon absorption is the relevant process, but eight-photon absorption cross-section values were not available. So, Stuart et al. [5] used evaluation of Keldysh’s expression for the multiphoton absorption rate:

$$W(I) = 9.52 \times 10^{10} \times I^8 \text{ cm}^{-3} \text{ ps}^{-1}$$  \hspace{1cm} (3)

where the intensity $I$ is in TW/cm$^2$.

In order to describe the photoionization rate accurately, we use Keldysh’s photoionization expression [8]

$$W_{p1}(E) = \frac{2 \omega}{9 \pi} \left(\frac{\omega m}{\sqrt{\gamma}}\right)^{3/2} Q(\gamma, \gamma) \exp \left(-\pi \gamma + 1\right) \frac{\kappa(\gamma_1) - E(\gamma_1)}{E(\gamma_2)}$$  \hspace{1cm} (4)

where $\omega$ is the laser frequency, $m$ is the reduced mass, $\kappa$, $E$ are complete elliptic integrals of the first and second kinds, respectively, and

$$\gamma = \frac{\omega \sqrt{m \delta}}{eE},$$

$$\gamma_1 = \frac{\gamma^2}{1 + \gamma^2},$$

$$\gamma_2 = \frac{1}{1 + \gamma^2},$$
\[ Q(\gamma, \chi) = \sqrt{\frac{\pi}{2\kappa(\gamma_2)}} \sum_{n=0}^{\infty} \exp\left\{ -n \frac{\kappa(\gamma_2) - E(\gamma_2)}{E(\gamma_1)} \right\} \Phi\left\{ \frac{\pi}{2} \sqrt{\frac{2(\chi + 1) - 2\chi + n}{\kappa(\gamma_2)E(\gamma_2)}} \right\}, \]

\[ \chi = \frac{2}{\pi} \frac{\delta}{\omega} \frac{\sqrt{1 + \gamma^2}}{\gamma} E\left( \frac{1}{1 + \gamma^2} \right), \]

\[ \Phi(z) = \int_{0}^{z} \exp\left( y^2 - z^2 \right) \, dy, \]

and \( \lfloor z \rfloor \) – the integral part of the number \( z \).

Figure 1 shows the general trend for the photoionization rate predicted by Keldysh’s model as a function of electric field for a wavelength of 1053 nm and 800 nm.

![Figure 1](image1.png)

Fig. 1. Typical photoionization rate as predicted by Keldysh’s model for a wavelength of 1053 nm (solid) and 800 nm (dotted).

![Figure 2](image2.png)

Fig. 2. Time dependence of Keldysh’s photoionization rate (solid) and multiphoton ionization rate of STUART et al. [1], [5] (dotted) for a 1053 nm, 100 fs Gaussian laser pulse (dash).
800 nm. Time dependent Keldysh’s photoionization rate and the multiphoton ionization rate based on the model of STUART et al. [1], [5] for a 1053 nm, 100 fs Gaussian laser pulse are presented in Fig. 2.

2.2. Avalanche ionization

Avalanche ionization is initiated when free electrons absorb laser energy through inverse Brehmstrahlung followed by impact ionization. Initial seed electrons originating from impurities or generated by ionization absorb several laser photons sequentially, moving to higher energy states in the conduction band [10]. During inverse Brehmstrahlung, seed electrons absorb laser photons by favourable collisions with other electrons and ions. If the electrons sustain enough favourable collisions, they will eventually gain sufficient energy for impact ionization of other electrons, freeing new electrons to repeat the process. This avalanche ionization process results in a geometric increase in the free-electron density [12], [13].

STUART et al. [1], [5] suggest linear scaling of the avalanche rate with laser intensity, i.e., $\eta = \alpha I$ ($\alpha$ – the avalanche ionization coefficient), as shown in Fig. 3. Some researchers [3], [14] have called this model into question. The linear relationship between the avalanche rate and the laser intensity is the consequence of two major assumptions:

- flux doubling: an electron in the conduction band impacts ionization of an electron in the valence band as soon as it has enough energy to do so. In other words, there are no electrons in the conduction band with energy higher than the conduction band minimum plus the band-gap energy (at least until the material is fully ionized, after which further electron heating can occur);
- unchanged shape of electron distribution: the energy distribution of electrons in the conduction band does not change shape as the electron number density grows.

However, studies based on Monte Carlo methods conclude that in both semiconductors [15] and wide-gap materials [16], [17], the shape of the electron distribution is a

Fig. 3. Intensity dependence of avalanche rate based on Thornber’s impact ionization (solid) and the model of STUART et al. [1], [5] (dotted).
function of the electric field and electrons can be found with energy greater than the ionization energy. Higher fields cause longer high-energy tails in the electron distribution. Therefore, the two assumptions are violated in a strong electric field.

We use Thornber’s expression [18], [19] to describe the avalanche process which is applicable for all electric field strengths:

\[
\eta(E) = \frac{v_{\text{drift}} e E}{\delta} \exp\left\{ \frac{E_I}{E(1 + E/E_{\text{phonon}}) + E_{KT}} \right\}
\]

Fig. 4. Impact ionization rate based on Thornber’s formula (solid) and avalanche rate of STUART et al. [1], [5] (dotted).

Fig. 5. Time dependence of Thornber’s impact ionization rate (solid) and avalanche rate of STUART et al. [1], [5] (dotted) for a 1053 nm, 100 fs Gaussian laser pulse (dash).
where $v_{\text{drift}}$ is the saturate drift velocity. We denote $E_{KT}$, $E_{\text{phonon}}$, $E_I$ as threshold fields for electrons to overcome the decelerating effects of thermal scattering, optical phonon scattering, and ionization scattering in one mean free path, respectively. Figure 3 depicts intensity dependence of the avalanche rate based on Thornber’s impact ionization and the model of Stuart et al. [1], [5]. Impact ionization rate based on Thornber’s formula and avalanche rate of Stuart et al. as a function of electric field are plotted in Fig. 4. Figure 5 shows time dependence of Thornber’s impact ionization rate and avalanche rate of Stuart et al. for a 1053 nm, 100 fs Gaussian laser pulse.

**2.3. Electron loss**

As free electrons are formed in the conduction band through photoionization and avalanche ionization, two processes will contribute to reduction of electron population: diffusion and recombination.

The diffusion rate $\eta_{\text{diff}}$ is given by [20]:

$$\eta_{\text{diff}} = \frac{\tau_0 \delta}{3m} \left[ \left( \frac{2.4}{w_0} \right)^2 + \left( \frac{1}{z_R} \right)^2 \right],$$

where the Rayleigh range

$$z_R = \frac{n_0 \pi w_0^2}{\lambda_0},$$

where $\omega_0$ – the beam waist, $m$ – the rest mass of an electron, $\tau_0$ – the mean free time between collisions, $\lambda_0$ – the laser wavelength in free space and $n_0$ – the refractive index of the medium. We use a Gaussian laser pulse for $\lambda = 1053$ nm with a beam waist $w_0 = 20$ $\mu$m irradiated on fused silica. This term is strongly dependent on the size of the focal volume, a small focal volume leads to higher electron diffusion. Unfortunately, accurate values of $\tau_0$ have not been measured for most materials. Though, a value of 1 fs has been estimated by others [12], [20], and is used in our present work. As for the electron recombination rate, an empirical value $\eta_{\text{rec}} = 2 \times 10^{-9}$ cm$^3$/s as measured by Doccchio [21] is applied in this work, which is also used by others [12]. Fan et al. [22] also used this expression for the electron diffusion and recombination, further theoretical modeling of such a decay process needs to be done in the future.

**2.4. Model for electron number density**

A model for the evolution of electron number density is presented that takes the effects of photoionization, avalanche ionization, electron-ion recombination, and electron diffusion into account. The temporal behaviour of the free electron density $n$ in the conduction band is described by the following equation:
Where \( E \) is electric field, \( \eta(E) \) is the avalanche rate described by Thornber’s impact ionization, and \( W_{pl}(E) \) is the Keldysh’s photoionization rate. The first two terms in Eq. (8) represent a gain of electron density. The last two terms in Eq. (8) refer to the loss in the number of free electrons due to diffusion and recombination. Since recombination requires two charged particles, it is proportional to \( n^2 \), on the other hand, electron diffusion depends linearly on \( n \). This model was used by other authors [23]–[25]. By numerical evaluation of Eq. (8), we can determine the time-dependent electron number density and predict damage threshold.

3. Results and discussion

3.1. Electron evolution and material damage

In our calculation, we use a 100 fs, 12 TW/cm² Gaussian pulse which is described in Eq. (2) incident on fused silica. By numerical evaluation of Eq. (8) for the time-varying electron density, Fig. 6 depict the evolution of electron number density induced by femtosecond laser pulse; the electron number density produced by photoionization alone is included for reference.

From the figures, we find that photoionization strongly depends on intensity, therefore, the electron production takes place principally at the peak of the pulse. Only after the laser pulse is gone is energy transferred from the electrons to the lattices. In this 100 fs duration, multiphoton ionization produces a substantial amount of free electrons, material no longer has the properties of dielectrics, it becomes conductor.

![Fig. 6. Total (solid) electron number density and that produced by photoionization alone (dotted) plotted for a 1053 nm, 100 fs Gaussian laser pulse (dash).](image-url)
Femtosecond laser-induced damage in dielectrics

and will absorb laser energy via inverse Bremstrahlung. So, the characteristic of laser–matter interaction becomes independent of the initial state of material, both dielectrics and metals have similar behaviour and morphology when machined with ultrashort laser pulses. For laser pulses of different duration, the relative fraction of photoionization to avalanche ionization will change, photoionization will contribute a relatively greater fraction of the electron number density with shorter pulses. Analyzing the figure, we find that avalanche ionization predominates even in the case of sub-100 fs pulses. If the laser field is high enough, photoionization rate reduces to the formula for the tunneling ionization, which alone produces the electron critical density $n_{cr}$ to cause damage.

In order to validate the results derived from Eq. (8), comparison of our present work with the results of Stuart et al. [1], [5] for a 100 fs, 1053 nm Gaussian pulse is depicted in Fig. 7. We can find that our present work is in quantitative agreement with the previous studies.

3.2. Mechanisms for long and short pulse

For sub-picosecond laser pulses, photoionization by the leading edge of the laser pulse provides seed electrons for avalanche ionization during the rest of the pulse. After seed electrons are produced, a small electron avalanche achieves the critical density plasma, this self-seeded avalanche makes short-pulse breakdown less dependent on defects in the material than long-pulse breakdown and therefore the threshold for short-pulse damage is deterministic. In this case, the electron-to-ion energy transfer time and the heat conduction time exceed significantly the pulse duration. Then the absorbed laser energy is going into the electron thermal energy, and the ions remain cold, making the conventional thermal expansion inhibited [26]. The energetic electrons created by the laser radiation pulled ions out of the materials. For pulses longer than a few tens of
picoseconds, energy is transferred from the laser-excited electrons to the lattice on a time scale of the pulse duration. This energy is then carried out of the focal volume by thermal diffusion. Thus, it is the relative rate of energy deposition and thermal diffusion that determines the damage threshold. Damage occurs when the deposited heat is sufficient to melt, boil, or fracture the dielectrics material [5], [10]. Simple calculations show that, in this case, the threshold fluence for optical damage scales as the square root of the pulse duration [23]. The source of the initial conduction band electrons that seed the avalanche ionization is very important for long pulse laser. Avalanche ionization is very efficient for such pulses because the long pulse duration allows more time for exponential growth of the electron density. Because avalanche ionization is so efficient, the laser intensity required to produce damage is not high enough to directly photoionizing electrons, so either thermally excited electrons or impurity and defect states provide the initial seed electrons for the avalanche. A high concentration of easily ionized impurity electrons lowers the threshold for optical damage compared to that of the pure material, making determination of the intrinsic breakdown threshold difficult [10], [19].

3.3. Uncertainty of the model

The loss term due to electron diffusion and recombination is not rigorous, other factors that cause a reduction in the electron population include energy loss from the pulse by scattering, linear absorption and nonlinear absorption, which are not included in the present model. During the breakdown process, plasma absorption dominates the energy loss and can dramatically alter the pulse profile, resulting in lower intensity. In addition, accurate mean-free time between collisions has not been measured for most materials. If these factors are taken into account accurately, higher peak intensity will be required in the model to produce the same critical electron density. Further theoretical model of ablation is needed to develop in the future.

4. Conclusions

In summary, ultrashort pulse laser has many advantages for many technologies. During optical breakdown, a high density of free electrons is formed in the material, which dominates energy absorption, and in turn, the material removal rate during ultrafast laser material processing. We present a new model to determine the time-dependent electron number density in fused silica by femtosecond laser. Keldysh’s photoionization rate and Thornber’s avalanche rate, in addition, a decay term due to free electrons diffusion and recombination are included in the new model. Based on the numerical evaluation of electron density, we examine the respective role of ionization, and avalanche ionization in ultrashort laser induced damage. In addition, ablation mechanism of dielectrics by femtosecond lasers is presented which is quite different from the thermal ablation by long pulses. Present results are in quantitative agreement with earlier study, demonstrating that we use an effective method to determine the produced electron number density and predict damage threshold.
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Multi-photon processes considering magnetic sublevels coherence

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The exact nonlinear theory of polarized radiation propagation is derived in adiabatic following approximation taking into account different populations and coherence of atom magnetic sublevels. The nonlinear refractive indices for circularly and linearly polarized waves in the resonant media with arbitrary angular moments $j_1$ and $j_2$ are obtained. The exact formulas for rotation angle of polarization ellipse axes (without deformation) are found on the sample of medium with $j_1 = 1/2$ and $j_2 = 3/2$. The influence of coherence of magnetic sublevels on multi-photon phenomena and their specific behavior is analyzed.

Keywords: multi-photon processes.

1. Introduction

The propagation of polarized intense laser radiation through a resonant medium is accompanied by specific nonlinear polarization effects, such as atomic sublevels splitting and shift, rotation of the polarization ellipse, etc. These phenomena are theoretically studied in different media in [1]–[4]. The general case of two-level system with arbitrary angular moments is considered in [3], [4]. Induced by an intense wave change of the weak probe signal polarization was first observed in alkali metal vapors in [5], [6]. This phenomenon, in particular, becomes the base of the high-resolution spectroscopy [7]. Interaction of quasi-monochromatic pulsed laser radiation with a two-level medium is considered in detail in monograph [8], nonlinear magnetic and optical coherence in coupled two-level systems is investigated in paper [9]. In paper [10], polarization dynamics of femtosecond pulses propagating in air is studied through computer simulation. A rich variety of dynamics that depends on the initial polarization state and power of the pulse is found. Taking into account magnetic sublevels coherence leads to the occurrence of new interference effects, particularly, to the population trapping [11]–[17]. For systems with degenerate levels, the consideration of saturation effects becomes essential since, in consequence of the optical pump of atoms, the saturation of absorption can occur at anomalously small radiation
intensities. For studying nonlinear interference phenomena in the field of polarized radiation, the adiabatic following approximation is very promising.

In this paper, the influence of different populations and coherence of magnetic sublevels on the propagation of radiation through the two-level resonant medium is explored. Section 2 discusses the propagation of polarized radiation through the medium with arbitrary angular moments \( j_1 \) and \( j_2 \). In Sec. 3, the influence of coherence on multi-photon effects is investigated on the sample of medium with \( j_1 = 1/2 \) and \( j_2 = 3/2 \).

2. Propagation of polarized radiation through the resonant medium with arbitrary angular moments \( j_1 \) and \( j_2 \)

Let us consider a two-level atom in the field of monochromatic wave specified by the vector potential

\[ A = A_1(z) \exp[i(kz - \omega t)] + A_1^*(z) \exp[-i(kz - \omega t)]. \tag{1} \]

Later on, it is convenient to turn to the circular components of waves \( A_\pm = A_x \pm iA_y \).

We assume that the atom has in the ground state an energy \( E_1 \) and an angular momentum \( j_1 \), while in the excited state \( -E_2 \) and \( j_2 \), respectively. For an isolated atom these states are degenerate with respect to the projection of the angular momentum.

The Hamiltonian operator \( \hat{H} \) of the atom in the field of radiation in dipole approximation is

\[ \hat{H} = \hat{H}_0 - \hat{d}\hat{E} \tag{2} \]

where \( \hat{H}_0 \) denotes the Hamiltonian operator of the isolated atom, \( \hat{d} \) is the electric-dipole-moment operator, \( \hat{E} \) is the electric-field vector, and

\[ \hat{E} = -\frac{1}{c} \frac{\partial \hat{A}}{\partial t}. \]

We find the solution of the Schrödinger equation

\[ i\hbar \frac{\partial \Psi}{\partial t} = \hat{H} \Psi \tag{3} \]

in the form

\[ \Psi = \sum_{m_1} a(m_1, t) \psi_{m_1} \exp \left[ -\frac{i}{\hbar} E_1 t \right] + \sum_{m_2} b(m_2, t) \Phi_{m_2} \exp \left[ -\frac{i}{\hbar} E_2 t + i\epsilon t \right] \tag{4} \]
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where $\psi_m$, $\Phi_\mu$ are the wave functions of the isolated atom ($m = \pm j_1, \pm(j_1 - 1), \mu = \pm j_2, \pm(j_2 - 1)$), $\varepsilon = (\omega_0 - \omega)$ is the resonance detuning.

The field of radiation (1) takes off the degeneracy of atomic levels and thereby, leads to the occurrence of $m = 2j_1 + 1$ new quasi-stationary wave functions of the system “atom + field”. For a circularly polarized wave ($A_1 = 0$, $A_1 = A$) these wave functions are

$$\Psi_m = \alpha_m \left[ \psi_m \exp\left(-\frac{i}{\lambda} E_1 t\right) + \frac{i \omega_0 d^* \exp(ikz)}{\varepsilon \lambda c (1 + \sqrt{1 + \frac{\varepsilon}{\varepsilon_m}})} A \right]$$

$$\times \left[ \frac{(j_1 - m)(j_1 + m + 1)}{j_1(j_1 + 1)(2j_1 + 1)} \delta_{j_1,j_2} - \frac{(j_2 + m)(j_2 + m + 1)}{j_2(2j_2 - 1)(2j_2 + 1)} \delta_{j_2,j_1 + 1} \right]$$

$$+ \frac{(j_1 - m)(j_1 - m - 1)}{j_1(2j_1 - 1)(2j_1 + 1)} \delta_{j_1,j_2 + 1} \right] \Phi_{m + 1} \exp\left(-\frac{i}{\lambda} E_2 t + i \lambda t\right) \exp\left(-\frac{i}{\lambda} \lambda_{1m} t\right)$$

(5)

where $d$ is the reduced dipole matrix element, and the following notations are introduced:

$$|\alpha_m|^2 = \frac{1 + \sqrt{1 + \frac{\varepsilon}{\varepsilon_m}}}{2 \sqrt{1 + \frac{\varepsilon}{\varepsilon_m}}}$$

$$\lambda_{1m} = \frac{\varepsilon}{2} \left(1 - \sqrt{1 + \frac{\varepsilon}{\varepsilon_m}}\right)$$

$$\varepsilon_m = \frac{\varepsilon - \frac{\varepsilon}{2j_1 + 1} \left[ \frac{(j_1 - m)(j_1 + m + 1)}{j_1(j_1 + 1)} \delta_{j_1,j_2} + \frac{(j_2 + m)(j_2 + m + 1)}{j_2(2j_2 + 1)} \delta_{j_2,j_1 + 1} \right]$$

$$+ \frac{(j_1 - m)(j_1 - m - 1)}{j_1(2j_1 - 1)} \delta_{j_1,j_2 + 1} \right]$$

$$\varepsilon = \frac{|d|^2 |E_1|^2}{\varepsilon_{2,2}}.$$
\[ \mathcal{P} = \sum_m \gamma_m \Psi_m. \]  

The coefficients \( \gamma_m \) are found from the initial data.

Substituting the average dipole momentum of the atom \( \langle d \rangle = \langle \mathcal{P} | d | \mathcal{P} \rangle \) into the Maxwell equation for the slowly varying amplitudes \( A_{1-} \), we find the refractive index of the medium for circularly polarized wave

\[ n^{(-)} = 1 + q \frac{c}{\omega_0} \sum_{m = n = -j_1}^{j_1} \rho_{mn} \left[ \frac{(j_1 + m + 1)(j_1 - m)}{\sqrt{1 + \varepsilon_m j_1(j_1 + 1)(2j_1 + 1)}} \delta_{j_1,j_2} \right. \]

\[ + \frac{(j_2 + m)(j_2 + m + 1)}{\sqrt{1 + \varepsilon_m j_2(2j_2 - 1)(2j_2 + 1)}} \delta_{j_2,j_1 + 1} \]

\[ + \left. \frac{(j_1 - m)(j_1 - m - 1)}{\sqrt{1 + \varepsilon_m j_1(2j_1 + 1)(2j_1 - 1)}} \delta_{j_1,j_2 + 1} \right] \]

where \( q = (\pi |d|^2 \omega_0 n)/\varepsilon c \) (\( n \) is the density of the atoms). We have introduced the density matrix \( \rho_{mn} = \gamma_m^* \gamma_n \) of coherent states \( \Psi_m \) and \( \Psi_n^* \).

According to the selection rules of angular momentum projections, \( d_z^{j_1,m_1} \) is proportional to \( \delta_{m_1,m_2} \), therefore for linearly polarized wave it is convenient to select the \( x \) axis along the wave propagation, while the \( z \) axis is along the polarization. The similar evaluations lead to the refractive index

\[ n_z = 1 + \frac{2gc}{\omega_0} \sum_{m = n = -j_1}^{j_1} \rho_{mn} \left[ \frac{m^2}{\sqrt{1 + \varepsilon_m j_1(j_1 + 1)(2j_1 + 1)}} \delta_{j_1,j_2} \right. \]

\[ + \frac{j_2^2 - m^2}{\sqrt{1 + \varepsilon_m j_2(2j_2 - 1)(2j_2 + 1)}} \delta_{j_2,j_1 + 1} \]

\[ + \left. \frac{j_1^2 - m^2}{\sqrt{1 + \varepsilon_m j_1(2j_1 + 1)(2j_1 - 1)}} \delta_{j_1,j_2 + 1} \right]. \]

Comparing expressions (7) and (8) with corresponding formulae in papers [3], [4], one can see that the consideration of non-uniform population of magnetic sublevels modifies the expressions of refractive indices. In particular case, when states \( \Psi_m \) are incoherent and uniformly populated, \( i.e., \rho_{mn} = \delta_{mn}/(2j_1 + 1) \), the results obtained coincide with the results of the above-mentioned papers [3], [4].
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We will study the influence of non-uniform population of magnetic sublevels coherence on the resonant rotation of polarization ellipse by the example of the medium with angular moments \( j_1 = 1/2 \) and \( j_2 = 3/2 \). For this case, it is convenient to represent the density matrix \( \rho_{mn} \) through Stock’s parameters \( \eta_1, \eta_2, \eta_3 \)

\[
\rho = \frac{1}{2} \begin{bmatrix} 1 + \eta_3 & \eta_1 - i \eta_2 \\ \eta_1 + i \eta_2 & 1 - \eta_3 \end{bmatrix}
\]  

where the diagonal element \( \eta_3 \) defines the different populations of magnetic sublevels in the field of the wave, whereas the non-diagonal elements \( \eta_1, \eta_2 \) characterize the coherence of magnetic sublevels.

The refractive indices for circularly components of elliptically polarized wave \( A_{\pm} \) are:

\[
n^+ = 1 + \frac{q_1 c}{\omega_0} \left[ \frac{3(1 + \eta_3)}{\sqrt{1 + \mu_1}} + \frac{1 - \eta_3}{\sqrt{1 + \mu_2}} \right],
\]

\[
n^- = 1 + \frac{q_1 c}{\omega_0} \left[ \frac{1 + \eta_3}{\sqrt{1 + \mu_1}} + \frac{3(1 - \eta_3)}{\sqrt{1 + \mu_2}} \right],
\]

\[
q_1 = \frac{\pi |d|^2 \omega_0 n}{12 c^2 \varepsilon},
\]

\[
\mu_{1,2} = \frac{2 \omega_0 |d|^2}{6 c^2 \varepsilon^2} \left( 3 |A^x|^2 + |A^y|^2 \right).
\]

Formula (10) describes rotation of the axes of the polarization ellipse (without deformation) through an angle \( \gamma \), where

\[
\gamma = q_1 z \left( \frac{1 + \eta_3}{\sqrt{1 + \mu_1}} - \frac{1 - \eta_3}{\sqrt{1 + \mu_2}} \right).
\]  

For simplicity, let us assume that the atom prior to the interaction was in the ground state \( 1 \). If states \( \psi_{\pm 1/2} \) are incoherent prior to the switching on of the interaction (\( \eta_1 = \eta_2 = \eta_3 = 0 \)) the rotation angle is

\[
\gamma = q_1 z \left( \frac{1}{\sqrt{1 + \mu_1}} - \frac{1}{\sqrt{1 + \mu_2}} \right).
\]
For atoms with initial state \( \psi_{+1/2}(\eta_3 = 1) \)

\[
\gamma = \frac{2q_1}{\sqrt{1 + \mu_1}}, \tag{13}
\]

It follows from (11) that in the general case of coherent states, the linearly polarized light (\( \mu_1 = \mu_2 = \mu \)) is exposed to polarization plane rotation. Whereas for incoherent states, as is obvious from (12), the linear polarization is not changed through the propagation.

3. Propagation of two waves through the system with \( j_1 = 1/2 \) and \( j_2 = 3/2 \)

To study multi-photon effects such as Rayleigh scattering, three-photon scattering, parametric four-photon interaction, let us assume that along with the intense monochromatic wave (1) a weak quasi-monochromatic wave \( A_2(z, t) \) propagates in the medium. We have for its potential:

\[
A = A_2(z, t) \exp(-i\omega t) + A_2^*(z, t) \exp(i\omega t),
\]

\[
\left| A_2(z, t) \right| \nabla \left| A_1(z) \right|.
\]

It is convenient for further consideration to expand the field \( A_2(z, t) \) to the Fourier integral

\[
A_2(z, t) = \int_{-\infty}^{+\infty} F(z, \omega') \exp \left[ i(\omega - \omega')t \right] d\omega',
\]

where \( \omega' \) is the carrier frequency of weak quasi-monochromatic wave.

In order to exclude four-wave parametric interaction that takes place when the intense and weak waves propagate in the same direction, let us first assume that they propagate in opposite directions. The equations of propagation for slowly varying amplitudes of \( F(z, \omega') \) in the field of strong linearly polarized along the x axes (\( \mu_1 = \mu_2 = \mu \)), wave (1) are:

\[
- \frac{\partial F_x(\omega')}{\partial z} + i \frac{\omega - \omega'}{c} F_x(\omega') = iD_1 \left[ F_y(\omega') - \eta_1 \eta_2 \right] F_x(\omega'),
\]

\[
- \frac{\partial F_y(\omega')}{\partial z} + i \frac{\omega - \omega'}{c} F_y(\omega') = iD_1 \left[ F_y(\omega') - \eta_1 \eta_2 \right] F_x(\omega'),
\]

\[
\tag{16}
\]
where

\[
D_1 = \frac{q_1 e \left( \sqrt{1 + \mu} + 1 \right)^2}{1 + \mu} \frac{1}{\omega_a - \omega'} - \frac{q_1 e \left( \sqrt{1 + \mu} - 1 \right)^2}{1 + \mu} \frac{1}{\omega_a - \omega'},
\]

\( \omega_a = \omega + e \sqrt{1 + \mu} \) is the pole of Stark shifted one-photon absorption, \( \omega_\sigma = \omega - e \sqrt{1 + \mu} \) is the pole of three-photon scattering process. \( D_1 \) and \( D_1/4 \) characterize the refractive indices of \( x \) and \( y \) polarization components of weak wave in Eq. (16), the terms \( \pm D_1 \eta_1/2 \) define the energy transfer from one weak wave component to the other one.

By solving the set of Eq. (16), the following solutions are obtained:

\[
F_{x,y}(\omega') = \exp \left[ -i \frac{5}{8} D_1 + i \frac{\omega - \omega'}{c} z \right] 
\times \left[ F_{x,y}^z(\omega') \cos(rz) \pm i \frac{3}{8} D_1 \frac{r}{F_{x,y}}(\omega') \sin(rz) \pm \eta_1 \frac{D_1}{2r} F_{y,x}^z(\omega') \sin(rz) \right],
\]

\[
r = \frac{D_1}{8} \sqrt{9 + 16 \eta_1^2}.
\]

It is obvious from expressions obtained that under the action of intense field the energy is transferred periodically from one weak wave component of polarization to the other. This is caused by magnetic levels coherence and disappears when \( \eta_1 = 0 \).

The equations of propagation for Fourier components \( \mathbf{F}(z, \omega') \) of wave (15) in the case when the strong and the weak waves propagate in the same direction (the intense wave is linearly polarized along the axes \( x, \mu_1 = \mu_2 = \mu \)) are given by

\[
\frac{\partial F_x(\omega')}{\partial z} + i \frac{\omega - \omega'}{c} F_x(\omega') = i D_1 \left[ F_x(\omega') - \frac{\eta_1}{2i} F_y(\omega') \right]
\]

\[
+ iD_2 \exp \left[ \frac{8iq_1 z}{\sqrt{1 + \mu}} \right] \left[ F_x^*(2\omega - \omega') + \frac{\eta_1}{2i} F_y^*(2\omega - \omega') \right],
\]

\[
- \frac{\partial F_y(\omega')}{\partial z} + i \frac{\omega - \omega'}{c} F_y(\omega') = i \frac{D_1}{4} \left[ F_y(\omega') - \frac{2\eta_1}{i} F_x(\omega') \right]
\]

\[
+ i \frac{D_2}{4} \exp \left[ \frac{8iq_1 z}{\sqrt{1 + \mu}} \right] \left[ -F_y^*(2\omega - \omega') + \frac{2\eta_1}{i} F_x^*(2\omega - \omega') \right] + iD_3 F_y(\omega')
\]
where:

\[
D_2 = \frac{2q_1 \varepsilon^2 \mu}{\sqrt{1 + \mu} \left( \omega - \omega' \right)^2 - \varepsilon^2 (1 + \mu)},
\]

\[
D_3 = 3q_1 \varepsilon \frac{1 + \sqrt{1 + \mu}}{2 \sqrt{1 + \mu}} \frac{1}{\omega'_a - \omega'},
\]

\[
\omega'_a = \omega + \frac{\varepsilon}{2} \left( \sqrt{1 + \mu} + 1 \right)
\]
is the pole of Stark shifted Rayleigh elastic scattering.

As is seen from (19), \(x\) and \(y\) components of weak radiation polarizations are connected in the medium at the frequency \(\omega'\) as well as \(2\omega - \omega'\). This process is due to the non-degenerate four-photon parametric interaction of the waves and is characterized by the nonlinear coefficient \(D_2\). Atom absorbs two photons of \(x\) polarization of the intense wave and emits two photons with different polarizations of weak wave transferring from one coherent state to the other.

The coefficient \(D_3\) defines the Rayleigh elastic scattering when an atom absorbs a photon of the strong field polarized along the \(x\) axis and emits a photon of the weak field polarized along the \(y\) axis. It must be noted that in the case of incoherent magnetic sublevels [1] when \(\eta_1 = 0\), this process does not take place in the adiabatic following approximation.

4. Summary

The complicated interferential polarization effects in resonant medium associated with atom magnetic sublevels coherence are found.

From the nonlinear refractive indices for circularly and linearly polarized waves in the resonant media with arbitrary angular moments \(j_1\) and \(j_2\) obtained, it follows that the linearly polarized light undergoes a rotation of the plane of polarization, whereas in the case of uniformly populated and incoherent states, the linear polarization remains unchanged in the process of propagation.

The allowance for different populations of the magnetic sublevels and their coherence as well changes the behavior of multi-photon processes. So, for the counterpropagating waves the common three-photon scattering is accomplished by energy transfer from one polarization component of the weak wave to the other.

For the propagation of intense and weak waves in the same direction, the picture of nondegenerate four-photon parametric interaction changes simultaneously with the occurrence of the other process. As distinct from incoherent case, the elastic Rayleigh scattering occurs. The atom absorbs a photon of strong wave with the \(x\) polarization and emits a photon of the weak wave with the \(y\) polarization transferring from one coherent state to the other.
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Spectral modification of supercontinuum light by means of fs-light pulses optimized in a closed learning loop

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First closed loop optimization experiments on the spectral envelope of white light produced by shaped fs pulses irradiated on a thin (2 mm) sapphire plate are presented. Thereby, the spectral position of the maximal white light intensity could overall be shifted by about 80 nm. The modification of this spectral characteristic due to non-linear effects in optimal control is discussed with regard to novel applications in laser development.

Keywords: supercontinuum generation, optimal control.

1. Introduction

The phenomenon of white light radiation is observed in gases, liquids, and solids. White light radiation is generated by means of a complex interaction of many processes stemming from the third-order nonlinear susceptibility, as well as from ionization and plasma generation [1]–[6]. Because of the strongly coupled time and space variation, this process is complex and therefore still a subject of current study [7]. White light is also an easy source of tunable short pulse radiation already realized in several laboratories [8]–[10]. The potential for applications would be greatly enhanced if the spectrum of the white light could be modified.

Several studies regarding the white light generation show that in the simplest approach the intensity-dependent index of refraction results in an instantaneous frequency at a distance $L$ within the medium, $\omega = \omega_0 - k_0 n^2 L dI/dt$ [7]. Due to the second term, pulse shaping could be used to influence the spectral components of the white light. A first study in this regard showed considerably different white light spectra for different particular pulse shapes [7]. This observation gives rise to the question whether the spectrum of the white light could be influenced in a controlled way. As a well suited method to achieve this goal closed loop optimization can be applied. In 1992, JUDSON and RABITZ [11] proposed closed loop experiments for
controlling experimental observables. Since then several teams have successfully performed various optimization experiments [12]–[15]. To date the optimization objectives included such processes as population transfer, fragmentation, ionization, and impulsive stimulated Raman scattering (ISRS). The method enables one to find an optimal pulse form by using an iterative process. Programmable pulse shapers were combined with optimization algorithms, which are capable of solving large dimensional search problems. Feedback loop optimization is well suited for this problem since the process has a high level of complexity. One can imagine that even for simple pulses there could be several regions of the same \( Ld/dt \). This would correspond to the same white light frequency, which could give rise to interferences and hence oscillations in the white light spectrum. The temporal phase of the excitation pulse may affect the spectral shape of the white light, as well. Moreover, considering the relatively high pulse intensities used other nonlinear processes may also contribute to the white light generation and may be influenced by the pulse shape. Thus, a new challenge arises in learning about these effects.

In the next section the functioning of the experimental setup is explained. The experimental results of the modification of the white light spectrum by optimal control are presented in Sec. 3. The article ends with conclusions.

2. Experimental setup

A femtosecond laser beam is focused by a plane-convex lens (50 mm focal length) onto a 2 mm thick sapphire plate. The emitted white light is collected in a glass fiber via a second lens located on the beam axis. This fiber is connected to a spectrometer (Ocean Optics) within the computer, which allows a fast monitoring and therefore online modification of the observed spectrum. In order to strongly diminish the fundamental laser light at 790 nm a BG 39 filter (Coherent) is located in front of the fiber. This is necessary to prevent the CCD array of the spectrometer from overload by the laser light.

The light of a fs Ti:Sapphire oscillator (Tsunami; Spectra Physics) is amplified in a chirped pulse regenerative/multipass amplifier (Quantronix). It provides pulses of 1.5 mJ energy and about 150 fs time duration. In order to have a diminished light intensity for the experiment, a grey filter was placed before the shaper (2–5 µJ pulse energy at the sapphire plate). This also prevents the shaper from destruction by too high pulse energies. The low pulse-to-pulse deviation of about 1% assures a good stability of the white light obtained, which is necessary for the optimization. Averaging over 150 ms improves the signal stability. The experiments are carried out at a central wavelength of \( \lambda_0 = 790 \) nm. The corresponding width of the laser spectrum is about \( \Delta \lambda \approx 8 \) nm at FWHM.

The pulse shaper consists of a liquid crystal modulator mask (CRI-SLM-256) [16]. It is placed in the Fourier plane of a zero dispersion compressor [17] – see Fig. 1. The
two liquid crystal arrays of the modulator mask allow independent spectral phase and amplitude modulation [18] with a discretization of 128 pixels. As the most general case all values are optimized (free optimization). Here only phase optimization is applied in order to keep the pulse energy constant.

The implemented closed loop experiment combines the spectral detection with a programmable pulse shaper, which is driven by a self-learning optimization algorithm based on evolutionary strategies (see Fig. 1). According to the experimental feedback
signal, the algorithm iteratively creates and selects pulse forms. Convergence is reached when there is no further progress in optimizing the signal, whereby the optimal pulse shape is found. In our case the desired spectral features determine the quality (fitness) of the pulse forms.

Each pulse form is determined by an array of 128 numbers (individual), representing the phase values. At the beginning individuals consisting of random numbers are created. Groups of individuals are called populations. The array values are modified by two operators called cross-over and mutation. After being written on the modulator, each pulse form of the first population creates a signal which represents its fitness. In the last step of the cycle, the best individuals are selected and serve as parent individuals for the next iteration. Details of the algorithm have been published elsewhere [19].

3. Experimental results

A typical learning curve of the optimization measurement (here for the spectral range 400–700 nm) is shown in Fig. 2. A significant rise in the integral spectral signal with increasing iteration (generation) number is clearly visible. The three values for each generation show the best, the mean and the worst of all values in one generation. At the beginning the white light intensity is very small because the phase values of the first generation are randomly chosen. This leads to complex pulse trains spread in time over several ps, which yields low white light signals. With successive iterations the signal exceeds the outcome of the initial pulse and converges after about 40 generations.

![Fig. 2. Progression of the white light signal for the frequency range 400–700 nm during optimization. At the beginning the yield is small since the initial pulses are randomly formed. After approximately 40 generations the algorithm reaches a maximum.](image-url)
The noise in the optimization curve presented is due to the fluctuation in the laser beam, whereby the nonlinear effects in white light creation amplify the fluctuations considerably. Theoretically, a smooth monotonous rise should be expected, since the optimization algorithm leaves the best individual of each generation unchanged. Nevertheless due to beam fluctuations, the best individual in one generation can provide a weaker signal in the following one. This will lead to a decrease in signal intensity, if additionally all other individuals are weaker than the previous best one. Therefore the slight negative variations in the curve of the best values can be understood.

In order to achieve a shift of the white light spectrum, the ratio of the spectral range 400–520 nm vs. 520–700 nm is optimized. Thereby, the ratio of the integral signals (fitness) is either minimized or maximized. The integral value for each spectral range is gained by integrating and subtracting the baseline signal. The 520 nm is chosen for the separation of the two spectral ranges since at this wavelength the area under the initial white light spectrum is divided into two almost equal parts. This allows both minimization and maximization at equally good starting conditions.

Figure 3 shows the white light spectrum recorded by the implemented spectrometer. The dotted line indicates the spectrum for the maximization of the ratio 400–520 nm vs. 520–700 nm. It starts at shorter wavelengths than the initial spectrum, shows a global maximum at 480 nm and declines smoothly to the long wavelength side. The white light spectrum for minimization of the above ratio is represented by a solid line. The curve rises at longer wavelengths and has its maximum at 560 nm. The
widths of the two spectra do not differ considerably. An auxiliary feature visible in Fig. 3 is the observation that the intensity at the laser wavelength of 790 nm is lower when the total white light intensity is higher. This might indicate the transformation of the laser light into the fundamental white light component, but other origins cannot be excluded.

The substantial difference of 80 nm between the two maximal white light intensities is surprising and not understood entirely up to now. Simple explanations like intensity and pulse duration dependences are not valid according to our additional preliminary studies. The intensity pulse form in time probably plays a major role for the spectral shift (claimed in [7]) but this needs further verification. A detailed investigation of the acquired optimized laser pulse forms was not possible with the experimental setup. Schumacher [7] observed that a fast-rise-time pulse generates a red shifted white light spectrum whereas a fast-fall-time pulse creates a blue shifted spectrum, which can be understood qualitatively by self-phase modulation. Yet, he has not achieved a quantitative agreement with model calculations, which indicates a more complex origin of white light generation. Other contributing nonlinear effects (i.e., self-steepening, self-focusing, stimulated Raman scattering, four-photon parametric generation) may also be influenced by the pulse shape, presumably leading to amplification of particular white light frequency components. This assumption certainly requires further investigation in the future.

4. Conclusions

We investigated the closed loop optimization of the spectral shape of white light induced by focusing fs pulses in a thin sapphire plate. In particular, a shift of the white light peak in both spectral directions could be enforced. The amount of the total spectral shift achieved was measured to be 80 nm. Extensive theoretical simulations should be undertaken in order to understand the observed features and the modification of the spectral components due to the optimal control process. The reported ability of tuning the white light maximum frequency could create great potential for new short pulse laser applications. For example, the process of seeding optical parametric generators for producing tunable radiation (OPO, NOPA) could be made considerably more efficient. Even the design of particularly shaped white light pulses in time and frequency may be achieved with the method presented. This could enable further potential applications in short pulse laser technique.

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Analysis of luminous flux transfer through a conical ring-core light guide

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Analysis of luminous flux transfer through a conical ring-core light guide is presented. Three optical guides of this kind are the main elements of an original instrument for measurement of the luminance distribution in the field of view constructed by the authors. It was found that in the case of the output surface perpendicular to the symmetry axis of the cone having vertical angles greater than 50° the luminous flux is not transmitted through this kind of light guide. The conical output surface with great vertical angle considerably improves the process of transferring the luminous flux through a conical ring-core light guide. Replacing a flat outlet surface of the light guide by a conical one makes the transferred luminous flux more uniform to some extent.

Keywords: luminous flux, radiative transfer, photometry.

1. Introduction

The measuring instrument for the assessment of the luminance distribution in the field of view developed by the authors [1], [2] requires the use of ring-core light guides in the form of cones with appropriately selected vertical angle and numerical aperture. The properties of transferring the luminous flux by such a light guide are generally not known [3], [4]. Hence the need to carry out an analysis of the physical phenomena occurring in such an element.

In the discussion below the following simplifying assumptions have been made:

1. The following dimensions of a light guide of a step-index profile are known (Fig. 1): the length \( l \), the diameter of the outlet front \( d \), half of the vertical angle of the cone \( \alpha \), refractive indices of the core \( n_1 \) and cladding \( n_2 \).

2. The source of light illuminating from a great distance the inlet surface of the light guide is of small dimensions and it produces constant intensity of lighting \( E \) in the plane perpendicular to the direction of propagation.

3. The symmetry axis of the light guide coincides with the \( z \) axis of the rectangular system of coordinates with its origin at \( 0 \).

4. The dimensions of the core and the cladding are many times greater than the wavelength. The values of the refractive indices \( n_1 \) and \( n_2 \) are constant (also as
functions of wavelength). The losses caused by reflection between the core and the cladding and the losses caused by absorption of the luminous flux in the core of the light guide are neglected. The luminous flux penetrating from the core to the cladding is absorbed or leaves the guide through the side surface.

5. The elementary luminous flux is represented by a vector. The element of the surface interacting with this vector is flat both when the luminous flux passes through the core-cladding boundary and when the flux is reflected.

6. The elementary luminous fluxes, leaving the light guide are summed.

7. Because of the waveguide symmetry with respect to the plane $yz$, it is enough to analyse only the operation of half of the inlet surface.

2. **Entrance of the luminous flux into the core**

The inlet surface of the light guide was divided into $k$ zones (rings) of equal width. The width $s_{z1}$ of a single zone, measured in the plane $xy$, is equal to (Fig. 2):

$$s_{z1} = \frac{r_2 - r_1}{k_Z} = \frac{r_o \cos^2 \alpha}{k_Z}$$

where: $r_1 = l \tan \alpha$ – inner radius of the cone base, $r_2 = r_1 + r_o \cos^2 \alpha$ – outer radius of the cone base, $k_Z$ – number of zones, $\alpha$ – half of the vertical angle of the conical light guide. The real width of the zone $s_{z2}$ will be greater:

$$s_{z2} = \frac{s_{z1}}{\cos \alpha} = \frac{r_o \cos \alpha}{k}.$$
The mean radius \( r_{sr} \) of all the zones is equal to

\[
r_{sr} = \frac{r_1 + r_2}{2} = r_1 + \frac{r_2}{2} \cos^2 \alpha.
\]

Half of the circumference of the circle with the radius \( r_{sr} \) was divided into such a number of \( m \) elements that the length of the elementary field did not differ greatly from its width

\[
m = \text{INT} \left( \frac{2\pi r_{sr}}{\pi} + 1 \right).
\]  

(3)

It is convenient to present the division of the front surface into elements in the polar coordinate system.

The radius \( r_k \) of the \( k \)-th zone is equal to:

\[
r_k = r_1 + \frac{s_{z1}}{2} + (i - 1) s_{z1}
\]

for \( i = 1 \) to \( k \),

(4)

\[
\lambda_k = (j - 1)s_{zK} + \frac{s_{zK}}{2},
\]

for \( j = 1 \) to \( m \)

(5)

where \( s_{zK} = \pi/m \) denotes increment of the angle \( \lambda_k \) corresponding to the mean length of the elementary surface.

The coordinates of the point \( K \), being the center of the elementary surface in the system of the rectangular coordinates, are determined from the dependences:

\[
x_K = r_k \cos \lambda_k, \quad y_K = r_k \sin \lambda_k, \quad z_K = (r_k - r_1) \tan \alpha.
\]

(6)
The field of the elementary surface $\Delta S_K$ on which the point $K$ is situated, is equal to

$$\Delta S_K = s_z r_K s_z K.$$  \hspace{1cm} (7)

The elementary luminous flux $\Delta \Phi_K$ hits the elementary surface at the angle $\alpha_K$. This angle can be determined when the vectors of the incident flux $\mathbf{P}$ and the normal $\mathbf{N}_K$ to the element of the inlet surface are known: $\mathbf{P} = (-\sin \varphi, 0, \cos \varphi)$ – unit vector, $\mathbf{N}_K = (-\sin \alpha \cos \lambda_K, -\sin \alpha \sin \lambda_K, \cos \alpha)$ – unit vector, normal to the surface at the point $K$.

The cosine of the angle $\alpha_K$ between these vectors is equal to:

$$\cos \alpha_K = \cos (\mathbf{p}, \mathbf{N}_K) = \sin \varphi \sin \alpha \cos \lambda_K + \cos \varphi \cos \alpha.$$  \hspace{1cm} (8)

Thus, the elementary luminous flux $\Delta \Phi_K = |\mathbf{P}|$, incident on the elementary surface is:

$$\Delta \Phi_K = E \cos \alpha_K \Delta S_K.$$  \hspace{1cm} (9)

If the illuminance $E$ is given in luxes and the field of the elementary surface in square metres, then the luminous flux will be determined in lumens.

Knowing the angle $\alpha_K$, at which the luminous flux hits the inlet surface of the light guide, we can determine the refractive angle $\beta_K$. Since $\sin \beta_K = (\sin \alpha_K)/n_1$ hence $\beta_K = \arcsin[(\sin \alpha_K)/n_1]$.

Now, we can determine Fresnel’s coefficient of reflection:

$$\rho_K = \frac{1}{2} \left( \sin^2 (\alpha_K - \beta_K) + \frac{\tan^2 (\alpha_K - \beta_K)}{\tan^2 (\alpha_K + \beta_K)} \right).$$  \hspace{1cm} (10)

The elementary luminous flux $\Delta \Phi_T$, which will enter the core of the light guide is equal to:

$$\Delta \Phi_T = (1 - \rho_K) \Delta \Phi_K.$$  \hspace{1cm} (11)

3. Propagation of the luminous flux in the light guide

The vector equation of the refractive beam [2] has the form:

$$\mathbf{T} = \left( \frac{\mathbf{P}}{n_1} - \frac{|\mathbf{P}| \cos \alpha_K}{n_1} \cdot \mathbf{N}_K + |\mathbf{P}| \cos \beta_K \cdot \mathbf{N}_K \right) (1 - \rho_K).$$

Thus the components of the vector $\mathbf{T}$, representing the elementary luminous flux after refraction are as follows:
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3.1. Case A

The vector \( T \), running from the point \( K \), meets the outer surface of the conical light guide at the point \( A \) (Fig. 1). In order to determine the position of this point the system of equations must be solved:

\[
\begin{align*}
\text{i) } & \quad \frac{x_A - x_K}{T_x} = \frac{y_A - y_K}{T_y} = \frac{z_A - z_K}{T_z} \quad \text{equations for the straight line passing through the point} \ K \ \text{and parallel to the vector} \ T, \\
\text{ii) } & \quad x^2 + y^2 + \left[(l - z)\tan \alpha + r_o\right]^2 \quad \text{equation of the cone (outer surface).}
\end{align*}
\]

As a result of substitution a quadratic equation is obtained:

\[
x_A \frac{T_y}{T_z} \left(\frac{T_x^2 - T_y^2}{T_z^2} - \tan^2 \alpha\right) + z_A \left(\frac{y_K - \frac{T_y}{T_z} z_K}{T_z} - (r_o + r_1)^2\right) = 0
\]

(13)

from which \( z_A \) is determined, and next the other coordinates of the point \( A \):

\[
x_A = \frac{T_x}{T_z} (z_A - z_K) + x_K,
\]

(14)

\[
y_A = \frac{T_y}{T_z} (z_A - z_K) + y_K.
\]
Point $A$ lies in the plane $z = z_A$, at the angular distance $\lambda_A$ with respect to the axis $x$

$$\lambda_A = \arctan \frac{y_A}{x_A}.$$  \hspace{1cm} (15)

The vector $\mathbf{N}_A$ (unit) normal to the outer cone surface at the point $A$ is as follows:

$$\mathbf{N}_A = (\cos \lambda_A \cos \alpha, \sin \lambda_A \cos \alpha, \sin \alpha).$$

On this basis we can determine the angle of incidence $\sigma_A$ of the beam $\mathbf{T}$ on the boundary of the cone:

$$\cos \sigma_A = \frac{T_x \cos \lambda_A \cos \alpha + T_y \sin \lambda_A \cos \alpha + T_z \sin \alpha}{\sqrt{T_x^2 + T_y^2 + T_z^2}}.$$  \hspace{1cm} (16)

Knowing the angle $\sigma_A$ we can judge from the following condition whether at the point under consideration the total internal reflection of the luminous flux occurs:

$$\frac{n_1}{n_2} \sin \sigma_A > 1.$$  \hspace{1cm} (16)

If condition (16) is not satisfied, the luminous flux penetrates into the cladding and (according to the assumptions) is absorbed; otherwise, the light beam $\mathbf{S}$ is reflected. The vector equation of the reflected luminous flux has the form [2]:

$$\mathbf{S} = \mathbf{T} - 2\mathbf{N}_A |\mathbf{T}| \cos \sigma_A.$$  \hspace{1cm} (16)

The components of the vector $\mathbf{S}$ are as follows:

$$S_x = T_x - 2|\mathbf{T}| \cos \sigma_A \cos \lambda_A \cos \alpha,$$

$$S_y = T_y - 2|\mathbf{T}| \cos \sigma_A \sin \lambda_A \cos \alpha,$$

$$S_z = T_z - 2|\mathbf{T}| \cos \sigma_A \sin \alpha.$$  \hspace{1cm} (16)

The value of the reflected luminous flux $|\mathbf{S}|$ is equal to the incident flux $|\mathbf{T}|$ since the reflection is without loss.

Equations of the straight line passing through the point $A$ and parallel to the vector $\mathbf{S}$

$$\frac{x_D - x_A}{S_x} = \frac{y_D - y_A}{S_y} = \frac{z_D - z_A}{S_z}$$

with the equation of the cone, describing the inner surface of the light guide core:
form a system of equations, whose solution is reduced to quadratic equation:

\[ x_D^2 + y_D^2 = (l - z_D)^2 \tan^2 \alpha, \]

\[
x_D \left( \frac{S_x^2}{S_z^2} + \frac{S_y^2}{S_z^2} - \tan^2 \alpha \right) + z_D \cdot 2 \left[ \tan^2 \alpha + \left( x_A - \frac{S_y z_A}{S_z} \right) \frac{S_x}{S_z} + \left( y_A - \frac{S_y z_A}{S_z} \right) \frac{S_y}{S_z} \right]
\]

\[ + \left( x_A - \frac{S_y z_A}{S_z} \right)^2 + \left( y_A - \frac{S_y z_A}{S_z} \right)^2 - l^2 \tan^2 \alpha = 0. \]

The coordinate \( z_D \) of the point \( D \), determined in this way allows the other coordinates to be calculated:

\[ x_D = \frac{S_x}{S_z} (z_D - z_A) + x_A, \]

\[ y_D = \frac{S_y}{S_z} (z_D - z_A) + y_A. \]

If the coordinate \( z_D \) satisfies the inequality \( z_D < l \) (where \( l \) is the length of the light guide), then the flux \( S \) falls on the inner surface of the light guide core, and in the opposite case – on the outlet surface of the guide. At the point \( D \) we must also determine the angle of incidence \( \sigma_D \) and make sure that the total internal reflection occurs. For this purpose the components of the normal vector \( N_D \) at the point \( D \) of the inner surface of the cone are determined (15), \( \lambda_D = \arctan(y_D/x_D) \).

The angle of incidence \( \sigma_D \) of the flux at the point \( D \) can be determined from the dependence:

\[
\cos \sigma_D = \cos \left( \mathbf{S}, \mathbf{N}_D \right) = \frac{-S_x \cos \lambda_D \cos \alpha - S_y \sin \lambda_D \cos \alpha - S_z \sin \alpha}{\sqrt{S_x^2 + S_y^2 + S_z^2}}.
\]

Next, the condition of the reflection of the luminous flux from inner surface of the core is checked:

\[ \frac{n_1}{n_2} \sin \sigma_D > 1. \]

If the above condition is satisfied, the elementary flux \( S \) will be reflected again from the cladding surface. In the opposite case it will be lost.
3.2. Case B

In the case of large values of the inclination angles $\phi$ of the incident ray $P$, the ray $T$ (after refraction) may hit the point $B$ of the inner surface of the core (Fig. 3). On the basis of the system of equations:

$$\frac{x_B - x_K}{T_x} = \frac{y_B - y_K}{T_y} = \frac{z_B - z_K}{T_z},$$

$$x_B^2 + y_B^2 = (l - z_B)^2\tan^2 \alpha$$

we can write the quadratic equation:

$$z_B^2\left(\frac{T_x^2}{T_z^2} + \frac{T_y^2}{T_z^2} - \tan^2 \alpha\right) + z_B^2\left[\tan^2 \alpha + \left(x_K - \frac{T_x}{T_z}z_K\right)\frac{T_z}{T_x} + \left(y_K - \frac{T_y}{T_z}z_K\right)\frac{T_y}{T_z}\right]$$

$$+ \left(x_K - \frac{T_x}{T_z}z_K\right)^2 + \left(y_K - \frac{T_y}{T_z}z_K\right)^2 - l^2\tan^2 \alpha = 0$$

(21)

from which the coordinate $z_B$ of the point $B$ is determined and next the other coordinates:

$$x_B = \frac{T_x}{T_z}(z_B - z_K) + x_K, \quad y_B = \frac{T_y}{T_z}(z_B - z_K) + y_K.$$

Fig. 3. Path of a light ray for great incidence angle $\phi$. 
If $z_D < l$, the angle of incidence $\sigma_B$ must be determined in order to judge whether the internal reflections occur. The normal vector $\mathbf{N}_B$ at the point $B$ has the components $\mathbf{N}_B(-\cos \lambda_B \cos \alpha, -\sin \lambda_B \cos \alpha, -\sin \alpha)$ and $\lambda_B = \arctan(y_B/x_B)$, thus

$$\cos \sigma_B = \frac{-T_x \cos \lambda_B \cos \alpha - T_y \sin \lambda_B \cos \alpha - T_z \sin \alpha}{\sqrt{T_x^2 + T_y^2 + T_z^2}}. \quad (22)$$

If the inequality $\frac{n_1}{n_2} \sin \sigma_B > 1$ is satisfied, the elementary flux is reflected at the point $B$.

The components of the vector of reflected $\mathbf{S}$ are as follows:

$$S_x = T_x - 2|T| \cos \sigma_B \cos \lambda_B \cos \alpha,$$

$$S_y = T_y - 2|T| \cos \sigma_B \sin \lambda_B \cos \alpha,$$

$$S_z = T_z - 2|T| \cos \sigma_B \sin \alpha$$

with the vector $\mathbf{S}$ coincident with the straight line described by the equations:

$$\frac{x_C - x_B}{S_x} = \frac{y_C - y_B}{S_y} = \frac{z_C - z_B}{S_z}$$

which intersects the outer surface of the core $x_C^2 + y_C^2 = [(l - z_C) \tan \alpha + r_0]^2$, at the point $C$.

The solution of the above system of equations consists in determining the elements of the quadratic equation:

$$z_C^2 \left( \frac{S_x^2}{S_z} + \frac{S_y^2}{S_z} - \tan^2 \alpha \right) + z_C \cdot 2 \left[ (r_0 + r_1) \tan \alpha + \left( x_B - \frac{S_x}{S_z} z_B \right) \frac{S_x}{S_z} + \left( y_B - \frac{S_y}{S_z} z_B \right) \frac{S_y}{S_z} \right]$$

$$+ \left( x_B - \frac{S_x}{S_z} z_B \right)^2 + \left( y_B - \frac{S_y}{S_z} z_B \right)^2 - (r_0 + r_1)^2 = 0. \quad (23)$$

Knowing the coordinate $z_C$ we can calculate the other coordinates of the point $C$ from the dependence:

$$x_C = \frac{S_x}{S_z} (z_C - z_B) + x_B, \quad y_C = \frac{S_y}{S_z} (z_C - z_B) + y_B.$$  

If the coordinate $z_C$ satisfies the inequality $z_C < l$, the angle of incidence $\sigma_C$ of the ray $\mathbf{S}$ at the point $C$ is determined from the following relation:
where: $\lambda_C = \arctan(y_C/x_C)$ – angular distance of the coordinate $x$ of the point $C$ in the plane $z = z_C$, $\mathbf{N}_C(\cos \lambda_C \cos \alpha, \sin \lambda_C \cos \alpha, \sin \alpha)$ – unit vector normal to the surface of the cone at point $C$.

If the inequality $\frac{n_1}{n_2} \sin \sigma_C > 1$ is satisfied, the total internal reflection occurs and the elementary luminous flux $\mathbf{S}$ under dismission becomes reflected again.

In this way, the phenomenon of reflection of the elementary luminous flux can occur many times along the light guide.

### 3.3. Case C

If the angle of incidence $\phi$ of the elementary light beam $\mathbf{T}$ is close to the angle $\alpha$ of the cone generator (Fig. 1) or coincides with it, then the luminous flux, after refraction, can hit directly the outlet surface of the light guide (Fig. 4). In this case, the coordinate $z_B$ of the point $B$ lies outside the exit surface of the cone ($z_B > l$), and the light beam hits this surface at the point $W$.

The conical light guide output surface can be hit by fluxes running directly from the point $K$ (Fig. 4) and by fluxes refracted at arbitrary points ($A, D, B, C$) considered above, as well as at other points of repeated reflections. The components of the incident vector ($\mathbf{T}$ or $\mathbf{S}$) enable the angle of incidence $\beta_w$ to be determined:

$$\cos \beta_w = \frac{T_z}{\sqrt{T_x^2 + T_y^2 + T_z^2}}$$

or

$$\cos \beta_w = \frac{S_z}{\sqrt{S_x^2 + S_y^2 + S_z^2}}$$

Fig. 4. Path of light beam when $\phi = \alpha$. 
If \( n_1 \sin \beta_W > 1 \), the inside reflection does not occur and the output angle \( \alpha_W \) of the ray from the light guide is \( \alpha_W = \arcsin(n_1 \sin \beta_W) \).

On the output surface there will take place Fresnel’s reflection which is defined by the reflection coefficient \( \rho_W \)

\[
\rho_W = \frac{1}{2} \left[ \frac{\sin^2(\alpha_W - \beta_W)}{\sin^2(\alpha_W + \beta_W)} + \frac{\tan^2(\alpha_W - \beta_W)}{\tan^2(\alpha_W + \beta_W)} \right].
\]  

(26)

The elementary luminous flux is as follows:

\[
\Delta \Phi_W = |W| = (1 - \rho_W)\Delta \Phi_p = (1 - \rho_K)(1 - \rho_W) \Delta S_K E \cos \alpha_K.
\]  

(27)

Summing up all elementary luminous fluxes \( \Delta \Phi_W \), we can determine the value of the luminous flux \( \Phi_\phi \) incident from the direction \( \phi \), which has passed through the light guide

\[
\Phi_\phi = 2 \sum \Delta \Phi_W.
\]  

(28)

The sum obtained must be doubled, since only half of the inlet surface is analysed (item 7 of the adopted simplifying assumptions).

### 4. Results of calculations

In the calculations the following data were assumed: \( l = 20 \text{ mm} \) – length of the conical cylindrical core light guide, \( d = 4 \text{ mm} \) – diameter of the outlet surface of the light guide, \( n_1 = 1.5400 \) – refractive index of the core, \( n_2 = 1.5181 \) – refractive index of the cladding, \( E = 1000 \text{ lx} \) – intensity of light.

The angle of inclination of the generator of the cone with respect to the symmetry axis of the light guide was changed in a step-like mode beginning from \( \alpha_0 = 15^\circ \) to \( 45^\circ \), every \( 5^\circ \). The entrance surface of the conical ring-core light guide was divided into \( k = 14 \) zones so that the width of the zone \( s_{22} \) was from 0.101 mm at the angle \( 45^\circ \) to 0.134 mm for the angle \( \alpha_0 = 15^\circ \), and the number of elementary light beams was from 794 948 to 239 232, respectively. The calculation results of the value of the luminous flux \( \Phi_\phi \) as a function of the angle \( \phi \) is shown in Fig. 5. At the angle \( \alpha_0 = 50^\circ \) the luminous flux does not pass through the light guide.

Computer simulation of a conical, ring-core light guide with steady inclination of the generator \( \alpha_0 = 30^\circ \) and changing diameter of the outlet front was also carried out. The diameter \( d_0 \) was changed step-like from 0.5 to 10 mm. The calculation results are shown in Fig. 6.

As follows from Fig. 5, at a high value of the angle \( \alpha_0 \) (above \( 35^\circ \)), the angle of incidence \( \beta_W \) of elementary beams on the output surface of the light guide is rather
Fig. 5. Luminous flux $\Phi_\varphi$ as a function of the incidence angle $\varphi$ for varying angle $\alpha_0$ for flat output surface.

Fig. 6. Luminous flux $\Phi_\varphi$ as a function of the incidence angle $\varphi$ for varying diameter $d_\varphi$. 
high and many beams are subjected to complete inner reflection. These elementary
light beams do not leave the light guide.

In order to reduce the probability of complete inner reflection on the outlet surface
of the light guide that surface in the form of a cone was considered (Fig. 7). In this
way, the angles of incidence $\beta_W$ of the beams hitting the outlet surface become
considerably reduced and the probability of complete inner reflection is smaller.
However, for this outlet shape, the vector $\mathbf{W}$ leaving the outlet surface, may hit it again.
This was not taken into consideration.

The coordinates of the point $W$ can be determined solving the system of equations:
\[ (x_W - x)/S_x = (y_W - y)/S_y = (z_W - z)/S_z \] — equations defining the straight line passing
through the point with the coordinates $x, y, z,$ parallel to the vector $\mathbf{S},$
\[ (x_W^2 + y_W^2)\tan^2\alpha_o = (z_W - l)^2 \] — equation of the outlet surface of the light guide. Thus,
we must calculate the roots of the quadratic equation
\[
z_W^2 \left[ \left( \frac{S_x^2}{S_z^2} + \frac{S_y^2}{S_z^2} \right) \tan^2\alpha_o + 1 \right] + z_W^2 \left[ \frac{S_x}{S_z} \left( \frac{x - S_x z}{S_z} \right) + \frac{S_y}{S_z} \left( \frac{y - S_y z}{S_z} \right) \right] \tan^2\alpha_o + l \right]
\[
+ \left( \frac{S_x}{S_z} - x \right)^2 + \left( \frac{S_y}{S_z} - y \right)^2 \right] \tan^2\alpha_o - l^2 = 0.
\]
Knowing the coordinate $z_W$ the other coordinates can be determined from the
dependence:

Fig. 7. Escape of an elementary luminous flux from the conical output surface.
At the point $W$ we define the unit vector $N_W$ normal to the outlet surface of the light guide: 

$$N_W = \left( -\cos \alpha_o \cos \lambda_W, -\sin \alpha_o \cos \lambda_W, \cos \alpha_o \right)$$

where $\lambda_W = \arctan(y_W/x_W)$.

The angle between the vectors $S$ and $N_W$ is the angle of incidence of the beam on the output surface of the light guide ($\beta_W$):

$$\cos \beta_W = \frac{-S_x \sin \alpha_o \cos \lambda_W - S_y \sin \alpha_o \sin \lambda_W + S_z \cos \alpha_o}{\sqrt{S_x^2 + S_y^2 + S_z^2}}.$$

If the inequality $n_1 \sin \beta_W < 1$ is not satisfied, there takes place the total internal reflection on the output surface of the light guide.

Next, the angle $\alpha_W$ of the escape of the elementary light beam from the light guide $\alpha_W = \arcsin(n_1 \sin \beta_W)$ and the coefficient $\rho_W$ Fresnel’s reflections occurring on the outlet surface (relation (26)) are determined. The components of vector $W$ leaving the light guide are defined by the dependences:

$$x_W = \frac{S_x}{S_z} (z_W - z) + x, \quad y_W = \frac{S_y}{S_z} (z_W - z) + y.$$

Fig. 8. Luminous flux $\Phi_\varphi$ as a function of the incidence angle $\varphi$ for varying angle $\alpha$ for conical output surface.
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This vector intersects the outlet surface at the point \( P \) (Fig. 7) with the coordinates:

\[
x_p = \frac{W_x}{W_z}(l + l_2 - z_w) + x_w, \quad y_p = \frac{W_y}{W_z}(l + l_2 - z_w) + y_w.
\]

Point \( P \) must lie on the surface of the circle with the diameter \( d_\alpha \), which is a condition of the escape of elementary luminous flux from the light guide:

\[
\sqrt{x_p^2 + y_p^2} \leq \frac{d_\alpha}{2}.
\]

The dependences (27) and (28) in this case are also valid.

The results of calculations of the values of the luminous flux \( \Phi_\varphi \) passing through the light guide with conical output surface, as a function of the angle of incidence \( \varphi \) are presented in Fig. 8.

5. Conclusions

This work can be summarized as follows:

1. In the case of the outlet surface perpendicular to the symmetry axis of the cone having greater vertical angles than 50° the luminous flux does not pass the analysed light guide (Fig. 5). The output of conical shape with great vertical angle (90° – \( \alpha \)) considerably improves the transfer of the luminous flux through a conical ring-core light guide (Fig. 8).

2. The luminous flux passing through the light guide increases with an the increase of the outlet diameter \( d \) since the outlet surface is greater.

3. Although a constant value lighting (1000 lx) in the plane perpendicular to the direction of the course of the luminous flux incident on the light guide has been assumed, the value of the luminous flux leaving the light guide is not steady and depends on the incidence angle \( \varphi \). At greater vertical angles (\( \alpha \geq 40^\circ \)) the value of the luminous flux decreases to 10% of the maximal value. Replacing a flat outlet surface of the light guide by a conical one makes the transferred luminous flux more uniform to some extent.
4. Comparing the results of calculations displayed in Figs. 5 and 8 shows that for conical entrance surface it is possible to use conical light guides with greater vertical angles.

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In this paper, we proposed that using a pair of fiber gratings in a section of the transmission fiber, a gain clamped broadband distributed fiber Raman amplifiers (DRA) can easily be made based on the utilization of the pumps’ interactions and uneven gain property along the fiber in the amplifier.

Keywords: fiber Raman amplifier, fiber Bragg grating, gain clamping, power transient.

1. Introduction

Broadband amplification and distributed amplification are two distinctive characteristics of Raman amplifiers (RA) revived and progressed quickly in recent years. Fiber Raman amplifiers can provide flat gain over a wide band if several pumps at various wavelengths are used together. Besides the broadband amplification, the distributed amplification, in principle, shows better system performances, especially in terms of noise [1].

But when the amplifiers are to be used in practical DWDM systems, the number and power level of the input channels may change randomly at times. Then, it is important to ensure that the equal performance of channels at different wavelengths can be achieved at a wider range of input signal power levels.

There have been reported two kinds of gain control methods for Raman amplifiers. One is monitoring the signals’ power and adjusting pumps’ power [2]; the other is using an all-optical feedback loop in a discrete Raman amplifier [3], [4]. But in a broadband distributed Raman amplifier (DRA), multi-pumps should be used to achieve broadband amplification; and in order to take advantage of distributed amplification, the transmission fiber is used as gain medium. This makes the dynamic gain control of broadband DRA amplifiers more difficult. The former gain control method needs complex feedback and auto pumps’ power control systems to change the pumps’ power. When multi-pumps are used, more than one monitoring channel should be used [2], and the cooperation of the pumps under different input conditions becomes a big
problem. Also the delay of the pump’s adjusting system can cause trouble as regards the amplifier’s transient effects [5]. The latter method uses wavelength selective couplers, band-pass filter and the amplifier to form a ring laser structure, and takes advantage of the Raman gain to generate lasing as the gain-clamped method often used in EDFAs. When the transmission fiber is used as a gain medium, the Raman gain coefficient of transmission fiber is much lower than that of the fiber used in discrete Raman amplifiers, and the transmission fiber is too long to form a ring structure. It is impossible to make an optical loop with low attenuation for the clamped wavelength in the optical feedback loop allows lasing to be sustainable. So, the latter method can only be used in discrete Raman amplifiers.

2. Principle

Two important properties of broadband DRA are worth paying attention to. The first one is that although the net Raman gain of a distributed fiber Raman amplifier is small, the amplification occurred mainly near the end of transmission fiber where the pumps’ power is high. The other important property is that there are great interactions among the pumps of the amplifier. Surely they can cause inhomogeneity in common Raman amplifiers, but they can also be utilized in clamping the amplifiers’ broadband gain profile.

The uneven gain property along the fiber and the interactions among pumps make it possible to get gain-clamped DRA by a sample all optical method. For a DRA backward pumped by high power laser diodes, we use fiber Bragg gratings as reflectors near the transmission fiber’s end (see Fig. 1). Then the section of the transmission fiber between the gratings becomes an optical cavity and can cause high-power lasing at the grating’s wavelength. Because of this the section of fiber has high pumps’ power which can give high Raman gain. The lasing operates in a saturation region of the amplifier providing uniform gain and noise performance for input signals, and the pump’s interaction can help the lasing to clamp the gain in the whole band. The power of signal, spontaneous Raman emission (ASE), lasing, and some pumps along the transmission fiber of this RA are shown in Fig. 2.

![Fig. 1. Structure of all optical-gain-clamped DRA.](image-url)
All optical method to achieve gain-clamping ...

The structure of the gain-clamped RA is shown in Fig. 1. The wavelength of the gratings we choose is 1584 nm. Because of the gain-clamping lasing work at shorter wavelength, the lasing can act as a pump for the signals for they just fill in the Raman gain bandwidth of the lasing. Then the amplifier’s gain profile will change with the lasing power. Furthermore, the lasing’s ASE noise makes the noise figure higher especially for the signal channels at shorter wavelength near the lasing (due to temperature dependent spontaneous Raman emission noise[6],[7]). These effects can be avoided if the lasing wavelength is longer than that of the signals.

3. Simulation results and discussion

During the simulation we use a numerical method to resolve the differential equations of RA complete numerical model [7] that include fiber loss, pump-to-pump, pump-to-signal, signal-to-signal Raman interactions, Rayleigh scattering, spontaneous Raman emission and its temperature dependence:

Fig. 2. Optical power in all optical gain-clamped DRA.

Fig. 3. Gain and NF of: a – DRA without gain clamping, b – optical gain-clamped DRA.
\[
\frac{dP_f(z, \nu)}{dz} = -\alpha(\nu)P_f(z, \nu) + \gamma(\nu)P_b(z, \nu)
\]

\[
+ \int_{\xi > \nu} \left\{ \frac{g_\nu}{A_{\text{eff}}} (\nu - \xi) \left[ P_f(z, \xi) + P_b(z, \xi) \right] P_f(z, \nu) \right\} \, d\xi
\]

\[
+ 2h \nu \frac{g_\nu}{A_{\text{eff}}} (\nu - \xi) \left[ P_f(z, \xi) + P_b(z, \xi) \right] \left[ 1 + \frac{1}{\exp \left( \frac{h(\xi - \nu)}{KT} \right) - 1} \right] \, d\xi
\]

\[
- \int_{\xi < \nu} \left\{ \frac{g_\nu}{A_{\text{eff}}} (\nu - \xi) \left[ P_f(z, \xi) + P_b(z, \xi) \right] P_f(z, \nu) \right\} \, d\xi
\]

\[
+ 2h \nu \frac{g_\nu}{A_{\text{eff}}} (\nu - \xi) \left[ P_f(z, \xi) + P_b(z, \xi) \right] \left[ 1 + \frac{1}{\exp \left( \frac{h(\xi - \nu)}{KT} \right) - 1} \right] \, d\xi
\]

where: 
- \( P_f(z, \nu) \) – forward power at frequency \( \nu \) and distance \( z \); 
- \( P_b(z, \nu) \) – backward power at frequency \( \nu \) and distance \( z \); 
- \( \alpha(\nu) \) – attenuation; 
- \( \gamma(\nu) \) – Rayleigh scattering coefficient; 
- \( g_\nu(\Delta \nu)g_(\xi - \nu) \) – Raman gain coefficient between frequencies \( \xi \) and \( \nu \); 
- \( A_{\text{eff}} \) – effective area of the fiber; 
- \( h \) – Plank’s constant, 
- \( K \) – Boltzman’s constant, 
- \( T \) – temperature of the fiber.

First, a common RA without gain-clamped method was investigated by simulation. The Raman amplifier is backward-pumped by three LDs at different wavelengths (see the Table) to get flat gain profile in 1530–1580 nm. The transmission fiber used as the gain medium is a standard single mode fiber. The gain and noise figures for different

<table>
<thead>
<tr>
<th>Pump wavelength [nm]</th>
<th>Pump power [mW] (conventional RA)</th>
<th>Pump power [mW] (GCRA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1420</td>
<td>200</td>
<td>220</td>
</tr>
<tr>
<td>1435</td>
<td>180</td>
<td>200</td>
</tr>
<tr>
<td>1468</td>
<td>490</td>
<td>470</td>
</tr>
</tbody>
</table>
input conditions are shown in Fig. 3a. The gain profile for small signal is very flat (gain ripple is ±0.6 dB). When the input signals change to 50 channels (at the wavelength from 1530 to 1579 nm with 1 nm channel spacing and signal power 1 mW/channel) from results obtained by numerical simulations we can clearly see the gain profile changes (Fig. 3a). The gain profile becomes tilt. The gain at shorter wavelength decreases more seriously. The gain variation for 1530 nm is more than 1 dB. Even when the total power of input channels is the same, the different wavelengths of the input channels can cause the gain profile to differ. This phenomenon makes the gain clamping of RA more complicated when the monitoring and adjusting method is used.

Figure 3b shows the simulation results for the gain-clamped RA (the FBG at the receiver end has 1 nm bandwidth and 99% reflection ratio at center wavelength 1584 nm, and the FBG inserted in the transmission fiber 20 km away from the output end has 20% reflection ratio) under the same input conditions with the conventional RA that has been disused above. Different input conditions do have very small effect.
on the gain profile and the noise figure; and the gain variation is less than 0.3 dB for all input conditions. The simulated gain saturation properties at different wavelengths of RA with and without gain clamping are shown in Fig. 4. The gain saturation properties have obviously been improved for the amplifier with the gain clamping method we proposed. Compared to the RA without pump clamping using the same total pump power, the decrease of the gain of the gain clamped amplifier is less than 1.5 dB.

The transient effects of the gain clamped distributed Raman amplifiers have also been analyzed by numerical method [8]. The surviving channel is at 1530 nm and the other 49 channels at 1531–1579 nm are adding/dropping channels, the power of the input signals is 1 mW/channel. Figures 5 and 6 show the simulation results of power transient of a conventional distributed Raman amplifier and the distributed Raman amplifier using the gain clamping method we have introduced. The lasing power (Fig. 7 shows the calculated lasing power before the FBG near the output end) for such an amplifier changed automatically to keep the gain stable. From the figures we can see that the transient effects of RA have been improved by means of using the gain clamping method (power variation of surviving channel decreases from 0.5 mW to

Fig. 6. Surviving channel in conventional DRA and gain-clamped DRA.

Fig. 7. Lasing power in all gain-clamped DRA.
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less than 0.15 mW). The overshot of adding channel and the power fluctuation of surviving channel have been suppressed efficiently. Through changing the fiber grating’s wavelength or reflectivity, the degree of gain clamping can be adjusted conveniently. Figure 8a, b show simulated results of the signal and lasing power of a RA with high degree gain clamping, from which we can see that the signal power of this RA is more stable, but the lasing power is higher and thus makes the amplifier have lower gain efficiency.

4. Conclusions

In summary, we have demonstrated a simple all optical method to get broadband distributed Raman amplifiers. The realization of the method is based on the uneven gain property and pumps’ interactions. Using this method the gain variation can be reduced effectively. It is a promising method to be used in broadband distributed fiber Raman amplifiers for it needs no complex monitor and control systems.

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References


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Laser Doppler vibrometry with acoustooptical frequency shift

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This paper describes our experiments and investigations on vibration measurements by laser interferometers. The main objective was to build a simple heterodyne interferometer which would allow vibrations of objects with light scattering surface to be measured. The optimisation procedure of optical setup, basic heterodyne interferometer and results are presented.

Keywords: LDV, vibrometry, vibration measurements, heterodyne interferometry.

1. Introduction

It often appears necessary in practice to measure object vibrations. The more rigorous norms of sound noise emission require from designers and constructors of devices taking into account the negative influence of vibrations on environment. Usually, this is done by application of accelerometers or other mechanical methods. However, all commonly used methods have one basic disadvantage – the mechanical contact with vibrating object is the necessary condition. The influence of sensor in the case of measurement of large object vibrations is not substantial, but if the vibrating object has got a small mass (e.g., a diaphragm of loudspeaker) such a sensor disturbs the value measured significantly. Besides, it is difficult to ensure the required physical contact between a transducer and a vibrating object in high temperatures and inaccessible places. Non-contact measuring methods utilising a coherent light (laser interferometry) have no such limitations. There are two main arrangements of the interferometric methods possible in vibration measurements: homodyne or heterodyne interferometry. The basic setup of Doppler shift measurement is the homodyne interferometer. This method is simple, but the necessity of determining the direction of movement makes the system much complicated. Measurement of movements or vibrations of the object that scatters light can be hard either. In practice, such a solution utilizes interference fringes counting method. This measurement is characterised by a small exactitude,
limited by resolution of the interference fringes (the use of additional signal processing allows the precision to be improved to 0.1 fringe) [1].

The main disadvantage of classical laser homodyne (single-frequency) interferometers applied in the laser vibrometry is the presence of intensity noise of laser radiation at low frequencies below 1 MHz. It causes that conversion of phase signal into intensity signal may produce a significant and harmful increase in the background noise levels. This problem may be overcome by the use of a heterodyne interferometer setup, where one of the beams is frequency shifted by passing it through frequency shifter (e.g., acoustooptic Bragg modulator) [2] imposing frequency shift in the deflected beam. For the case of equal amplitudes of interfering beams in both arms of interferometer, the normalised signal from the photodetector is

$$s(t) = \left\{ 1 + \sin\left[ 2\pi v_B t + \phi(t) \right]\right\}$$

where $\phi(t)$ is the difference in phase of two arms and the $v_B$ is the frequency shift in, e.g., Bragg modulator. This frequency should be well outside the 1/f noise region. As a result, a significant increase in the signal-to-noise ratio of over 20 dB is obtainable.

When phase fluctuations $\phi(t)$ are periodic (stationary periodic vibrations), then they cause modulation of the intermediate frequency:

$$v_{IF} = \left\{ v_B + \frac{d\phi(t)}{dt} \right\}.$$  

Standard phase modulation detection techniques may be used to extract the modulation. Stable offset frequency for heterodyne interferometry is required. It is quite easily obtainable by using Bragg modulator [3] with single or double Doppler shift $v_B$, depending on the heterodyne setup. Typical Bragg frequency shifts are a few tens of MHz and flicker 1/f noise region is much lower than intermediate frequency.

2. Basic heterodyne interferometers

Our main target was to build a simple heterodyne interferometer which could be applied to measure vibrations of scattering light surfaces. The main criteria of choosing a proper setup were minimum number of optical elements needed and simple aligning of the setup.

A simple heterodyne interferometer with single frequency shift is shown in Fig. 1. In the arrangement presented laser beam crosses the Bragg cell and splits into two beams – zero and first-order ones [4]. The first of the beams falls on the measured object after passing the beamsplitter. Having been scattered on the surface the light is reflected from the beamsplitter and reaches a photodetector. The Bragg order beam after reflection in mirrors passes through the beamsplitter and interferes with the first beam on the photodetector. The FM modulated signal on the Bragg shift frequency $v_B$
Laser Doppler vibrometry ... is received as a result of heterodyning. In this setup the separation between zero and first Bragg order beams is the main problem because of the small angle between beams. Additionally, the Bragg order beam (in this setup, reference one) is weakly modulated in amplitude due to deflection effect. As a result, the undesirable signal appears on a photodetector and masks the signal originated from vibrating object. To avoid this problem the frequency shift can be doubled by passing the beam twice through the Bragg modulator [5]. This arrangement is exceptionally simple (Fig. 2). The light beam crossing through the Bragg cell splits it into two, the zero order of diffraction – the reference beam (frequency $\nu_o$) and the first order of diffraction – the measuring beam (frequency $\nu_o - \nu_B$). After reflection from the reference mirror and vibrating object respectively, beams are crossing through the Bragg cell again. The measuring beam falls on a photodetector (frequency changed by the Doppler phenomena $\nu_o - \nu_B \pm \nu_D$), whereas the reference beam deflects in the first order diffraction (frequency $\nu_o \pm \nu_B$).

Fig. 1. Single Bragg shift configuration.

Fig. 2. Double Bragg shift configuration.
The interference signal received on photodetector is \((\nu_o + \nu_B) - (\nu_o - \nu_B \pm \nu_D) = 2\nu_B \pm \nu_D\), and it forms the carrier at double Bragg frequency. Doppler-modulated signal gives information about the object’s speed. In this case, disturbance caused by deflection effect does not influence signal from vibrating object. Unfortunately, the presence of even residual standing ultrasonic wave in the modulator causes undesirable amplitude modulation of beams on double Bragg frequency appearance – the same as the heterodyne signal. The next simplification of the setup can be made by means of a laser output mirror [6] – see Fig. 3. The reference beam passes through the Bragg modulator without frequency shift and the signal beam is deflected to the first order of diffraction (frequency \(\nu_o + \nu_B\)).

The beam scatters from the object and the frequency of the returning signal is \(\nu_o + \nu_B \pm \nu_D\). After being Bragg shifted again the signal is reflected from the laser mirror and then passes through the Bragg modulator (without change of frequency) and gets on the photodetector, where it interferes with the reference beam. As a result of interference the received signal frequency is \((\nu_o + \nu_B \pm \nu_D) - \nu_o = 2\nu_B \pm \nu_D\) (the same as in the double Bragg shifted configuration). The disadvantages of this setup are quite similar to the previous one. Additionally, light coming back to the laser can disturb significantly its work conditions and cause a great increase in optical noise.

3. Undesirable effects

As a result of experiments with simple heterodyne interferometers we found two undesirable phenomena occurring in acoustooptic Bragg cells. The first one is a deflection effect, which is shown in Fig. 4. It appears in two cases: running and standing ultrasound wave. The light beam propagates in medium almost perpendicularly to the gradient of refraction index \(n\) and it deflects at the angle \(\theta\)

\[
\theta = L \text{ grad}(n)
\]

where \(L\) is the active length of Bragg cell. Beam deflection can cause intensity modulation at ultrasound frequency \(\nu_B\). In the case of the running ultrasound wave
propagating along the $z$ axis the refraction index $n$ is a sinusoidal variable,

$$n(t, z) = n_0 + \Delta n \sin(Kz - \Omega t)$$  \hspace{1cm} (4)

where: $K$ – the ultrasound propagation coefficient, $\Omega = 2\pi \nu_B$ – the angular frequency of ultrasonic wave. In this case, $\text{grad}(n)$ is equal to:

$$\text{grad}(n) = \frac{2\pi \Delta n}{\Lambda} \cos(Kz - \Omega t)$$  \hspace{1cm} (5)

where $\Lambda$ is the ultrasound wavelength.

From Eqs. (3) and (5), change of deflection angle occurs with acoustic wave frequency $\Omega$. Maximum angle of deflection is given by:

$$\Delta \theta_{\text{max}} = 4\pi \frac{L}{\Lambda} \Delta n.$$  \hspace{1cm} (6)

The effectiveness of the deflection depends on relation between ultrasonic wavelength and width of light beam. The gradient of refraction index is not constant across the beam diameter and the divergence of the laser beam occurs as a result of it [7]. The angle of divergence is

$$\delta \theta_c = \Delta \theta_{\text{max}} - \Delta \theta \bigg|_{z = D/2 + \nu_B t} = 4\pi \frac{L}{\Lambda} \Delta n \left(1 - \cos \frac{\pi D}{\Lambda} \right)$$  \hspace{1cm} (7)

where $D$ is a diameter of the light beam, $\nu_B$ is a velocity of the ultrasound wave.

Different part of the beam reaches active surface of a photodetector in consequence. It causes the modulation of photocurrent frequency $\Omega$. This effect can be reduced by
enlarging the light beam diameter, but this is unfavourable with regard to the limited size of the photodetector area and active area in the acoustooptic Bragg cell.

The second undesirable effect from the point of view of laser vibrometry is so-called standing wave effect. Bragg modulators can utilize standing or running ultrasonic wave. In the case of standing wave the intensity of the \( m \)-th order diffraction beam is proportional to a square of the \( m \)-th order Bessel function with an argument dependent on time

\[
I_m = I_0 J_m^2 \left( \frac{\Gamma_0 \cos(\Omega t)}{2} \right)
\]

where \( \Gamma_0 \) is the coefficient dependent on intensity of ultrasonic wave and length of ultrasound interaction with the light. The intensity of light in zero diffraction order is:

\[
I_0 = J_0^2 \left( \frac{\Gamma_0 \cos(\Omega t)}{2} \right)
\]

After spectral analysis of expression (9) one can obtain:

\[
J_0^2 \left( \frac{\Gamma_0 \cos(\Omega t)}{2} \right) = \left[ J_0^4 \left( \frac{\Gamma_0}{2} \right) + 2 \sum_{a=1}^{\infty} J_a^2 \left( \frac{\Gamma_0}{2} \right) \right] \\
- \left[ 4 \sum_{a=0}^{\infty} J_a^2 \left( \frac{\Gamma_0}{2} \right) J_{a+1}^2 \left( \frac{\Gamma_0}{2} \right) \right] \cos(2\Omega t) \\
+ \left[ 2 J_1^4 \left( \frac{\Gamma_0}{2} \right) + 4 \sum_{a=0}^{\infty} J_a^2 \left( \frac{\Gamma_0}{2} \right) J_{a+2}^2 \left( \frac{\Gamma_0}{2} \right) \right] \cos(4\Omega t) - \ldots
\]

Fig. 5. Standing wave effect.
Expression (10) shows that the beam intensity in zero order diffraction is amplitude modulated frequency $2\Omega$ and higher even harmonics. The occurrence of the intensity modulation in standing ultrasound wave appears because the time domain standing wave is equivalent to motionless phase grating with parameter $\Delta n$ changing in time domain (Fig. 5). The depth of modulation in zero order depends on the value $\Gamma_0$, however all non-zero diffraction orders are amplitude modulated with a hundred-percent efficiency and frequencies of that modulation are $2\Omega, 4\Omega, etc.$

In the case of partial standing wave the observed effect is smaller (a decrease of modulation depth in all diffraction orders). The occurrence of even residual acoustic standing wave causes such undesirable intensity modulation of output beam. Hence, the spectral analysis of the second harmonic in the Bragg diffracted beam is a useful method for establishing usability of Bragg cells.

4. Configuration

After the analysis of the above configurations and effects which can disturb a heterodyne signal, important conclusions can be drawn about the principles of construction and optimisation of simple heterodyne interferometers:

– the reference beam which dominates in interference signal should not cross through the Bragg modulator to avoid its amplitude modulation;

– the very important matter is to prevent the light from returning to the laser and disturbing its work. This can be done by applying polarising optics.

As a result of the above analysis a practical model of heterodyne interferometer was developed (Fig. 6). The linearly polarised He-Ne laser was applied as a source of coherent light in the setup. The ratio between measurement and reference beams can be settled by changing polarization angle. The optimal value is 50:50. Light from the laser reaches polarisation beamsplitter to create two beams. The reference one after reflection from the mirror crosses beamsplitter and falls on the photodetector. The

![Fig. 6. Developed configuration.](image-url)
quarter waveplate in connection with the mirror provides change of light polarization into orthogonal and ensures its passing through the beamsplitter instead of being reflected from it. The measuring beam passes through the Bragg cell (the frequency $\nu_o + \nu_B$), the set of lenses and then strikes incident the object. The set of lenses plays two roles – it increases the aperture of the setup and it focuses the beam on the object. For typical 3 meter measuring distance, the spot diameter on the object is about 100 µm. The light is scattered from the object surface and a small amount of it goes back to the system. Next, the light passes through the Bragg cell again (frequency $\nu_o + 2\nu_B \pm \nu_D$). The scattered light has not defined polarization which causes that it is not necessary to use the quarter waveplate in the arm of the interferometer. It presence does not change the level of the heterodyne signal obtained from the setup. The additional advantage of this arrangement is that light reflected from lenses surface does not reach a photodetector and does not disturb measured signal. The utilization of the polarising optics causes reduction of optical power losses and the backcoupling to the laser is eliminated as well. As a photodetector sensitive p-i-n photodiode was applied. The signal level in this arrangement is about 20 dB higher than that on the arrangement from Fig. 2. The signal to noise ratio was over 20 dB for white sheet of xero paper as an object and measuring distance over 3 m. It was impossible to obtain such S/N ratio in other configurations.

The next step of our studies was to develop demodulation circuits. The signals coming from laser Doppler vibrometer have a wide spectrum. In the case of the model investigated in this work the frequency of signal is 160 MHz (double Bragg shift). This carrier wave is frequency modulated by the Doppler signal. The deviation of frequency depends linearly on the movement speed of measured object. In the case of using the He-Ne laser with wavelength $\lambda = 632.8$ nm the deviation is 1.58 MHz per 1 m/s of the velocity [8]. As a result, it is necessary to use FM demodulator which accepts a wide frequency spectrum. In the case of large amplitude of vibration the best choice is usage of a phase locked loop [9]. On the other hand, if sensitivity is a primary parameter – detection of very small vibrations – the demodulator should be a narrow-band one. For example, for the same laser and movement speed 1 µm/s the Doppler frequency change is 1.58 Hz. The standard radio FM demodulator technique can fulfil proper and accurate demodulation. As is shown a large spread of signal frequencies coming from vibrometer requires at least two different demodulators to cover the whole measurement range.

5. Vibrometer calibration

To measure vibrations it is necessary to scale the electronics circuits. This can be done by using an object (standard) whose amplitude of vibrations is well known. It can be an exciter whose amplitude of vibrations is properly scaled thanks to measuring it in standard Michelson interferometer [10]. For scaling purposes, the exciter was driven with sinusoidal voltage and produced vibrations. Signals coming from the photodetector were observed at the oscilloscope. These signals were represented by
interference fringes as a result of optical length change in one of the arms in the interferometer. The amplitude of vibrations can be easily determined by counting the number of interference fringes occurring in half period of the vibrations. The voltage driving the exciter was used as a reference signal. After scaling of the exciter the next step was to calibrate the vibrometer (Fig. 7a). The exciter was driven by sinusoidal signals for which it had been scaled before. Assuming the linear dependence of vibration amplitude and value of the voltage driving the exciter it allowed us to make a scaling diagram (Fig. 7b) for different driving frequencies. The value of signal coming from vibrometer demodulator corresponds to the velocity of object. In the case of sinusoidal vibrations with settled frequency, the amplitude of vibrations is
proportional to the speed of movement. It permits calibration of the output signal from demodulator directly in the values of vibration amplitude. This method is inconvenient because of the necessity of graduation for every measured frequency and due to vibrations being limited exclusively to sinusoidal ones. To obtain calibration independent of the character of vibration and its frequency, the calibration procedure should rely on the velocity values because it is the natural signal from the Doppler vibrometer. This can be done for sinusoidal vibrations. The dependence between amplitude and velocity of vibrations in this case is given by the simple formula:

\[ \text{amplitude} \propto \text{velocity} \]

Fig. 8. Sample measurements made using the model of laser Doppler vibrometer: a – pulse response, b – two dimensional scan of loudspeaker’s diaphragm.
Laser Doppler vibrometry ...

\[ V(t) = \omega A \cos(\omega t) \]  

(11)

where \( V(t) \) is the vibration velocity, \( A \) – the vibration amplitude, \( \omega \) – the vibration angular frequency. Taking advantage of this one can determine sensitivity of vibrometer in velocity domain. For the setup described and the case of sinusoidal vibrations (Fig. 7b) it was 0.2 [V/(m/s)]. This value is correct for all types of vibrations, provided that spectrum of vibrations is contained in the frequency range of FM demodulator. To obtain amplitude signal from the vibrometer directly an integration of the output velocity signal (analogue or digital) should be applied.

6. Experimental results

Figure 8 shows some measurements performed by the vibrometer being constructed. Figure 8a demonstrates typical pulse response of a loudspeaker taken at the central point of the diaphragm. Trace 1 on the scope record is its velocity response and trace 2 is a driving signal of the loudspeaker. This vibration signal carries information about mechanical resonance frequency which can be determined by spectral analysis of that signal. Figure 8b presents two dimensional scan of vibrations of the loudspeaker’s diaphragm at 1 kHz excitation. It is an example of mechanical modes analysis. It was performed by manual scanning from point to point. Measurement points were marked on the loudspeaker to provide satisfactory pointing accuracy about 1 mm with the raster size of 1 cm. In this case pointing precision is limited by hand-held accuracy, especially when the surface has a complex shape.

It should be considered that all measurements presented are scaled on velocity of the vibrating object (directly from FM demodulator without any integration circuits). Other parameters of vibrations can be obtained by mathematical analysis, the amplitude by integration of the velocity signal, the acceleration by derivation of the velocity signal. Measurements were performed with the use of an arrangement of demodulation based on PLL loop.

7. Conclusions

The main principles of how to build a simple and low cost heterodyne vibrometer have been shown. Two undesirable effects appearing in the Bragg modulator which are commonly utilized in the laser vibrometry have been presented. As has been shown the standing wave effect can be very a useful method for determining the Bragg cell applicability in this area. The workable model of the simple heterodyne interferometer which allows measurement of vibrations of objects with light scattering surface and the main principles of demodulation circuits design have been demonstrated. The calibrations of arrangement were presented as well. Some measurements of loudspeaker’s diaphragm vibrations with application of designed vibrometer were performed.
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Application of the 3-frame interferometry and the crater replica method for investigation of laser accelerated macroparticles interacting with massive targets in the Prague Asterix Laser System (PALS) experiment

In the present paper results from our experiments with macroparticles, accelerated at first to high speeds by the PALS iodine laser and subsequently hitting massive targets and creating craters, are presented. The main aim of these investigations concerned the influence of wavelength on the efficiency of macroparticles acceleration and creation of craters. To this end, two different harmonics of the PALS laser beam ($\lambda_1 = 1.315 \mu m$ and $\lambda_3 = 0.438 \mu m$) and several types of targets (simple massive planar Al targets as well as much more elaborated double targets consisting of $6 \mu m$ thick Al foils or disks placed in front of the massive target at the distance of either $200 \mu m$ or $500 \mu m$) were used. All these targets were irradiated by the iodine laser beam with its parameters very much the same for both harmonics: the energy of $130 J$, the focal spot diameter of $250 \mu m$, and the pulse duration of $400 ps$. Velocities of accelerated extracted foil fragments or disks as well as electron density distributions of plasma streams were determined by means of the 3-frame interferometry. Shapes and volumes of craters were obtained employing the crater acetate cellulose replica technology and microscopy measurements. The data from these experiments provided valuable information concerning the ablative plasma generation and crater creation processes.

Keywords: laser produced plasma, three-frame interferometry, optical microscopy, macroparticle, single and double targets, crater, classical inverse bremsstrahlung absorption, shock wave, efficiency of laser radiation absorption, ablation loading efficiency.
1. Introduction

The laser-planar target experiments play a fundamental role in the study of laser-produced plasma physics and the problems related to inertial confinement fusion (ICF). They represent the starting point for numerous physical phenomena analysis and provide information necessary for research work in the domain of complex configurations such as implosion experiments with spherical targets on which the idea of laser fusion is based.

Experiments utilizing ablatively accelerated planar targets can model large pellet shells in their early implosion phase. Instead of imploding a pellet, a disk target can be accelerated and treated as a section of a sphere (until convergence effects dominate). Another very important problem which can be investigated in planar experiments is the value of initial entropy injected under the laser beam action into the solid part of targets – the key characteristics to find the conditions of high compression of the fuel in ICF. One of the main advantages of experiments with the disk targets lies in the fact that they make the diagnostic of the cold inner surface of spherical targets possible, otherwise rather difficult to access in experiments with the real pellets. The main areas of thin foil experiments are usually the physics of laser-plasma interaction (radiation absorption, heat transport, electron distribution) and ablation acceleration physics (generation of the ablation pressure and the shock wave, hydrodynamic efficiency, energy transfer, Rayleigh–Taylor instability, laser pulse uniformity requirements and so on), e.g., [1]–[3].

Some other applications of the experiments using simple or more complex planar targets, very useful in fusion research, are, e.g., the possibility of extremely high pressure generation and ablative acceleration of very fast macroparticles. Such fast macroparticle is one of the most effective types of igniting drivers [4] for the fast ignition approach [5], [6] in ICF.

When thin foil accelerated by means of the laser beam to reach velocity of more than $10^7$ cm/s collides with the target at rest, a very strong shock wave and a pressure of 100 Mbar can be generated.

The craters created by the accelerated foil can provide some very useful data explaining many features of interest.

The two main aims of our investigation were determination of the macroparticles acceleration and craters creation efficiency depending on:

– origin of macroparticles (an extracted foil fragment or prefabricated disk),

– wavelength of the laser beam.

In the first case, our natural expectation was that, contrary to the extracted foil fragment variant, the prefabricated disk attached to a mylar foil can be accelerated with negligible energy loss required for its extraction and practically no lateral heat conduction loss.

The second aim was connected with a significant influence of incident radiation wave length on the efficiency of laser light absorption, parameters of evaporated
plasma and, as a consequence, on the energy of macroparticles. Moreover, different laser radiation wavelengths under certain conditions can prove the dominant role of physically different mechanisms of laser radiation absorption. Indeed, at the laser intensity, for example, $5 \times 10^{15} \text{ W/cm}^2$, the value of $I \lambda^2$ can be changed about one order of magnitude due to a change of the wavelength from $\lambda_1 = 1.315 \mu\text{m}$ to $\lambda_3 = 0.438 \mu\text{m}$ (i.e., from $I \lambda_1^2 = 9 \times 10^{15} \text{ W}\mu\text{m}^2/\text{cm}^2$ to $I \lambda_3^2 = 10^{15} \text{ W}\mu\text{m}^2/\text{cm}^2$). The smaller value of $I \lambda^2$ corresponds to a domination of the classical inverse bremsstrahlung absorption mechanism of laser radiation, while in the case of $I \lambda^2 = 10^{15} \text{ W}\mu\text{m}^2/\text{cm}^2$ the resonance absorption mechanism plays the dominant role (with the fast electrons responsible for the absorbed laser energy transfer into the target [7], [8]). The main goal of this experiment was to study physical processes connected with irradiation of double targets. For this reason, complex targets irradiated by the laser intensity corresponding to classical inverse bremsstrahlung absorption mechanism ($I = 0.66 \times 10^{15} \text{ W/cm}^2$) were investigated. Nevertheless, the investigation of crater creation under the action of sufficiently high laser intensity at the different wavelengths allows us, in principle, to study an influence of the resonant absorption and the fast electron generation on the parameters of laser-produced plasma [9].

2. Experimental set-up

The experiment was carried out with the use of the PALS iodine laser facility [10]. The laser system, together with the 3-frame interferometry set-up, is shown schematically in Fig. 1. Plasma was generated by the laser beam with its diameter at the vacuum chamber entrance window about 290 mm focused by means of an aspherical lens with focal lengths of 627 mm and 600 mm for the first and the third harmonic, respectively. Targets were irradiated by the iodine laser beam: $E_L = 130 \text{ J}$ for both harmonics, the focal spot diameter of 250 $\mu\text{m}$, and the pulse duration of 400 ps.

The choice of 130 J of the laser energy for both harmonics was partially influenced by the third harmonic laser energy limitation (the maximum energy available in the third harmonic was about 180 J). On the other hand, however, it was found that the laser energy over certain threshold can result in overheating of macroparticles (particularly disks due to their limited volume) thus leading to their disintegration before an impact. The selected laser energy seemed to be safe enough for both the foils and the disks with thickness of 6 $\mu\text{m}$.

To study the plasma expansion and macroparticle acceleration, a 3-frame interferometric system with automatic image processing was used. The diagnostic system was illuminated by the third harmonic of the iodine laser. Due to mechanical constrains, the minimum delay between subsequent frames could not be shorter than 3 ns. The time instant of 0 ns corresponds to the maximum of heating laser pulse.

In these experiments several different types of targets (simple massive planar Al targets as well as much more elaborated double targets consisting of 6 $\mu\text{m}$ thick Al
foils or disks placed in front of the massive target at the distance of either 200 \( \mu \text{m} \) or 500 \( \mu \text{m} \) were used. Double targets with the gap of 500 \( \mu \text{m} \) were used for the accelerated macroparticle velocity determination (this gap was large enough for registration of two subsequent frames during the macroparticle flight). However, from the point of view of the crater creation itself, such a long distance seemed to be rather unprofitable due to the possibility of accelerated (and heated) macroparticle premature disintegration. Thus, for this purpose a shorter distance of 200 \( \mu \text{m} \) was set.

The constructions of the double targets used are presented in Fig. 2. Disks were attached to a supporting mylar foil on the side opposite to the incoming laser beam. To keep the same conditions of the target heating as in the disk case, the front side of all the remaining double targets was covered by the same mylar foil with thickness of 2.5 \( \mu \text{m} \). The target type shown in Fig. 2b was used for the crater creation, while the type in Fig. 2c served for the disk velocity measurement. Shielding of the disk by the Al foil with a hole a bit smaller than the disk area prevented heating and acceleration of the mylar foil itself. Otherwise, the mylar foil, heated by even a very weak laser radiation falling outside of the disk, would undergo its own acceleration to velocities much higher than \( 10^7 \text{ cm/s} \), thus screening the disk (the motion of which is considerably slower) from observation. For the crater creation by the disk, however, the double targets without the shielding Al foil were used as the edge of the hole in the covering foil was also heated and its fragments would participate in the crater creation.
To irradiate the area of the disk as completely as possible (simultaneously avoiding any irradiation of the off-disk region) the focal spot diameter was taken equal to 250 µm (i.e., slightly smaller than the disk diameter). Such irradiation scheme was applied to all remaining targets.

3. Results of interferometric measurements

One of the most important problems connected with the crater creation by very fast macroparticles is determination of their velocities at the moment of an impact. Some information about methodology of macroparticles velocity measurement was presented earlier. For the time being, we would like to pay some attention to a particular problem, which appeared during our experiments. It will be discussed on the basis of Fig. 3. As was already mentioned earlier, for macroparticle velocity measurement the distance of 500 µm between the foil (or the disk) and the massive target was taken. It can be seen in Fig. 3a that the macroparticles start moving about 1 ns after the laser action. So, to see clearly the macroparticle position after its start in the first frame, we had to delay this frame by 2 ns with respect to the heating beam. This delay and relatively high velocities of macroparticles accelerated by the third harmonic caused, however, that the distance of 500 µm turned out to be not sufficiently long for this velocity measurement (see Fig. 3b). Nevertheless, in the case of the third harmonic this measurement allowed us to estimate the velocity of accelerated macroparticles.

Somewhat contrary to our initial expectation, the macroparticle average velocity determined on the basis of several interferometric sequences seems to be approximately the same for both types of the double targets. This velocity for the first harmonic is equal to $(6 \pm 0.2) \times 10^6$ cm/s. In the case of the third harmonic, however, the situation is a bit more complex. The sequence of interferograms in Fig. 3b shows that the macroparticle in the period of 3 ns can travel a distance of 400–500 µm. Thus we can conclude that the macroparticle velocity amounts to $(1.3–1.7) \times 10^7$ cm/s. Although, even if our velocity measurement is not absolutely precise, it clearly shows that under...
Fig. 3. Two sequences of interferograms showing the ablative plasma expansion and the foil motion for the first harmonic (a) and the third harmonic (b).
the same irradiation conditions the velocity of macroparticles for the third harmonic is 2.1–2.7 times greater than that for the first harmonic.

To determine the electron density distribution on the basis of the phase shift one, the special numerical methods have been prepared [11]. In the case of axial symmetry of the plasma, the relation between the phase of probing radiation and the electron density in a selected cross-section $z$ of the plasma is expressed by the well-known Abel integral equation:

$$\int_{-\infty}^{\infty} \frac{f(x)}{\sqrt{x^2 + \Delta t^2}} dx = f(0)$$

Fig. 4. Sequences of the electron density isodensitograms for the first harmonic of the laser radiation and the three types of the targets: single massive target, double target with foil, and double target with disk. The white fields in the vicinity of the target surface mean the opacity zones.
where: \( S(y) \) – the phase distribution \([\text{rad}/2\pi]\), \( y \) and \( r \) – the coordinates corresponding to one another in an interferogram plane and a real plasma cross-section, respectively,

\[
S(y) = 2 \int_{y}^{1} f(r) \frac{r}{\sqrt{r^2 - y^2}} \, dr
\]  

Fig. 5. Sequences of the electron density isodensitograms for the third harmonic of the laser radiation and the three types of the targets: single massive target, double target with foil, and double target with disk. The white fields in the vicinity of the target surface mean the opacity zones.
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\[ f(r) = 4.46 \times 10^{-14} \lambda R n_e(r), \]
while \( n_e(r) \) – electron density distribution \([\text{cm}^{-3}]\), \( \lambda \) – wavelength of laser radiation \([\text{cm}]\), \( R \) – radius of plasma for the selected cross-section \([\text{cm}]\).

Calculation of the \( n_e(r) \) in many cross-sections along the axis “\( z \)” allows the full electron density distribution \( n_e(r, z) \) to be reconstructed, which can be presented in different graphical forms.

Since the macroparticle acceleration results from plasma ablation, the interferometric investigation of electron density distributions during the early stage of its expansion can provide some interesting information about the ablation and acceleration processes.

In Figures 4 and 5 the sequences of the electron density distributions in the form of isodensitograms at different moments of plasma expansion for all of the tested target types irradiated by the two wavelengths are shown. The plasma stream boundary is represented here by the electron density contour \( n_e = 1 \times 10^{18} \text{ cm}^{-3} \). Information about the changes of number of electrons in interesting regions of the plasma stream could be provided by the diagrams of the linear electron density corresponding to these electron density distributions, presented in Fig. 6.

On the basis of the interferometric measurements we have obtained the following information.

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**Fig. 6.** Linear electron density distributions for the tested targets: M – single massive target, M+F – double target with foil, and M+D – double target with disk as well as for the first harmonic and the third harmonic at different moments of plasma expansion.
The first harmonic. At $\Delta t = 2$ ns the distribution of $n_e(r, z)$ and diagrams of $N_e(z)$ are nearly identical for all the types of the targets. Some differences are only seen at the later moment ($\Delta t = 8$ ns) and particularly concern:
- the size of thin plasma clouds,
- the growth of $n_e$ and $N_e$ in the vicinity of the massive target.
This means, that in the later period the differences in the targets construction play more important role.

The third harmonic. Differences in $n_e(r, z)$ for the different types of the targets are already seen at the moment of 2 ns. While the outer shapes of the plasma streams are similar in all the cases, forms of the dense plasma outflows for the single massive target and the double targets differ considerably. In the case of a single massive target this outflow is elongated along the axis, meanwhile in the other cases the axial plasma stream is shorter and is additionally equipped with a plasma ring located close to the target (seen in the form of wings in the electron density distributions).

In the subsequent period the differences between the single target and the double ones grow and concern both the thin plasma (i.e., size and shape of the plasma stream) and the dense plasma, represented by the inner equidensity lines.

A comparison of the linear electron density diagrams for the two harmonics reveals that the electron number in the region close to the target ($z < 0.05$ cm) is smaller for the case of the third harmonic.

4. Characteristics of craters

In order to obtain information about the shape and dimensions of individual craters, their replicas were made of acetate cellulose. To reconstruct quantitatively the crater shape the crater replica microphotography was taken. Then the crater shape in a chosen cross-section was digitized to be data for further computations.

The created craters as well as their replicas for the targets tested and the two harmonics of the laser radiation are shown in Figs. 7 and 8 (here, $R_c$ and $H_c$ mean the radius and the depth of the craters, respectively). The shapes of the craters from Figs. 7 and 8 are presented in Fig. 9. As some of these shapes deviate substantially from an axial symmetry, they are shown in two mutually perpendicular projections. Based on these shapes the following conclusions can be drawn:
- the craters obtained as a result of a direct laser beam-massive target interaction have approximately a hemispherical shape,
- in the case of double targets, the shapes of craters are similar to each other, but for the first harmonic the craters are shallower and less symmetrical whereas for the third one the crater shapes are close to hemispherical ones.

The essential difference between the first and the third harmonic cases concerns the volumes of craters. Due to the shape irregularity of some craters, their volumes were split into four quarters and the respective volume of each quarter of the crater was established independently. The volumes of the craters presented in Figs. 7–9 are given in the Table.
Fig. 7. Craters (left, top view) and crater replicas (right, side view) photographs for the first harmonic of the laser radiation and the three types of the targets: single massive target (a), double target with foil (b), and double target with disk (c).

Fig. 8. Craters (left, top view) and crater replicas (right, side view) photographs for the third harmonic of the laser radiation and the three types of the targets: single massive target (a), double target with foil (b), and double target with disk (c).
Fig. 9. Crater shapes and dimensions for the first (a) and the third (b) harmonics of the beam radiation, where: M – single massive target, F+M – double target with foil, and M+D – double target with disk.

Fig. 10. Illustration of the mylar foil participation in the crater creation by the disk. Notations: D+M – double target with disk, and MF+M – double target with mylar foil only (without disk).
Table. Set of the volumes of craters (in cm$^3$) for the three types of the targets.

<table>
<thead>
<tr>
<th></th>
<th>Single massive target</th>
<th>Double targets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foil + massive target</td>
<td>1.03x10^{-4}</td>
<td>0.64x10^{-4}</td>
</tr>
<tr>
<td>Disk + massive target</td>
<td>0.62x10^{-4}</td>
<td>0.62x10^{-4}</td>
</tr>
<tr>
<td>1st harmonic</td>
<td>7.46x10^{-4}</td>
<td>3.31x10^{-4}</td>
</tr>
<tr>
<td>3rd harmonic</td>
<td>3.05x10^{-4}</td>
<td>3.05x10^{-4}</td>
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This table clearly shows that for the same harmonics the differences in the volumes of craters between the two types of double targets are relatively small, whereas much more noticeable differences between the volumes of craters are observed in the case of different harmonics. For the massive targets the crater volumes created by the third harmonic are about 7.2 times larger than those created by the first one. In the case of the double targets, the volumes of craters for the third harmonic are about 5 times larger than those for the first one. Differences in efficiencies of the creation of craters by means of the foil fragments and disks with respect to the creation of craters by the direct laser beam action amount to 60% and 40–45% for the first and the third harmonic, respectively.

Finally, we would like to make some notes about our tests concerning the possibility of participation of the holding mylar foil in the crater creation by the disk. Since in this case the mylar foil was not covered by the Al foil (see Fig. 2b), the outer part of the mylar foil was accelerated as well and could participate in the crater creation together with the disk itself. To estimate this unwanted effect, one laser shot was fired onto the double target with the disk removed using the third harmonic. The crater obtained by this mylar foil only impact and, for comparison, the crater generated by the disk, are shown in Fig. 10. As can be seen in this figure, participation of the outer part of the laser beam beyond the disk in the crater creation is quite negligible (grey fields on the cross-section of the mylar foil impact crater). Of course, assumption of independent participation of the holding mylar foil in the crater creation in the presence of the disk is not entirely correct. A relatively weak shock wave generated by the mylar foil located outside of the disk is within the reach of the main, much stronger shock wave generated by the disk, so in such a case only a minor amplification of the main shock wave occurs. Thus this method of the determination of the holding mylar foil participation in the crater creation can provide only a very limited piece of information.

5. Discussion of the experimental results

At the beginning we would like to discuss the problem of the direct crater creation by means of the laser beam. The crater volume depends on a fraction of the laser beam energy $E_L$ which is transferred to the shock wave. The process of the laser energy transfer consists of two stages. The first one concerns the laser radiation absorption $E_{ab}$ by an evaporated part of the target. The efficiency of this absorption is expressed by the ratio $K_{ab} = E_{ab}/E_L$. The second one is connected with a transfer of the ablative plasma energy into the shock wave propagating into a non-evaporated solid part of the
target. The ratio of the shock wave energy $E_{sw}$ and the laser radiation absorption $E_{ab}$ is the ablation loading efficiency $\sigma = E_{sw}/E_{ab}$ [8]. Hence, the crater volume $V_{cr}$ is proportional to the product of these two efficiencies, $V_{cr} \propto K_{ab}\sigma E_L$ [9].

As was shown in [9], in the case of the classical inverse bremsstrahlung, the absorption of the laser radiation taking place in the region of the critical plasma density $\rho_{cr}$ and the ablation loading efficiency $\sigma$ are given by:

$$\sigma = \frac{\sigma_1}{1 + 0.65 \frac{\xi}{R_L}}$$

(2)

$$\sigma_1 = \frac{0.94}{(\gamma_s + 1)^{1/2}} \left( \frac{\rho_{cr}}{\rho_0} \right)^{1/2}$$

(3)

$$\rho_{cr} = 1.83 \times 10^{-3} \frac{A}{Z\lambda^2}$$

(4)

where: $\rho_0$ – solid state density, $R_L$ – radius of laser beam focal spot, $\lambda$ – wavelength of the laser radiation, $A$ – atomic weight of the plasma ions, $Z$ – average degree of ionization of the plasma ions, $\gamma_s$ – the adiabatic index in the condensed matter ($\gamma_s = 5/2$ for Al), $\xi$ – size of the plasma torch given by

$$\xi = 1.2 \left( \frac{K_{ab} I}{\rho_{cr}} \right)^{1/3} \tau$$

(5)

while $I$ denotes laser radiation intensity, $\tau$ – laser pulse duration.

The dependence of ablation loading efficiency on the laser radiation wavelength consists of two components. The first one is expressed by a proportion $\sigma \propto \rho_{cr}^{1/2} \propto \lambda^{-1}$. This strong dependence is connected with an increase of the plasma torch density on the solid target–plasma boundary when the laser radiation wavelength decreases. The second component of the growth of ablation loading efficiency $\sigma$ with a decrease of wavelength $\lambda$ results from a weakening role of the two dimensional plasma torch expansion. According to Eq. (5) $\xi \propto \rho_{cr}^{1/3} \propto \lambda^{2/3}$ and the factor of the ablative loading efficiency decreasing due to two dimensional expansion, decreases when the wavelength increases. On the basis of the well known results it is obvious that at the inverse bremsstrahlung absorption the mass of the plasma torch $m$ increases when the laser wavelength decreases, but, on the contrary, its velocity $v_{cr}$ (and temperature $T$) decreases when laser radiation decreases. These relations are expressed by the formulas [12]:

...
where $\gamma$ denotes the adiabatic index in the plasma torch.

So, it means that in the case of the third harmonic the laser radiation is absorbed in a denser plasma region in comparison with that for the first one (see formula (4)). Thus the mass of the expanding plasma in the case of the third harmonic is larger (see formula (6)), but its temperature and, of course, its expansion velocity, are smaller, since $\xi \propto v_{cr} \propto \lambda^{2/3}$ (see formula (7)).

The results calculated using formulas (2)–(5) have shown that the ablation efficiency in the case of the third harmonic is 5 times greater than that for the first harmonic. Meanwhile, this efficiency, determined on the basis of the experimental data, is 7.2 times greater. Taking into account the crater volume $V_c \propto K_{ab} \sigma E_L$, the ratio of the crater volumes for the third and the first harmonic is given by:

$$
\frac{V_{c3}}{V_{c1}} = \frac{\sigma_3}{\sigma_1} \frac{K_{ab3}}{K_{ab1}}.
$$

Because the experimental data give this ratio equal to 7.2, while from the theoretical calculation $\sigma_3/\sigma_1 = 5$, well now $K_{ab3}/K_{ab1} = 7.2/5 = 1.4$. This estimation allows us to conclude that in the case of the third harmonic the absorption efficiency of the laser radiation is higher than that for the first harmonic by the factor of 1.4.

The important conclusion can be drawn from the comparison of the crater volumes in the case of the double targets. The efficiency of the energy transfer during the non-elastic impact of the accelerated macroparticle with the wall in rest $\beta$ (the fraction of the macroparticle energy transferred to the wall) depends on the relation between the densities of the macroparticle $\rho_p$ and the wall $\rho_w$ as well as the adiabatic indexes of both materials. According to the calculation in [13], for the same densities of colliding elements ($\rho_p = \rho_w$) and the adiabatic index $\gamma_s = 5/2$ for Al, the efficiency of the macroparticle energy transfer $\beta = 0.58$. This value is in a good agreement with the experimental value of the relative efficiency of the crater creation by means of the macroparticles with respect to that in the case of direct laser beam action for the first harmonic, which amounts to about 60%.

This agreement of the theoretical and experimental results testifies that both the foil fragment and the disk conserve their compact form and the initial Al density until
their collision with the massive part of the target. In the case of the third harmonic the above mentioned relative efficiency of the crater creation is equal to 0.4–0.45. This lower efficiency allows us to conclude that in this case macroparticles undergo decomposition and they have their density at the impact moment smaller than the value of the initial Al density in solid state.

The considerably lower relative efficiency of the energy transfer to the massive part of the target at the collision moment in the case of the third harmonic results from the much larger energy of the shock wave propagating in the macroparticles in comparison with the case of the first harmonic. It is in agreement with the experimental results for the massive targets and the theoretical prediction of a strong growth of ablation loading efficiency with a decreasing laser radiation wavelength. Indeed, when the laser radiation of the third harmonic acts, the shock wave energy and, in consequence, the temperature of the macroparticles (both foil and disk) are several times higher than those for the first harmonic. Therefore, the decomposition of the macroparticles before collision in the third harmonic case runs considerable faster.

Macroparticle velocity to an order of magnitude is equal to the velocity of the shock wave which is proportional to the ablation pressure of plasma torch \( v_{sw} \propto (P_a / \rho_0)^{1/2} \), where \( P_a \propto \rho_{cr} v_{cr}^2 \). So, it is simple to obtain from Eq. (7) that the macroparticle velocity \( u \propto K_{ab}^{1/3} \lambda^{4/3} \). Taking into account that the absorption efficiency of the third harmonic laser radiation is higher than that for the first harmonic by the factor of 1.4 we can easily ascertain that the macroparticle velocities in the case of the third harmonic should be higher by the factor of 1.6 than that in the case of the first harmonic. Such simple estimations qualitatively explain the experimental results of the macroparticle velocities determined by the interferometric measurements.

Moreover, because the ratio of the crater volumes for the third and the first harmonic is equal to about 5, assuming the same masses of the macroparticles for both wavelengths, the ratio of the velocities of macroparticles should be equal to the square root of 5 \((i.e., \approx 2.24)\). Taking into account that the relative efficiency of the crater creation for the third harmonic is about 25% smaller in comparison with that for the first one, the velocities of the particles in the case of the third harmonic should be close to the upper limit of the experimentally determined velocity range.

The conclusion, resulting from the linear electron density diagrams, where the electron number in the vicinity of the target appears smaller in the case of the third harmonic in comparison with the first harmonic case, can be explained in the following way. From the above discussion the temperature of the plasma torch \( T \propto \lambda^{4/3} \) (see formula (7)). In our experimental conditions the ratio of the wavelengths of the first and the third harmonic equals 3, so the plasma temperature for the first harmonic is 4.33 times higher. As the interferometric measurements provide the electron density and the amount of free electrons in the plasma stream, the plasma temperature is a very important parameter from the point of view of the ionization degree. The higher ionization degree in the case of the first harmonic means the higher free electron
number. Hence, one gets the higher linear electron density in the vicinity of the target for the case of the first harmonic.

Differences in the \( n_e(r, z) \) distributions related to the first and the third harmonic are connected with some properties of the PALS iodine laser beam. In comparison with a solid laser, the iodine laser, having its active medium in a gaseous form, has a more non-uniform distribution of the intensity across the cross-section of the laser beam. In the case of the low output laser energy, below 180 J, the intensity distribution has approximately a flat character along the radius. For higher output laser energy the laser amplifiers cause a decrease in the center of this distribution, the depth of which increases with increasing laser energy. This situation concerns only the first harmonic of the laser radiation. In the case of the third harmonic, to have any laser energy, the output laser energy should be 2–3 times larger, mainly due to the wavelength transformation efficiency which is in the range of 30–50%. The concave character of the intensity distribution is even more pronounced in the case of the third harmonic because of the nonlinear transformation of the DKDP crystal.

Similarity of the character of the plasma expansion for all the targets tested in the case of the first harmonic is connected with the flat intensity distribution of the laser radiation. This homogeneous irradiation of the double targets allows the flat form of the foil (and disk) to be conserved and the plasma expansion can be realized analogously to that for the single massive target.

In the case of the third harmonic, the outer forms of the plasma streams for all the targets are similar. One can assume that the initial conditions of the plasma emission were similar, too. However, differences concern the inner dense plasma which is emitted a bit later. The elongated shape of the dense plasma in the case of the massive target is specific for the annular irradiation of the flat target [14]. Even if the irradiation conditions for both types of double targets were the same as in the previous case, the shape of the dense plasma is essentially distinct. This difference can be only explained by the foil target deformation during the laser action. This deformation should have a ring like form, in accordance with the laser beam radiation intensity distribution. Reconstruction of plasma emission from target surface thus deformed is in a good agreement with the actual electron density distribution.

Even if the target material evaporation and plasma ablation processes last a very short time (during the laser beam action), expansion of the ablative plasma lasts a relatively long time (longer than the period of our interferometric observation).

The growth of the electron density in the vicinity of the single massive target at the time of 8 ns after the laser action for both harmonics can testify to the appearance of a new plasma source. This source can only be connected with the crater creation. At this time the accelerated macroparticles run away far from their initial positions and such growth in the double targets case does not appear.

Our expectation concerning higher efficiency of the disk, in comparison with the foil, in the process of the crater creation has not been fulfilled. The efficiencies of
both methods under our experimental conditions are comparable. For the same wavelength of the laser radiation both the velocities of the two kinds of the macroparticles and the volumes of the craters generated by them are approximately the same. Thus one can come to the conclusion that the kinetic energies of these macroparticles and, going even further, their respective masses as well, are the same. Because the macroparticle mass is connected with its diameter, one can assume that the extracted foil fragment diameter is approximately equal to that of the disk. This conclusion is confirmed by the similar shapes of the craters for the both kinds of macroparticles.

6. Conclusions

Our investigations have shown a very useful role of the interferometric method for visualization of the laser produced plasma expansion and determination of the dynamics of the plasma and the accelerated macroparticles. This active plasma diagnostic method, although rather complex (both technically and methodologically), is irreplaceable in such experiments.

On the other hand, the relatively simple replica method for measuring the crater parameters also provides many interesting and important information about interaction of the laser beam or macroparticles with a massive target. Both these methods give evidence about the processes of laser energy transformation into the energy of the shock wave in solids, energy transfer from a laser-driven accelerated macroparticle to the massive target, and even the absorption efficiency of the laser radiation in plasmas. Combination of these diagnostic methods allows obtaining a nearly full picture of the processes analyzed.

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References

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Fluorescence method for the determination of oil identity

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The paper presents an objective method for distinction and identification of oils based on the comparison of total fluorescence spectra of their hexane solutions. The spectra comparison was conducted by means of the determination of the differences between the normalized functions, which describe the internal fluorescence efficiencies of the solutions of the substances under investigation. A criterion of oil identity, which involves limited accuracy of spectra measurement, has been defined. The efficiency of the method was tested on 51 lubrication oils.

Keywords: fluorescence, petroleum, oil identification.

1. Introduction

A continuously growing use of crude oil and its products by industry results in progressing pollution of the natural environment despite the development of new protective systems. Crude oils, fuels and other petroleum derivatives, further referred to as oils, and the products of their transformation have adverse impact on natural environment, which has been presented by many authors (e.g., [1]–[5]). Pollution of environment with oil is violation of law. This problem is especially apparent in the marine environment, the protection of which is regulated by the MARPOL Convention [6]. An efficient system of pollutant detection and punishment of perpetrator of the pollution play a special role in the system of environment protection. Identification of pollutant as the settlement identity of the pollutant and oil sampled from suspected source is a fundamental element of such a system [7].

The necessary conditions of identification method are its objectiveness and effectiveness, however the speed and easiness of determination are also important. Oil is a mixture of many components, mainly hydrocarbons and their derivatives and thus the commonly used methods of their identification are based on the determination of their chemical composition, mainly by means of gas chromatography and mass spectrometry [8], [9]. Independently of the measures being taken in order to improve these methods along with the development of the instrumental analyses, simple
methods, especially objective ways of the determination of oil identity are being investigated.

The possibility of using fluorescence to detect and identify crude oil was reported in the early 1970s [10]–[13]. Studies on the distinction of oil types using the fluorescence spectra were conducted [14]–[18]. It was also confirmed that spectra of particular oil types are different.

A necessary condition for determining the identity of oils based on fluorescence spectra is their distinction. Particular oil types differ in spectral range of the fluorescence, in intensity of the phenomenon and in shapes of the spectra. These differences, which are significant in relation to substances representing different oil types, can be less distinct in the case of different products of one type. The first aim of this work was to determine the range of spectra distinction and to state whether different oils are always characterized by different spectra. Confirmation of the distinction facilitated the preparation of the methodology for distinguishing spectra of particular oils, which further led to the definition of the objective criteria for the determination of the spectra identity. The analyses of total fluorescence spectra of 63 oils, including 51 defined products, which belong to one group – lubrication oil, were made.

2. Methods

2.1. Physical backgrounds

Internal spectrum – an objective parameter describing luminescence spectrum of luminophore – is represented by spectral internal efficiency of luminescence \( \Psi \) [19]. It is a function of wavelength of emitted radiation \( \lambda_f \) (the index “f” refers throughout this paper to fluorescence and the index “ex” to exciting radiation):

\[
\Psi(\lambda_f) = Y \psi(\lambda_f)
\]  \quad (1)

where \( Y \) denotes energetic efficiency of luminescence and the function \( \psi \) is differential \( \psi(\lambda) = \phi(\lambda)d\lambda \), which fulfils the following condition:

\[
\int_{0}^{\infty} \phi(\lambda)d\lambda = 1.
\]  \quad (2)

Internal spectrum could be presented in this manner for exciting radiation of any wavelength. If the luminescence is excited by light beam going along an axis \( X \) and having defined wavelength \( \lambda_i^{ex} \) and intensity \( I_i^{ex} \), value of the spectral internal efficiency of fluorescence of the wavelength \( \lambda_j^{f} \) could be presented by the following expression:

\[
\Psi_{ij} = \frac{d I_i^{f}}{-d I_i^{ex}} \psi_{ij}
\]  \quad (3)
where $\text{d} I^f_{ij}$ denotes the intensity of the radiation emitted by an element of volume of the luminophore having longitude $\text{d}x$. The product of the intensity of radiation $\text{d} I^f_{ij}$ and the differential $\psi_{ij}$ is the intensity of luminescence of wavelength $\lambda^f_j$.

$$\text{d} I^f_{ij} = \text{d} I^f_{ij} \psi_{ij}. \quad (4)$$

A decrease in the intensity of exciting radiation is described by the expression

$$-\text{d} I^\text{ex}_{ij} = I^\text{ex}_{ij} \alpha_i \text{d}x \quad (5)$$

where $\alpha$ denotes the light absorption coefficient, so Eq. (3) can be described as follows:

$$\psi_{ij} = \frac{\text{d} I^f_{ij}}{I^\text{ex}_{ij} \alpha_i \text{d}x}. \quad (6)$$

The spectral internal efficiency of fluorescence is a function of two factors: wavelengths of luminescence and exciting radiation. The fluorescence could be described by function $\Phi$. The function $\Phi$ is defined as a quotient of the intensity of radiation of defined wavelength emitted by the element of luminophore volume and the intensity of radiation exciting the luminescence

$\omega_{ij} = \frac{\text{d} I^f_{ij}}{I^\text{ex}_{ij}} = \alpha_i \psi_{ij} \text{d}x. \quad (7)$

The function presents the total spectrum of particular substance and covers the whole characteristics of its luminescence and absorption properties. The function $\Phi$ is differential. It is proportional to the integral $w$ calculated from function $\Phi$ by the whole space, in which fluorescence occurs. The integral $w$ is also a function of $\lambda^\text{ex}_{ij}$ and $\lambda^f_{ij}$ and describes intensity of luminescence emitted by the whole sample in relation to intensity of exciting radiation. The expression

$$w_{ij} = \int \frac{\text{d} I^f_{ij}}{I^\text{ex}_{ij}} = \int_0^l \alpha_i \psi_{ij} \text{d}x \quad (8)$$

presents value of the function $w$ at definite point ($\lambda^\text{ex}_{ij}$, $\lambda^f_{ij}$) in the case of luminophore of longitude $l$ along the direction of excitation.

### 2.2. Measurements

The spectra measurements were carried out using the spectrofluorimeter Fluorat-02 Panorama. A narrow flux of the radiation runs through the center of a square cuvette of side dimension $l = 1$ cm and excites luminescence. The intensity of radiation emitted
from the entire cuvette length perpendicular to the excitation radiation is measured. The set measures directly a dimension \( F \) being the ratio of fluorescence light intensity and the intensity of the exciting radiation and thus parameter \( F \) is proportional to function \( w \) (formula (8)). In order to determine the spectral function absorption of both the excitation and emitted radiation was included. Thus the independent measurements of radiation transmission spectra in the entire spectral range were made for each sample. The values of the spectral function \( w \) at any point \((\lambda_i^{\text{ex}}, \lambda_j^{\text{f}})\) were determined from the following formula:

\[
wij = \left(F_{ij} \ln \frac{T_i^o}{T_i} \sqrt{\frac{T_j^o}{T_j} - F_{ij}^o} \right) \theta_{ij}.
\]

(9)

In the above dependence \( F \) is a result of the measurement of the luminescence of the solution investigated, \( T \) denotes transmission of radiation through the solution and \( T^o \) – transmission through the solvent. \( F^o \) denotes the result of background measurement for pure hexane, so the formula includes also the scattering of exciting radiation. Parameter \( \theta \) is a coefficient of spectral characteristics of the measurement device and is described as follows:

\[
\theta_{ij} = \frac{I_{ij}^f}{I_i^{\text{ex}}}. 
\]

(10)

During the studies different types of crude oil in the form of their hexane solutions of concentrations below 20 mg/dm\(^3\) were investigated. The measurements of luminescence spectra were made every 5 nm in the wavelength range from 260 to 500 nm for these solutions. Those spectra were excited by the radiation of wavelengths from the range of 210 to 300 nm, every 10 nm. The derived values of the spectral function create a matrix of 490 elements (10 rows and 49 columns). For computation purposes the number of elements was limited to 398, and the values of the spectral function were applied for the following wavelengths of excitation and emission radiation:

- \( \lambda_i^{\text{ex}} = 210 \) nm \( \lambda_i^{\text{f}}: 270\text{–}400 \) nm
- \( \lambda_i^{\text{ex}} = 230 \) nm \( \lambda_i^{\text{f}}: 270\text{–}440 \) nm
- \( \lambda_i^{\text{ex}} = 250 \) nm \( \lambda_i^{\text{f}}: 270\text{–}480 \) nm
- \( \lambda_i^{\text{ex}} = 270 \) nm \( \lambda_i^{\text{f}}: 270\text{–}500 \) nm
- \( \lambda_i^{\text{ex}} = 290 \) nm \( \lambda_i^{\text{f}}: 290\text{–}500 \) nm
- \( \lambda_i^{\text{ex}} = 300 \) nm \( \lambda_i^{\text{f}}: 300\text{–}500 \) nm

Such a range refers to the spectral area, in which the fluorescence of the solutions occurs in (Fig. 1).
Fig. 1. Examples of the normalized total spectra $\Phi$ of hexane solutions of oils.
3. Assumption for the identification

In order to use these spectra to identify oil types their predicted modifications, which result from the natural changes of oil properties, should be taken into consideration. The processed oils have stable composition and properties in natural conditions, this stability may be assumed for normal conditions of their storage or transportation over a period of some time for which the identification of a pollutant is sensible. Weathering of crude oil is a natural process especially intensive in its early stage, which results in the change of petroleum composition and properties [20]. Identification of crude oil

Fig. 2. Functions \( w \) representing fluorescence spectra of solutions of concentration 6 mg/dm\(^3\) of fresh and weathered crude oils from: a – North Sea, excited with light of wavelength \( \lambda^{ex} = 220 \) nm, b – Baltic (\( \lambda^{ex} = 250 \) nm), and c – Romashkino (\( \lambda^{ex} = 280 \) nm).
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makes sense only on assumption that naturally weathered oil is still the same substance as the fresh one.

The changes are also visible in fluorescence, due to the increase of luminescence intensity of oil solutions as a result of crude oil aeration. The value of the spectral function is a measure of the increased intensity, while the spectral shapes remain unchanged (Fig. 2). This requires the limitation of spectra analyses to investigation of spectra shapes and fluorescence areas. For solutions of small concentrations, which satisfy the Beer law, the spectral function is linearly dependent on the oil concentration in hexane and thus the spectrum which is described by such a function can be normalized [21]. The application of the natural normalization criterion for this function, i.e., the value of the integral over the entire area of changes is not possible due to the fact that only a part is known. Knowledge of the entire area would involve the conduction of the impossible measurements of luminescence induced by radiation of wavelengths shorter than 210 nm. Thus an auxiliary criterion was assumed regarding the normalization of the total spectrum through matching the function values at any point \((\lambda_{ex}^{a}, \lambda_{f}^{a})\) with the maximum values in the whole area in which the measurements were made. This way the function \(\Phi\) was obtained:

\[
\Phi = \frac{w}{\max \{ w_{ij} \}} = \frac{\omega}{\max \{ \omega_{ij} \}}.
\]

It describes correctly the shape of the total fluorescence spectrum of oil solution. Relative difference between such normalized spectra can be defined:

\[
D = \frac{\sum_i \sum_j |\Phi_{ij}^{a} - \Phi_{ij}^{b}|}{\sum_i \sum_j \Phi_{ij}^{a}}.
\]

The parameter \(D\) determines the relative difference of spectra – fluorescence regions and spectra shapes – for two oils “a” and “b” (in relation to the “a” oil spectrum). Also, this parameter describes the identity, which is greater with the smaller value of the parameter \(D\). In the case of oil the solution of which have the same spectra the difference is \(D = 0\).

4. Distinction of fluorescence spectra

The difference, as defined by formula (12), may be used to identify oils provided that the identity of spectral shapes and areas is identical with the substance identity. This

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*This seems to be the only way to normalize the spectral function since the fluorescence spectral regions of particular oils overlap but not entirely. It is impossible to find one fixed point \((\lambda_{ex}^{a}, \lambda_{f}^{a})\), for which the value of the spectral function could normalize the spectra of all oil types.*
problem is reduced to two issues: distinction of spectra (do particular oils have always spectra of different shapes?) and an issue of the identity of crude oil. The problem of distinction was tested for lubricating oils. For 51 determined products measurements of spectra of their solutions were made and the spectral function \( w \) was determined and then normalized. For each of the oils the difference \( D \) was determined between the normalized total spectrum of its solution and the spectrum of solutions of every other oil.

Table 1. Relative differences \( D \) between the normalized total spectra of lubrication oil hexane solutions.

<table>
<thead>
<tr>
<th>Oil</th>
<th>( D )</th>
<th>Oil</th>
<th>( D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aral basic Elf turbo</td>
<td>5.4</td>
<td>Marinol (A) Hipol GL4 (B)</td>
<td>4.2</td>
</tr>
<tr>
<td>Aral multi VAT super</td>
<td>4.5</td>
<td>Marinol (B) Marinol (C)</td>
<td>10.3</td>
</tr>
<tr>
<td>BP syntetic Statoil Lazer</td>
<td>30.9</td>
<td>Marinol (C) Marinol (D)</td>
<td>9.4</td>
</tr>
<tr>
<td>Castrol coral 2 Hipol GL4</td>
<td>16.9</td>
<td>Marinol (D) Marinol (E)</td>
<td>3.1</td>
</tr>
<tr>
<td>Castrol EP Marinol (A)</td>
<td>7.7</td>
<td>Marinol (E) Marinol (D)</td>
<td>3.0</td>
</tr>
<tr>
<td>Castrol GTX Elf competition</td>
<td>8.2</td>
<td>Mobil super Mobil super diesel</td>
<td>5.0</td>
</tr>
<tr>
<td>Castrol Lightec VAT SynTech</td>
<td>10.8</td>
<td>Mobil super diesel Mobil super</td>
<td>5.3</td>
</tr>
<tr>
<td>Castrol Magnetec Mobil 1 rally form</td>
<td>21.3</td>
<td>Mobil super S Mobil super diesel</td>
<td>9.0</td>
</tr>
<tr>
<td>Elf competition Castrol GTX</td>
<td>8.8</td>
<td>Mobil-1 rally form Castrol Magnetec</td>
<td>26.9</td>
</tr>
<tr>
<td>Elf SJ Elf sporti (A)</td>
<td>4.5</td>
<td>Mobil-1 turbo dies. Mobil 1 rally form</td>
<td>73.8</td>
</tr>
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<td>Shell diesel Elf sporti (A)</td>
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<tr>
<td>Elf sporti (B) Elf sporti (C)</td>
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<td>Shell plus Shell super</td>
<td>5.9</td>
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<tr>
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<td>6.9</td>
<td>Shell super Shell plus</td>
<td>5.8</td>
</tr>
<tr>
<td>Elf sporti (D) Lotos special</td>
<td>11.0</td>
<td>Shell ultra Mobil 1 rally form</td>
<td>29.7</td>
</tr>
<tr>
<td>Elf turbo Elf sporti (A)</td>
<td>3.6</td>
<td>Statoil classic Lotos mineralny</td>
<td>6.7</td>
</tr>
<tr>
<td>Hipol 15F (A) Hipol GL4 (A)</td>
<td>3.1</td>
<td>Statoil Lazer VAT SynGold</td>
<td>19.7</td>
</tr>
<tr>
<td>Hipol 15F (B) Hipol 15F (A)</td>
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<td>Statoil power Lotos diesel</td>
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</tr>
<tr>
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<td>Statoil super VAT SynTech</td>
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<td>Hipol GL4 (A) Hipol 15F (A)</td>
<td>3.1</td>
<td>Texaco diesel 1 Texaco Halvoline</td>
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<td>Texaco Greise Elf competition</td>
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<td>Hipol MF Hipol 15F (B)</td>
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<td>VAT SynGold Lotos sintetic</td>
<td>12.2</td>
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<tr>
<td>Lotos mineral Statoil classic</td>
<td>6.3</td>
<td>VAT SynTech Statoil super</td>
<td>5.5</td>
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<tr>
<td>Lotos sintetic VAT SynGold</td>
<td>12.1</td>
<td>VAT turbo Texaco Halvoline</td>
<td>8.2</td>
</tr>
<tr>
<td>Lotos special Texaco Halvoline</td>
<td>9.5</td>
<td>Average value of ( D )</td>
<td>10.2</td>
</tr>
</tbody>
</table>

*The original commercial names of oils have been used in the paper. Letters in parentheses denote different samples of the same product (from different sources).
Table 1 presents the products along with their oil of origin for which the difference was the smallest. Comparison of the normalized spectra of lubrication oil types proves their good statistical distinction – an average difference between the most similar spectra was 10.2%. Such distinction is confirmed by studies of other product types: diesel oils, whose normalized spectra differ by 9.2% on average, heavy fuels (14%), and crude oils (20%).

These results do not prove yet the distinction of the oils under study, since for over 1/4 of them there are products whose spectra differ by less than 5%. The question is: what is the uncertainty with which this difference is determined? The limited accuracy of the measurements is the source of the uncertainty and appoints the accuracy of the determination of the spectral function. The uncertainty with which the normalized spectral function $\Phi$ is determined has been obtained through the investigation of the difference $D$ in cases of independently made spectral measurements for the same oil type. Such studies have been made for 3 pairs of lubrication oils, which showed the closest similarities (Aral-multi, Elf-sporti (A), Elf-turbo, Hipol 15F (A), Hipol GL4 (A) and VAT-super) and for both fresh and weathered crude oil from the Baltic Sea and the North Sea. A number of solutions have been made for each oil, for which independent spectra measurements have been made and the normalized spectral function $\Phi$ has been determined. Using the normalized functions the average function was derived. The average function is by assumption close to the real spectrum of the given oil. Then the differences between such averaged function and the particular functions (relating to the particular measurements) were derived. These differences are presented in Tab. 2. The average error of the derivation of the spectral function, which has been determined using all results, is 1.68% with standard deviation of 0.74%. The uncertainty with which the difference $D$ has been determined between

<table>
<thead>
<tr>
<th>Oil</th>
<th>N</th>
<th>Range of $D$ [%]</th>
<th>$D_{av}$ [%]</th>
<th>$\delta D$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aral-multi</td>
<td>12</td>
<td>1.01–4.15</td>
<td>1.54</td>
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<tr>
<td>Elf-sporti (A)</td>
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<td>0.92–3.71</td>
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<td>0.73</td>
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<td>0.73–3.75</td>
<td>1.75</td>
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<td>Hipol 15F (A)</td>
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<td>0.85–3.37</td>
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<tr>
<td>VAT-super</td>
<td>12</td>
<td>1.46–2.52</td>
<td>1.78</td>
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<td>Fresh Baltic crude oil</td>
<td>22</td>
<td>1.49–3.57</td>
<td>2.23</td>
<td>0.48</td>
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<tr>
<td>Weathered Baltic crude oil</td>
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<td>1.30–3.33</td>
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<td>Fresh North Sea crude oil</td>
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<td>0.52–1.66</td>
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<tr>
<td>Weathered North Sea crude oil</td>
<td>12</td>
<td>0.61–2.11</td>
<td>1.29</td>
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particular spectra is higher. It may be assumed that accuracy of determination of the difference between the spectra of solutions of one oil (obtained as results of singular measurements) is two times higher than the accuracy of derivation of the function $\Phi$. The average accuracy of the difference $D$ determination, evaluated using the above results, equals 3.35% with the standard deviation of 1.48%. This means that the most probable differences between two normalized spectra of the given oil solution obtained from singular measurements will range from 1.85 to 4.85%.

The 22 normalized spectral functions $\Phi$ of the Baltic crude oil solutions were randomly divided into two groups (A and B), with 11 spectra in each. Within each of these groups the functions were averaged and the differences between the averaged function and functions related to particular samples were calculated. Eight divisions were made and in each of the cases the average differences between the averaged function and the basic functions within each of the groups ($D^{av}_A$ and $D^{av}_B$) were determined, as well as standard deviations of these differences ($\delta D^{av}_A$ and $\delta D^{av}_B$), and the differences between the averaged functions ($D_{AB}$ – versus the averaged function from group A, and $D_{BA}$ – versus the averaged function from group B). The results of these calculations, which are presented in Tab. 3, show that even in cases of relatively large measurement series the averaged spectral functions obtained for the same oil type are different. The differences ($D_{AB}$ and $D_{BA}$) between the averaged spectral functions of the same substance solutions are significant (even up to 2%), still they are always smaller than the average differences ($D^{av}_A$ and $D^{av}_B$) derived from the original functions. The numerous measurements revealed that also the shapes of spectra of solutions of weathered and fresh crude oil are very similar. The difference between the averaged, normalized spectral functions of fresh and weathered Baltic crude oil solutions was 1.4%, and this difference for fresh and weathered North Sea

<table>
<thead>
<tr>
<th>Trial</th>
<th>$D^{av}_A$ [%]</th>
<th>$D^{max}_A$ [%]</th>
<th>$\delta D_A$ [%]</th>
<th>$D_{AB}$ [%]</th>
<th>$D^{av}_B$ [%]</th>
<th>$D^{max}_B$ [%]</th>
<th>$\delta D_B$ [%]</th>
<th>$D_{BA}$ [%]</th>
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<td>2.17</td>
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<td>4</td>
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<td>8</td>
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<td>0.73</td>
<td>2.23</td>
<td>3.30</td>
<td>0.56</td>
<td>0.74</td>
</tr>
<tr>
<td>Average (A and B)</td>
<td>2.17</td>
<td>0.58</td>
<td>0.96</td>
<td></td>
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</table>
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These differences are also smaller than the average differences calculated for spectra of the same substance, obtained from particular measurements. This is connected with the fact that the averaged spectrum is closer to the real spectrum, if it originates from a large number of spectra and in the case of identical substances the differences between their spectra will be smaller along with an increase of the number of measurements. A different situation was confirmed for lubrication oils. The differences derived between the averaged oil spectra for Aral-multi and VAT-super (2.2%), Hipol-15F and Hipol-GL4 (2.1%) and Elf-sporti and Elf-turbo (3.7%) are greater than the average differences counted for singular spectra of particular oil. These differences are presented in Tab. 4.

5. Method of identification

The analysis of the results allows us to consider two oils as identical, if the differences between their averaged spectra, obtained from numerous measurements, are smaller than the average difference of singular total spectra of each of them. Determination of oil identity through comparison of their spectra requires a number of measurements appropriate for statistical description of the results. Confirmation of the difference between total spectra of solutions of two oils obtained from singular measurements, which exceeded some value (here 8%), rules out their identity.

The above information allows us to design a two-stage procedure for the determination of oil identity. The first, reconnaissance stage, involves the measurements of total spectra of solutions of the oils and the determination of related spectral functions. These functions are normalized to maximum values (formula (11)) and a relative difference between them is determined (12). Confirmation of the difference, which exceeds some critical value, leads to the statement that the oils under investigation are different. In another case the determination of the identity requires further studies (second stage). For each of oils it is necessary to prepare a number of solutions, measure their spectra and determine spectral functions which then must be

Table 4. Differences between the averaged oil spectra ($D_{12}^{av}$ and $D_{21}^{av}$) indicate the average differences of functions which relate to particular samples versus the averaged functions of the 1st and 2nd oil, $D_{12}$ and $D_{21}$ – differences between the averaged spectra of these oils calculated versus the 1st and the 2nd sample, respectively.

<table>
<thead>
<tr>
<th>Oil 1</th>
<th>Oil 2</th>
<th>$D_{12}^{av}$ [%]</th>
<th>$D_{21}^{av}$ [%]</th>
<th>$D_{12}$ [%]</th>
<th>$D_{21}$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aral-multi</td>
<td>VAT-super</td>
<td>1.54</td>
<td>1.78</td>
<td>2.19</td>
<td>2.17</td>
</tr>
<tr>
<td>Elf-sporti (A)</td>
<td>Elf-turbo</td>
<td>1.93</td>
<td>1.75</td>
<td>3.71</td>
<td>3.83</td>
</tr>
<tr>
<td>Hipol-15F (A)</td>
<td>Hipol-GL4</td>
<td>1.69</td>
<td>0.97</td>
<td>2.10</td>
<td>2.07</td>
</tr>
<tr>
<td>Baltic crude – fresh</td>
<td>Baltic crude – weathered</td>
<td>2.23</td>
<td>1.91</td>
<td>1.37</td>
<td>1.38</td>
</tr>
<tr>
<td>North Sea crude – fresh</td>
<td>North Sea crude – weathered</td>
<td>0.96</td>
<td>1.29</td>
<td>0.81</td>
<td>0.81</td>
</tr>
</tbody>
</table>
normalized and next derive the average function. For this average function the
difference between it and the functions relating to particular measurements must be
determined and then the average value of these differences should be determined. Next
the difference between the averaged functions of both oils compared must be
determined. If its value is smaller than the calculated average differences (between the
functions obtained from particular measurements and appropriate average functions),
then tested substances are identical. Otherwise the oils are different.

The studies made suggest some practical remarks (however solutions discussed in
this work did not comply with following recommendations). Due to the differences in
spectral areas, in which solutions of particular oil types show fluorescent properties,
it seems sensible to individually match the excitation and emission radiation
wavelengths to the area in which the solution of pattern oil fluoresces. This would
minimize the impact of measurement errors on final results. Figure 1 shows that for
majority of oils the ranges of fluorescence wavelengths, for which the measurements
and calculations were made significantly deviate from the range in which the
phenomenon occurs. For some types of fuels and crude oil it would be sensible to
extend the spectral area investigated and to conduct measurements with the excitation
radiation of wavelengths above 300 nm.

Similar concentrations of solutions of oils being compared also could minimize
the impact of measurement errors. Determination of concentration is not necessary in
preparing the solution. Compliance with Beer law is the only condition since this is
the precondition for both correct derivation of the spectral function as the sense for
comparison of spectra shapes. The recommendation for making solutions of similar
concentrations results from the impact of the background confirmed during studies.
This impact appears while normalizing the spectra of solutions of the same substance
of very different concentrations.

6. Summary

The method of determining the identity of two oils based on determination of the
difference between the total spectra of their solutions is fully objective and efficacious.
It is also relatively simple and rapid. The efficiency of the method dependent on
discrimination of the spectra of the oils has been confirmed by testing different lube
oils making up about 70% of a group of such products available on Polish market.
However, the oils tested make up only a fragment of the great number of petroleum
products and it is not conceivable to find two different oils of the same spectra. This
allows us to state, that the method presented is efficacious in determination of identity
of oil samples.

The investigations prove that particular kinds of oil – products of individual
manufacturers having defined names and the same specifications – can differ. This
points to limited possibility of determination of oil type in the case of testing a sample
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taken from environment and proves inexpedience of making a catalogue of the spectra. On the other hand, these differences augment the probability of finding a pollution maker, for example in case of many suspects using the same products.

Application of this method for determining the source of petroleum pollution in particular parts of natural environments needs individual studies. These must determine the influence of environmental conditions on the fluorescence properties of oils, rapidity of probable modifications of their spectra and the range of time connected with this, in which the identification can be possible. Irrespective of probable modifications, it seems that the method could be useful in determination of the identity of fresh pollutant and the oil of suspected maker.

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Creation and diagnostics of stable rainbow optical vortices

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An on-axis computer-synthesized hologram-based technique is introduced to create white-light “rainbow” optical vortices, which are stable with respect to environmental disturbances under long-distance propagation of singularity supporting beams. Regularities governing the radial alternation of colors at highly directed rainbow vortices are discussed. The original diffraction technique for detecting phase singularities is applied to reveal and diagnose the polychromatic vortices.

Keywords: singular optics, optical vortices, spatial coherence, white-light interference, mutual spectral purity, Young’s interference experiment.

1. Introduction

Singular optics recognized as a new chapter of modern physics in the nineties of the last century [1], [2] has been developed until recently exclusively under approximation of complete coherence of light. In this framework, the phase singularities of common complex amplitude of completely spatially coherent, monochromatic waves were the subject of consideration alone. Vector generalization of the singular optical paradigm for completely coherent optical fields is discussed in [1]. Only at the threshold of the third millennium, this field of investigations was richly extended to partially coherent fields supporting phase singularities. In this context, two general cases are considered, namely, partially spatially coherent quasi-monochromatic beams supporting the phase singularities of spatial correlation functions [3], [4], and spatially coherent polychromatic fields supporting phase singularities of the spectral components [5]–[9].

It is shown in [5], [6] that the phase singularities manifest themselves at polychromatic field, while the spatial coherence of this field is high enough to provide amplitude zeroes via completely destructive interference for any spectral component. In this case, incomplete destructive interference takes place for spectral components close to the one of vanishing amplitude. As a result, relatively large spectral interval of the initial
spectrum is considerably (though not completely) suppressed. This leads to noticeable coloring of initially “white” beam, and such coloring can be spatially non-uniform. Note that the local spectral modifications result from spatial non-coincidence of the phase singularities for different spectral components.

Several techniques for generation of polychromatic optical vortices have been proposed. Generation of the beams bearing a “white” optical vortex using a uniaxial crystal and a polarizer has been recently reported [10]. Another approach is based on diffraction of a femtosecond laser pulse with relatively narrow spectral band 10 nm at off-axis computer-synthesized hologram (CSH) computed for reconstruction of single-charged Laguerre–Gaussian (LG) doughnut modes at the first diffraction orders [11]. The pulse is decayed into spectrum, which impinges onto holographic grating. Then, after transformation into monochromatic vortices, such beams are spatially combined as a result of diffraction of spectrally decayed beam at the second grating with proper spacing that provides compensation of diffraction dispersion at the initial CSH. A disadvantage of this technique is that the spatial stability of the resulting polychromatic vortex depends considerably on mutual adjusting of two gratings (in period, orientation, etc.), so that a small misaligning of the arrangement results, under propagation, in decaying of the polychromatic vortex in spectral ones. Our previous experiments [12] with large-band polychromatic (virtually, white) light covering the visible range of electromagnetic radiation confirmed that polychromatic vortex might be obtained only at the plane where the singularity producing off-axis CSH is imaged, and even in this case one must operate with the gratings of low spatial frequency to minimize diffraction dispersion.

Here we introduce an on-axis CSH-based technique to create white-light (to judge from appearances, “rainbow”) optical vortices, which are the co-axial superposition of the spectral vortices and are stable under their long-distance propagation. In this study, we essentially take into account the advantages of on-axis CSHs fruitfully used recently to solve another problem of singular optics [13]. Regularities governing the radial alternation of colors at highly directed rainbow vortices are discussed and demonstrated. The original diffraction technique for detecting phase singularities [4], [14]–[16] is for the first time successfully applied to reveal and diagnose the polychromatic vortices.

2. Experiment

The arrangement for generation and diagnostics of a rainbow polychromatic vortex beam using an on-axis CSH is shown in Fig. 1. As the primary source of a white-light radiation (1), we use a zirconium incandescent gas-lamp DATS-50 (Russian production) with argon filling with 0.3 mm-diameter circular luminous body (spectrum of radiation of the source is shown in Fig. 2). By a lens 2 (see Fig. 1), the source is imaged at \(3.5 \times 10^{-2}\) mm-diameter circular pinhole at an opaque screen 3. The size of a pinhole,
which is virtually the secondary source of polychromatic light, is critical for generation of rainbow vortices. Namely, taking into account the Van-Cittert–Zernike theorem [17], it must be small enough to provide considerable spatial coherence of the field at the plane of a collimating lens 4 (and behind it) for all spectral components of the probing beam. Focal length of this lens in our experiment is \( f = 60 \) mm, thus the
coherence area of the beam behind lens 4 estimated by the first zero of the first-order Bessel function of the first kind \[18\] varies in diameter \( s = 1.22\lambda/\theta \), (\( \theta \) being the angular dimension of the secondary source estimated from the center of the collimating lens), from \( s_b \approx 0.8 \) mm for blue spectral component (\( \lambda_b = 380 \) nm) to \( s_r \approx 1.6 \) mm for red one (\( \lambda_r = 760 \) nm). Diaphragm 5 behind a collimating lens 4 selects 0.8 mm-diameter central part of the beam. Then, the partially spatially coherent collimated polychromatic singularity-free beam impinges onto an on-axis CSH 6 (see insertion, Fig. 1) computed for reconstruction of the single-charged doughnut LG modes at the first diffraction orders. An opaque screen 7 with a small diaphragm passes the minus-first diffraction order of the radiation diffracted by a CSH, namely a polychromatic optical vortex.

Let us note the important differences of our experiment from ones described in [13] and in [18]. When one works with a quasi-monochromatic optical radiation [13], the diameter of the diaphragm 7 must be as small as possible to minimize the contributions from the 0th and the +1st diffraction orders into analyzed vortex beam. In contrast, one operates with enough large a diaphragm to generate a spatially incoherent source that is presumed into Van-Cittert–Zernike theorem [17], [18]. Our considerations are quite different. On the one hand, we also strive for excluding the background caused by the contributions from the 0th and the +1st diffraction orders, which can camouflage the central optical vortex. Hence, the diaphragm 7 cannot be large. On the other hand, we must provide for passing all spectral components of polychromatic vortex beam bearing considerable energy, which are focused at different distances behind a CSH due to diffraction dispersion. Hence, the diaphragm 7 cannot be too small, being adjusted for the central, conventionally “green”, spectral component of the probing radiation. As a compromise, we use 2 mm-diameter diaphragm 7 at opaque screen adjusted to let fully pass green, while blue and red are passed considerably, too. In our experiment, the focal length of a snail-like Fresnel grating (inserted in Fig. 1), \( f_g = r^2/\lambda_g \) (\( r \) being the radius of the central Fresnel zone), for green (\( \lambda_g = 550 \) nm) is 70 mm.

As is known [2], paraxial free-space propagation of a monochromatic vortex beam of LG-mode is accompanied by diffraction spreading of the beam following the rule:

\[
\begin{align*}
  w_z &= w_0 \left[ 1 + \left( \frac{z^2}{\pi w_0^4} \right) \lambda^2 \right]^{1/2}
\end{align*}
\]

where \( w_0 \) being the waist parameter of the beam estimated by the \( e^{-1} \) intensity level for \( z = 0 \), and \( w_z \) – the width of the beam at the distance \( z \) from the caustics waist. It is clear that in the case of polychromatic vortex, \( i.e., \) co-axial superposition of elementary spectral vortices, diffraction spreading must cause “rainbow” effect. Namely, the cross-section of the polychromatic vortex beam propagating behind a diaphragm 7 looks like a ring rainbow, whose periphery is red-colored, while violet and blue are concentrated close to the common axis.
However, observation of free-space propagating rainbow vortex is impracticable owing to rather fast geometric spreading of the beam. That is why we use a collimating length $8$ (in our experiment, with a focal length $f_c = 180$ mm). The use of this lens causes the following peculiar transformation of the structure of the rainbow vortex. At free-space propagating vortex beam (as well as just behind the collimating lens 8), one really observes rainbow vortex with the above mentioned radial alternation of colors caused by diffraction dispersion. But the further propagation of the beam is accompanied by the inversion in the order of the radial alternation of colors. The reason for the color inversion lies in competition of diffraction (caused by a CSH 6) and refraction (caused by a lens 8) dispersion, which are of opposite signs. As a result, blue goes out to the periphery of the beam, while red is concentrated close to the beam axis (to the central optical vortex). What is important, despite color inversion the rainbow vortex occurs to be spatially stable at long-distance propagation. Namely, in our laboratory environment we observed stable (neither spatially nor spectrally decayed) rainbow vortices up to 80 m, even when the case of environmental disturbances kind of the combined influence of rapid heating and ventilator wind were applied. In the last case, the optical axis is fluctuated, but the central vortex is stable.

Further, due to incomplete spatial coherence of the beam and imperfect elimination of the contributions from the 0th and the +1st diffraction orders, the central vortex is observed at considerable incoherent background. It hampers direct visualization of the polychromatic vortex, as it is possible in the case described in [13]. Moreover, the standard interferometric technique for detection of phase singularities [2], which is generally accepted and highly efficient in coherent singular optics, is inapplicable in our case, while no any reference wave can be mutually coherent with all spectral components of the analyzed polychromatic beam simultaneously. That is why we apply the diffraction technique for revealing phase singularities introduced recently in [4], [14]–[16] for diagnostics of optical vortices in partially spatially coherent but monochromatic singular beams.

An opaque strip 9 (a 1 mm-diameter metallic needle) in Fig. 1 is placed in front of the vortex beam symmetrically to its center. Behind the screen, within its geometrical shadow, one observes and registers with a CCD-camera 10 interference fringes arising from a superposition of wavelets from the screen edges. So, the fringes result from interference of wavelets from different points of the vortex beam itself, rather than from interference of this beam with a complementary reference wave. Rigorously speaking, in this way one obtains the data on the vortex of the spatial coherence function rather than on the vortex of ordinary complex amplitude of completely coherent singular beam tested using a reference wave. Being applied to polychromatic beams, this technique provides observation of white-light interference, while the requirement of mutual spectral purity of the disturbances at the probing points of the beam [17] is wittingly satisfied in the case of interest owing to the axial symmetry of the problem. Besides, due to very small diffraction angles of the interfering edge waves, chromatic blurring of white-light interference fringes is also small. Thus,
specific bending of interference fringes following the arctan law [14] reflects helical phase of the spatial coherence function of polychromatic vortex beam and, indirectly, the vorticity of all spectral components of this beam.

Figure 3 illustrates the free-space propagating rainbow vortex originating from the diaphragm 7 in Fig. 1, (Fig. 3a) and the result of the diffraction diagnostics of the axial vortex (Fig. 3b). One can clearly see in Fig. 3a snail-like twirling of the beam near the core, as well as red periphery of the beam. As has been mentioned above, incoherent background camouflages the central vortex as well as the radial color distribution. Note that the central vortex can be put in evidence using a dark-field technique eliminating the regular background. But it is remarkable that even without applying a dark-field imaging technique, the use of an opaque strip at the beam of interest provides unambiguous confirmation of the presence of the central vortex (by the bending of

Fig. 3. Free-space propagating rainbow vortex (a), and its diffraction testing (b).

Fig. 4. Collimated rainbow vortex (a), and its diffraction testing (b).
interference fringes) as well as the radial alternation of colors governed by the diffraction dispersion, as is seen from Fig. 3b.

At last, in Fig. 4 we demonstrate the (spatially stable) rainbow polychromatic vortex with the inverted color alternation detected at the distance 5 m from the collimating lens 8 (see Fig. 1) and the result of its diffraction diagnostics. Bending of the interference fringes in Fig. 4b corresponds to right-hand (clockwise) twirling of the phase of the spatial coherence function of the single-charged LG mode and the same twirling of the phase of complex amplitudes of the spectral components of a polychromatic vortex beam. In Fig. 4a, one can see high spatial homogeneity of the stable vortex beam.

3. Conclusions

Summarizing, we have introduced a simple technique for creation of rainbow polychromatic optical vortices using a point-like white-light source and on-axis CSH technique. We have traced the transformations of the stable free-propagating rainbow vortices. The diffraction technique for revealing and diagnostics of vortices at partially coherent beams has been for the first time applied to polychromatic beams supporting phase singularities.

Considered here partially coherent singular beams are of interest, in part, in the problem of so-called optical traps and tweezers, as well as in the problem of optical telecommunications. In the first of the applications mentioned, it is attractive to use cheap non-laser, noninvasive sources for manipulating with microparticles. As for optical communications, it has been shown recently [19] that:

– a partially coherent beam may possess the directivity not conceding the directivity of any term of an expansion of the beam into the series of fully coherent (here, monochromatic) constituting modes, and

– some parameters of partially coherent beams occur more stable in respect of environmental disturbances than the corresponding parameters of completely coherent beams.

References


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Electrical substitution cryogenic radiometer based spectral responsivity scale between 250–2500 nm wavelengths

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This paper presents the spectral responsivity scale between 250–2500 nm developed by the National Metrology Institute of Turkey (UME). For that purpose silicon photodiode based trap detector and electrically calibrated pyroelectric radiometer (ECPR) that were calibrated against primary level absolute electrical substitution cryogenic radiometer (ESCR) were used as transfer standards. Using highly collimated and stabilized \(10^{-5}\) lasers, absolute optical powers and absolute responsivity of trap detector were measured with an uncertainty of the order of \(10^{-4}\). In visible (VIS) region responsivity scale was set by means of the models for the reflectance and internal quantum efficiency. In the ultraviolet (UV) and near-infrared (NIR) regions spectrally flat \((0.1\%)\) ECPR was used.

Keywords: electrical-substitution cryogenic radiometer, electrically calibrated pyroelectric radiometer, trap detector, radiant source, optical power, responsivity.

1. Introduction

In metrology the candela, the SI unit of luminous intensity, is determined in terms of the absolute responsivity of detector from the absolute measurement of optical power using ESCR’s working at liquid helium temperatures \((4.2\,\text{K})\) [1]–[5]. At this temperature, the uncertainty in the optical power measurements is of the order of \(0.005\%\) [1], [6]. Responsivity standards have been changed significantly in the past two decades from single element detectors to reflection or transition type trap detectors. Compared to other optical radiation detectors, trap detectors constructed from silicon photodiodes have better optical properties [7], [8]. Having good stability, uniformity, linearity and spectral response to intensity from \(10^{-13}\,\text{W/cm}^2\) to \(10\,\text{mW/cm}^2\), these detectors have been used in many applications starting from the deep UV through the VIS to the NIR.

In UV and NIR regions there are some problems with decreasing internal quantum efficiency of silicon photodiodes, which brings about high errors in the interpolation
of spectral responsivity using the physical models developed [9]–[11]. In order to measure optical radiation in UV and IR regions ECPR have been used as a transfer standard. It can measure the total power and irradiance of cw sources. The most common application for the ECPR is to transfer an absolute radiometric calibration to another detector or light source with a high degree of accuracy. This is accomplished by measuring either the total power (watts) of collimated sources that under fill the detector aperture or the irradiance (W/cm\(^2\)) of extended sources that overfill the detector aperture. The exceptionally flat spectral response ensures that broadband sources are measured with the same accuracy as monochromatic light, allowing the ECPR to calibrate VIS and IR detectors, standard lamps, blackbody emitters, laser power meters, UV exposure meters, etc. [12].

In this paper, we present both optical power and responsivity measurements necessary for the realization of spectral responsivity scale. Our absolute responsivity scale is based on home-made reflection type reference trap detectors calibrated against ESCR. The scale was obtained using suitable mathematical models for the calculated internal quantum efficiency from measured absolute responsivity values and reflectance measurements at the laser wavelengths. The relative responsivity scale is based on ECPR absolutely calibrated against ESCR at laser wavelengths and the spectral responsivity scale of ECPR was expanded by measuring the change in the pyroelectric detector’s reflectance.

2. Absolute optical power measurements

The optical power measurement system consists of two parts; a laser power stabilizer (LPS) system and an electrical substitution cryogenic radiometer (ESCR), which is designed such that it can be used with a beam of collimated and vertically polarized light with powers ranging from a few µW to 1 mW. As radiant sources we used Ar\(^+\) (488 nm and at 514.5 nm), Nd:YAG (with second harmonic 532 nm), and He-Ne (632.8 nm) lasers in the measurement. LPS shown in the Fig. 1 was used for compensation of fluctuations in the optical power and generation of a geometrically well-defined Gaussian laser beam. The LPS consists of an electro-optic modulator (EOM), temperature controlled monitor photodiode and control electronics. Vertically polarized laser light incident on the EOM first passes through a liquid crystal, which alters the beam transmittance, then passes through a beam splitter, which transmits 98% of the incident light. The optical beam after passing through the EOM is partially reflected from the wedged fused-silica beam splitter. A precision monitor photodiode receives the reflected beam from the wedged window and sends it back to the EOM. This feedback loop controls the optical beam to get a constant signal power output. Using this technique the output stability level reached was of the order of 6×10\(^{-5}\). The spatial profiles of the beams were purified with spatial filter. The diameters of the beams after spatial filter are close to 3 mm (1/e\(^2\) points).

The entrance window of the ESCR is a fussed silica glass aligned to Brewster angle. Before measurements the window was released and cleaned using suitable solutions.
Electrical substitution cryogenic radiometer... like ethanol, and lens paper with drop and drag method [13] and its transmittance measured was better than 99.9%. Laser beams enter the cavity by passing through quadrant photodiodes operating at photovoltaic mode. The scattered part of signals that have not entered to the absorbing cavity are incident on the quadrant photodiodes. These signals were calculated and applied as a correction factor to the measured power from cavity. The temperature of absorbing cavity was reduced to 4.2 K. The low operating temperature causes a dramatic increase in the sensitivity of the thermal sensor, reduces the cavity’s thermal capacity and minimizes the effect of background radiation. The temperature of the cavity was measured using the temperature sensors on the cavity.

The operating principle of ESCR is that during the alternate radiant and electrical heating cycles electrical power is adjusted by the current passed through series resistance so that the temperature recorded by the temperature sensor is the same for electrical and optical heatings. Then, the radiant power is equated to the measured quantity of electrical power. The measured electrical power is calculated from the following equation [14]:

\[ P_E = V_h \times I = V_h \frac{V_r}{R_s} \]  

where \( R_h \) and \( R_r \) (\( V_h \) and \( V_r \)) are heater and reference resistances (voltages), respectively; \( I \) is the current.

Since the optical radiation \( P_{opt} \) can be reduced by scattering \( S(\lambda) \), by the window transmittance \( \tau(\lambda) \), by the optical electrical non equivalence \( N \) and imperfect cavity...
absorbance $\alpha(\lambda)$, the measured optical power should be corrected for these parameters [15]. The resultant optical power can be obtained as follows:

$$P_{\text{corr, opt}}(\lambda) = \frac{1}{\tau(\lambda)} \left[ \frac{NP_{\text{opt}}}{\alpha(\lambda)} + S(\lambda) \right]. \tag{2}$$

The optical power measurements of the laser beams used are shown in Fig. 2. In order to minimize errors from power fluctuations, measurement cycles were repeated twenty times.

3. Responsivity measurements

3.1. Absolute responsivity measurements

Because silicon photodiode based reflection type trap detectors have good spatial uniformity, good temporal stability, low nonlinearity, low noise equivalent power, low reflectance, polarization independency, high quantum efficiency and predicted spectral responsivity in the visible region they have been used to link the high accuracy measurements obtained from the ESCR system to the other optical power measurement systems in the visible region [16], [17]. It is known that silicon photodiodes are used
Electrical substitution cryogenic radiometer ...

for photon detection between 250–1100 nm wavelength and internal quantum efficiency is relatively flat (nearly equal to 100%) between 350–850 nm wavelengths, which allows the spectral responsivity to be interpolated throughout this range [18]. In UV and NIR regions, there are some problems with decreasing internal quantum efficiency of silicon photodiodes, which brings about high errors in the interpolation of spectral responsivity using the physical models developed [9]–[11]. In order to measure optical radiation in UV and IR regions pyroelectric radiometers (detectors) have been used as a transfer standard for the ultraviolet, visible and mid-infrared ranges [19].

In order to measure the absolute responsivities, both detectors were placed on the computer-controlled stage (Fig. 1) and after the measurement of optical power with ESCR the detector moved in the beam position and photocurrents were measured. Dividing photocurrent to the optical power absolute responsivities of detectors at 488, 514.5, 532, and 632.8 nm laser wavelengths were obtained.

3.2. Extrapolation and interpolation of absolute responsivities of trap detector

In order to realize spectral responsivity scale, trap detectors consisting of three Hamamatsu S1337-11 windowless photodiodes were constructed as shown in Fig. 3. The number of photodiodes and their geometrical arrangements are determined so as to remove the polarisation sensitivity of trap detectors to incident light and reduce the high reflection losses of silicon photodiodes, which is about 30% in the visible region [20], and thereby increase external quantum efficiency. In this kind of trap detectors incoming beam undergoes five reflections where most of the radiation is being absorbed by photodiodes and the remaining beam returns back along the incoming beam. It is for these reasons that the response of the detector to incident light can be determined more effectively.

Fig. 3. Internal structure of trap detector and photodiode.
The spectral responsivity $R(\lambda)$ of trap detector in terms of reflectance and internal losses is given by [21]

$$R = \left[1 - \rho(\lambda)\right]\left[1 - \delta(\lambda)\right]\frac{e\lambda}{hc}$$

(3)

where $e$ is the elementary charge, $\lambda$ – the wavelength in vacuum, $h$ – the Planck constant, $c$ – the speed of light in vacuum, $\rho(\lambda)$ – the spectral reflectance, $\delta(\lambda)$ – the internal quantum deficiency of trap detector, and $1 - \delta(\lambda)$ is the internal quantum efficiency.

Spectral responsivity of a trap detector as given in Eq. (3) is defined with the reflectance and the internal quantum efficiency. These parameters have to be modelled so as to interpolate and extrapolate the absolute responsivity data.

3.2.1. **Measurement and modelling of reflectance losses**

The reflectance of trap detectors was measured by means of Ar$^+$ (at 488 nm and 514.5 nm wavelengths), Nd:YAG (at 532 nm wavelength) and He-Ne (at 632.8 nm wavelength) lasers using the set-up shown in Fig. 4a. A stabilized laser beam was aligned towards a point close to the rim of a mirror. The beam reflected from the mirror fell on the trap detector and direct signal was measured with this detector. Then the beam reflected from the trap detector was measured using another trap detector. The measurements were repeated by interchanging the detectors. The repeatability in these

![Fig. 4. Laser based spectral reflectance measurement set-up; A1, A2, A3 and A4 are the apertures (a). Illustration of reflectance of silicon photodiode based trap detector. Solid line indicates the theoretical model fitted to the measured values and black points indicate the measured data at laser wavelengths (b).](image-url)
measurements was of the order of $10^{-4}$. The measurement results are displayed in Fig. 4b. In order to use these measured reflectance values in Eq. (3) for the realization of spectral responsivity scale these reflectance values have to be modelled. This is achieved as follows. In a trap detector, incoming beam undergoes five reflections due to the geometry of device (Fig. 3). The angle of incidence of these reflections is twice 45° for the perpendicular ($s$) plane of polarization, once normal incidence, and twice 45° for the parallel ($p$) plane of polarization. Thus, the reflectance of a trap detector is written as

$$\rho_{\text{ref}} = \rho(0°)\rho_s^2(45°)\rho_p^2(45°). \quad (4)$$

To calculate these $\rho(0°)$, $\rho_s(45°)$ and $\rho_p(45°)$ reflections Fresnell’s reflection, transmission equations and known refractive indices of Si and SiO$_2$ were used [20].

The calculated reflectance data were fitted to the measured data by adjusting the oxide thickness parameter. The difference between measured and calculated data after the best fit was at $10^{-4}$ level. The oxide thickness after the fitting was obtained as 28.08 nm. As shown in Fig. 4b, the reflectance of trap detector above about 400 nm is below 0.5%, which means that in this region the responsivity is insensitive to the reflectance changes, but increases at short wavelengths.

### 3.2.2. Calculation and modelling of internal quantum efficiency

Quantum efficiency is defined as the ratio of countable events produced to the number of photons incident on the detector. The most important advantage of silicon based trap detectors is that internal quantum efficiency (IQE) is close to unity in the visible range [22]–[24]. This means that the spectral responsivity of trap detectors would not be changed in time and it would be linear over many orders of irradiance. Any deflection from unity is called internal quantum deficiency, which is assumed to be due to the trap charges at the SiO$_2$/Si interface [8]. This charge attracts the electrons (the minority carrier in this region) and reduces the number reaching the depletion region. To calculate the quantum efficiency of the photodiode it is necessary to quantify this loss mechanism. The IQE is related to the responsivity by the equation:

$$\text{IQE} = 1 - \delta(\lambda) = \frac{Rhc}{\left[1 - \rho(\lambda)\right]e\lambda}. \quad (5)$$

In Eq. (5) the absolute responsivity values were obtained by measuring the trap detector response in terms of either current or voltage against the absolute optical power from ESCR at power stabilized Ar$^+$ (488 nm, 514.5 nm), Nd:YAG (at 532 nm) and He-Ne (632.8 nm) lasers. Using these responsivity values and reflectance $\rho$ values measured as described in Sections 3.1 and 3.2.1 the IQE values were calculated. In order to expand the IQE between 350 to 850 nm (Fig. 5a), the model described by Geist [22], [25], [26] was used, which is given as:
This equation is fitted to the IQE values obtained from Eq. (5) by adjusting the parameters $K, A_1, A_2, \lambda_1$ and $\lambda_2$.

Having seen that both the internal quantum efficiency and the reflectance of trap detector obtained effectively by the combination of measurements and the models developed, the spectral responsivity scale can be realized using Eqs. (3), (4) and (6). Comparing the calculated and measured values, a good agreement between them was observed. Hence we decided to use this model for the realization of spectral responsivity from 350 to 850 nm as shown in Fig. 5b.

3.3. Spectral responsivity measurement of ECPR

Pyroelectric materials are inherently spectrally flat over a broad wavelength range. Therefore the spectral responsivity of a pyroelectric detector is dependent on reflectance of the face electrodes and other materials placed on the detector surface to efficiently convert optical energy into thermal energy. The absorbance of the gold black coating determines the relative spectral responsivity, provided the transmission through the detector is negligible and the reflectance is low. Then the relative spectral response of the detector can be determined from a spectral reflectance measurement, assuming that the gold black transmittance is negligible and the relative detector response is proportional to 1 minus the measured reflectance and transmittance. Using the law of conservation of energy the absorption $\alpha_p(\lambda)$ is given as [27]

$$\alpha_p(\lambda) = 1 - \rho_p(\lambda) - \tau_p(\lambda)$$

where $\rho_p(\lambda)$ is the reflectance and the transmittance $\tau_p(\lambda)$ is assumed to be equal to zero. The predicted spectral responsivity $R_p(\lambda)$ of the pyroelectric detector is [27]
Electrical substitution cryogenic radiometer ...

\[ R_p(\lambda) = \left[ 1 - \rho_p(\lambda) \right] \frac{A}{W} \]  

where \( CF_p \) is the pyroelectric calibration factor (absolute responsivity), which scales the output signal to the optical power received by the pyroelectric detector, being determined by calibrating ECPR against ESCR. The values of \( CF_p \) at laser wavelengths shown in Fig. 7a are about 1.005.

3.3.1. Reflectance measurements of ECPR

The spectral reflectance of ECPR between the wavelengths of 250 and 2500 nm was measured using a measurement facility consisting of a double monochromator, a collimator, integrating sphere, a lock in amplifier, ECPR head and detector system as shown in Fig. 6. As a light source a flat tungsten filament quartz halogen lamp was used and it was operated at constant current mode, which kept the output from the lamp stable to within a few tenths of percent over a measurement period.

Light was focused onto the entrance slit of double-monochromator by using a spherical concave and cylindrical concave mirrors. The purpose of the cylindrical mirror is to correct astigmatism of the optical system. The light emitted by the source and reflected by the condensing mirrors passes through the order-sorting filter wheel after reaching the entrance slit. The order sorting filters are interchanged at nominal wavelengths to stop the radiation from higher-order diffraction of spectrum that would overlap with the working range. The flat mirror diverts the light from the slit to the first collimating mirror, which in turn illuminates the diffraction grating by a parallel white light beam.

The gratings disperse the light and a narrow-band (near-monochromatic) beam is focused to the entrance slit of the second half by the second collimating mirror and via

Fig. 6. Reflectance measurement set-up for the determination of the reflectance of pyroelectric detector (A1 and A2 are the apertures).
a rotary flat mirror. In the second monochromator, the beam is again dispersed and
directed to the exit slit by collimating and flat mirrors. The light beam emerging from
the slit is collimated using collimators. In order to prevent scattered light small limiting
circular apertures were used. The collimated beam was incident on the pyroelectric
detector and reflection standard white barium sulphate (BaSO₄) plate, respectively.
Reflected beams from the pyroelectric detector and reference standard were collected
in the integrating sphere separately and these signals were recorded using silicon
photodiode trap detector, InGaAs and HgCdTe detectors depending on wavelength
range together with lock-in amplifier and chopper system. The spectral reflectance of
ECPR at each wavelength was calculated from the averages of 50 measured reflectance
values (Fig. 7b). The reflectance ρ at wavelength λ was calculated using the following
equation:

\[ \rho = \frac{A_{pd}(\lambda) - A_b(\lambda)}{A_{sd}(\lambda) - A_b(\lambda)} \]  

where \( A_{sd}(\lambda) \) is the average reflected signal of the white standard, \( A_{pd}(\lambda) \) – the average
reflected signal of the pyroelectric detector, and \( A_b(\lambda) \) is the average signal of the
background.

Assuming the gold black transmittance of pyroelectric detector is negligible we
have determined the relative spectral response (Fig. 8a) of the detector from a spectral
reflectance measurement using Eq. (8). In the 250–2500 nm region the reflectance
measurements indicated that the gold black coating is spectrally flat within 0.1%.

Employing the pyroelectric radiometer allows us to derive responsivity scale
between 250 and 400 nm using silicon photodiode detector and between 800 and
1800 nm using germanium (Ge) and indium galium arsenide (InGaAs). The relative responsivity of these detectors was measured using substitution method. The substitution method uses a reference detector (RD) to transfer its responsivity to test detector (TD) as given in the following equation

$$R_{TD}(\lambda) = \frac{V_{TD}}{V_{RD}} \frac{G_{RD}}{G_{TD}} R_{RD}(\lambda)$$

(10)

where $V_{TD}$ and $V_{RD}$ are the output voltages of test and reference detectors, respectively, $G_{RD}$ and $G_{TD}$ are gains for transimpedance amplifiers. Here, reference detector is the pyroelectric detector. $R_{TD}(\lambda)$ is the responsivity of reference detector at the specified wavelength.

4. Conclusions

Absolute optical power measurements were achieved at UME by using ESCR at the temperature of liquid helium with $1.2\times10^{-4}$ uncertainties. In the measurements, Ar$^+$, He-Ne and Nd:YAG lasers were used as radiant sources. To generate a geometrically well-defined Gaussian laser beam a laser power stabilizing system that kept the power level stable at about 0.006% was employed.

After optical power measurements, trap and pyroelectric detectors were calibrated against ESCR at Ar$^+$, Nd:YAG and He-Ne laser wavelengths in order to establish absolute responsivity scale. Reflectance and internal quantum efficiency of the trap detector was also obtained at the same laser wavelengths. The calculated values for the reflectance and internal quantum efficiency fitted the measured values with an uncertainty of $10^{-4}$ level. Using the interpolated and extrapolated reflectance and
internal quantum efficiency values the spectral responsivity scale was established in the spectral ranges of 350–850 nm.

The spectral responsivity scale from 250 to 350 nm and from 850 to 2500 nm was realized using ECPR traceable to the ESCR. ECPR was calibrated against ESCR at vertically polarized tuneable Ar+ (488 nm, 514.5 nm) and Nd:YAG (532 nm) and fixed He-Ne (632.8 nm) laser wavelengths with a standard uncertainty of 1.7%. At four laser wavelengths an average calibration factor of 1.005 was found. The relative spectral responsivity of ECPR between 250 and 2500 nm region was determined from spectral reflectance measurement. The reflectance measurements indicated that pyroelectric detector is spectrally flat within 0.1%.

References

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Hybrid objective with corrected chromatism in visible spectrum

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The possibilities of chromatic aberration correction in a hybrid (diffractive-refractive) objective are discussed. It is possible to design a hybrid triplet objective free from chromatic aberration in the wavelength range 0.45–0.85 µm practically covering the whole visible spectrum. To that end one of the lenses should be made of special glass, but not necessarily of fluorite. For illustration purposes objectives of relative aperture 1:3 and maximum field-of-view angle \( w = 5^\circ \) have been designed and their aberrations presented.

Keywords: hybrid objective, chromatic aberration, apochromatic correction, superachromatic correction.

1. Introduction

A very good correction of chromatic aberration is of major importance in designing optical imaging element (objective). The quality of chromatic correction depends on the complexity of optical system considered. In particular, in three-element objective apochromatic correction and superachromatic correction are possible. As shown by HERZBERGER [1] superachromatic correction, \( i.e., \) equality of focal lengths for four wavelengths: \( \lambda_e = 1014.0 \, \text{nm}, \quad \lambda_F = 486.1 \, \text{nm}, \quad \lambda_C = 656.3 \, \text{nm} \quad \text{and} \quad \lambda_j = 365.0 \, \text{nm}, \) assures that secondary spectrum is practically equal to zero in the whole visible spectrum. Unfortunately, both types of corrections require using a special glass. Moreover, in the case of superachromatic correction special condition concerning parameters of glasses used has to be satisfied.

In this paper, we discuss the feasibility of chromatic aberration correction in a hybrid objective in the widest possible range of wavelength spectrum.

A hybrid triplet objective is understood here as an optical system composed of two classic (refractive) lenses with spherical surfaces and a diffractive microstructure deposited either on one of the above-mentioned lens surfaces or on a separate, stand
alone, thin surface (Fig. 1). The diffractive structure has geometry analogous to the interference pattern obtained by interference of two spherical waves having different radii of curvature [2], [3]. Imaging properties of a such structure is fully defined by two parameters: $z_\alpha$ and $z_\beta$ (Fig. 2), corresponding to the radii of curvature.

2. Apochromatic correction

Let us consider an infinitely thin triplet objective of focal length normalized to unity. The apochromatic correction means that focal distances for three wavelengths: $\lambda_F$, $\lambda_C$ and $\lambda_d$ are equal. The necessary conditions assuring such corrections are as follows [4]–[6]:

$$\frac{\Phi_1}{\nu_1} + \frac{\Phi_2}{\nu_2} + \frac{\Phi_3}{\nu_3} = 0,$$

(1)
Hybrid objective with corrected chromatism ...

\[
\frac{\Phi_1}{\nu_1} P_{1,d} + \frac{\Phi_2}{\nu_2} P_{2,d} + \frac{\Phi_3}{\nu_3} P_{3,d} = 0, \quad (2)
\]

\[
\Phi_1 + \Phi_2 + \Phi_3 = 1 \quad (3)
\]

where \(\Phi\), \(\nu\) and \(P_{\lambda d}\) denote focusing power, Abbe number and partial dispersion for the wavelength \(\lambda_{d}\), respectively. Subscripts 1, 2, 3 denote the number of optical element constituting triplet objective. The analog of Abbe number and partial dispersion for diffractive structure are defined as [4]–[6]:

\[
\nu = \frac{\lambda_d}{\lambda_F - \lambda_C}, \quad (4)
\]

\[
P_d = \frac{\lambda_F - \lambda_d}{\lambda_F - \lambda_C}. \quad (5)
\]

The set of Eqs. (1)–(3) has non-zero solutions if the following condition is met

\[
W_1 = \begin{vmatrix}
1 & 1 & 1 \\
\frac{P_{1,d}}{\nu_1} & \frac{P_{2,d}}{\nu_2} & \frac{P_{3,d}}{\nu_3} \\
\end{vmatrix} \neq 0, \quad (6)
\]

and the solutions are given by:

\[
\Phi_1 = \frac{-c \nu_1}{c(\nu_3 - \nu_1) + \nu_2 - \nu_3}, \quad (7)
\]

\[
\Phi_2 = \frac{\nu_2}{c(\nu_3 - \nu_1) + \nu_2 - \nu_3}, \quad (8)
\]

\[
\Phi_3 = \frac{(c - 1) \nu_3}{c(\nu_3 - \nu_1) + \nu_2 - \nu_3} \quad (9)
\]

where

\[
c = \frac{P_{2,d} - P_{3,d}}{P_{1,d} - P_{3,d}}. \quad (10)
\]

In Table 1 four hybrid objectives made of different sets of optical glasses and designed according to formulas (7)–(10), i.e., fulfilling the condition of apochromatic correction, are presented. In columns 3 and 4 of the table the values of determinant \(W_1\)
as well as focusing powers of particular components are given. The dependence of focal length on the wavelength illustrating chromatic aberration for all objectives mentioned above is plotted in Fig. 3.

Objective No. 1 is a classic hybrid apochromate designed without use of special glasses [6]. In the next three objectives the glass BK3 (Schott) is substituted by special glass FK54 (Schott) or fluorite. From the curves presented in Fig. 3 we conclude that chromatic aberration of hybrid objectives including the lens made of special glass or fluorite is compensated in a much wider wavelength range than in typical apochromate. In practice, secondary spectrum is negligible in the wavelength range

### Table 1. Values of determinants $W_1$ and $W_2$ and focusing powers of particular components of selected triplet objectives designed as apochromats or superachromats.

<table>
<thead>
<tr>
<th>No.</th>
<th>DOE/glass</th>
<th>Apochromatic correction</th>
<th>Superachromatic correction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$W_1$</td>
<td>$\phi$</td>
</tr>
<tr>
<td>1</td>
<td>DOE</td>
<td>7.531×10^{-3}</td>
<td>1.826</td>
</tr>
<tr>
<td></td>
<td>BK3</td>
<td>4.046</td>
<td>-0.834</td>
</tr>
<tr>
<td></td>
<td>SF5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>DOE</td>
<td>7.611×10^{-3}</td>
<td>1.429</td>
</tr>
<tr>
<td></td>
<td>FK54</td>
<td>7.212</td>
<td>-0.436</td>
</tr>
<tr>
<td></td>
<td>SF5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>DOE</td>
<td>4.619×10^{-3}</td>
<td>1.574</td>
</tr>
<tr>
<td></td>
<td>fluorite</td>
<td>6.592</td>
<td>-0.579</td>
</tr>
<tr>
<td></td>
<td>LaSF15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>DOE</td>
<td>6.039×10^{-3}</td>
<td>1.417</td>
</tr>
<tr>
<td></td>
<td>fluorite</td>
<td>7.626</td>
<td>-0.422</td>
</tr>
<tr>
<td></td>
<td>SF5</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>5.578×10^{-3}</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>8.703×10^{-3}</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.564</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 3. Longitudinal chromatic aberration curves for triplet hybrid objectives designed as apochromates.
Hybrid objective with corrected chromatism ...

0.45 nm ≤ λ ≤ 0.95 nm. Moreover, we notice that focusing powers of the objective components are relatively small, which is very promising for monochromatic aberration correction.

3. Superachromatic correction

After Herzberger we define apochromatic correction as identity of effective focal lengths for four wavelengths

\[ f'_1 = f'_2 = f'_3 = f'_4 \]  \hspace{1cm} (11)

where \( \lambda_1 = \lambda_t = 1014.0 \) nm, and \( \lambda_2 = \lambda_j = 365.0 \) nm.

The set of equations assuring such correction has the form [6]

\[ \Phi_1 + \Phi_2 + \Phi_3 = 0, \] \hspace{1cm} (12)

\[ \Phi_1 P_{1, \lambda_1} + \Phi_2 P_{2, \lambda_1} + \Phi_3 P_{3, \lambda_1} = 0, \] \hspace{1cm} (13)

\[ \Phi_1 P_{1, \lambda_2} + \Phi_2 P_{2, \lambda_2} + \Phi_3 P_{3, \lambda_2} = 0, \] \hspace{1cm} (14)

\[ \Phi_1 + \Phi_2 + \Phi_3 = 1 \] \hspace{1cm} (15)

where \( P_{1, \lambda} \) and \( P_{2, \lambda} \) are partial dispersions for the wavelengths \( \lambda_1 \) and \( \lambda_2 \), respectively. The necessary condition for Eqs. (12)–(15) to have non-zero solutions is

\[ W_2 = \begin{vmatrix}
1 & 1 & 1 \\
\Phi_{1, \lambda_1} & \Phi_{2, \lambda_1} & \Phi_{3, \lambda_1} \\
\Phi_{1, \lambda_2} & \Phi_{2, \lambda_2} & \Phi_{3, \lambda_2}
\end{vmatrix} = 0. \] \hspace{1cm} (16)

As already mentioned it is possible to find sets of three glasses (including fluorite) assuring superachromatic correction. In the present work we want to find two glasses, which, together with diffractive structure, give similar correction. To do this we have calculated the determinant \( W_2 \) (see Eq. (16)) for all glasses from Schott catalogue (including fluorite) plus diffractive structure in the wavelength range typical of superachromate and we found that none of the combinations investigated gives satisfactory result. We conclude that it is impossible to find exact solution for hybrid lens assuring superachromatic correction in the wavelength range \( \lambda_t \leq \lambda \leq \lambda_j \).
We can, however, limit the wavelength of interest to the slightly narrower range extended from \( \lambda_1 = \lambda_c = 0.435 \, \text{nm} \) to \( \lambda_2 = \lambda_s = 0.852 \, \text{nm} \), which in practice covers the whole visible spectrum. In the following part of this paper we call such correction (equity of focal lengths for four wavelengths, but in smaller spectral range) “quasi-superachromatic”. To assure such correction the focusing powers of particular components of hybrid objective are calculated from Eqs. (7)–(9), where

\[
c = \frac{P_{2,s} - P_{3,s}}{P_{1,s} - P_{3,s}}.
\]  

(17)

Fig. 4. Longitudinal chromatic aberration curves for triplet hybrid objective on: fluorite and LaSFN15 glass (a) and fluorite and SF5 glass (b) designed as apochromate and quasi-superachromatic.
For further analysis we select pairs of glasses and diffractive structure for which determinant $W_2$ (see Eq. (16)) calculated in limited wavelength range is as small as possible, and simultaneously focusing powers of particular optical elements are not substantially greater than 1. Two best combinations are given in Tab. 1 as lens no. 3 and lens no. 4. In column 5 the values of determinant $W_2$ are given, and column 6 contains focusing powers of particular components of hybrid quasi-superachromate objectives.

The differences between apochromatic and quasi-superachromatic solutions for the objectives No. 3 and No. 4 are illustrated in the graphs of longitudinal chromatic aberration presented in Fig. 4a and b. Figure 4a presents chromatic aberration of the hybrid objective built from the fluorite and LaSFN15 glass. It can be seen that irrespective of whether the design is based on apochromatic or quasi-superachromatic solutions the secondary spectrum is practically the same. In the case of the objective built from fluorite and SF5 glass the design according to quasi-superachromatic solution does not assure correction of chromatic aberration at all. This means that the value of determinant $W_2$ is too great and the focusing powers calculated from Eqs. (13)–(16) are in fact completely accidental, and do not assure proper correction of chromatic aberration.

The above results suggest that quasi-superachromatic correction is possible if the value of determinant $W_2$ is not greater than $\sim 5 \times 10^{-4}$. The secondary spectrum in the whole wavelength range under consideration is negligibly small. Moreover, the values of focusing powers of all elements are very close to those calculated from apochromatic condition.

4. Examples oh hybrid objectives with corrected chromatic aberration and conclusions

In order to verify the presented solutions and their practical usefulness we designed several real objectives based on the data presented in Tab. 1. For practical reasons all the design objectives have focal length equal to 100 mm. Monochromatic aberrations are minimized as much as possible by appropriate choice of the curvatures of refractive lenses as well as parameters describing the diffractive structure. The construction parameters of these objectives are presented in Tab. 2. Their imaging quality can be evaluated from the aberration characteristics presented in Figs. 5–8. The curves describing spherochromatic aberration for the wavelengths from the range $\lambda_c$ to $\lambda_s$ as well as meridional and sagittal curvature are presented there. The amount of comma can be deduced from spot diagrams.

The analysis of imaging characteristics of the lenses designed on the basis of the presented solutions allows us to conclude that:

– by substituting one of the normal glasses with special glass or fluorite the correction of spherochromatic aberration can be better – the relative aperture as high as 1:3 can be obtained;
### Table 2a Construction parameters of apochromatic objective No. 1.

<table>
<thead>
<tr>
<th>$R$ [mm]</th>
<th>$d$ [mm]</th>
<th></th>
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<tbody>
<tr>
<td>+45.40</td>
<td>0</td>
<td>DOE: $z_\alpha = -13.50$ mm, $z_\beta = -14.469$ mm</td>
<td></td>
</tr>
<tr>
<td>+45.40</td>
<td>8.0</td>
<td>BK3</td>
<td></td>
</tr>
<tr>
<td>-83.10</td>
<td>1.0</td>
<td>SF5</td>
<td></td>
</tr>
<tr>
<td>42.48</td>
<td>36</td>
<td>aperture stop</td>
<td></td>
</tr>
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</table>

### Table 2b Construction parameters of apochromatic objective No. 2.

<table>
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<tr>
<td>$\infty$</td>
<td>0</td>
<td>aperture stop</td>
<td></td>
</tr>
<tr>
<td>+49.98</td>
<td>0</td>
<td>DOE: $z_\alpha = -20.00$ mm, $z_\beta = -19.974$ mm</td>
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</tr>
<tr>
<td>+49.98</td>
<td>6.0</td>
<td>FK54</td>
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<td>-79.00</td>
<td>0.5</td>
<td>air</td>
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</tr>
<tr>
<td>-65.00</td>
<td>1.0</td>
<td>SF5</td>
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<tr>
<td>-117.00</td>
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### Table 2c Construction parameters of superachromatic objective No. 3.

<table>
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<td>$\infty$</td>
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<td>+49.90</td>
<td>0</td>
<td>DOE: $z_\alpha = -20.00$ mm, $z_\beta = -19.976$ mm</td>
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</tr>
<tr>
<td>+49.90</td>
<td>7.0</td>
<td>fluorite</td>
<td></td>
</tr>
<tr>
<td>-63.22</td>
<td>0.5</td>
<td>air</td>
<td></td>
</tr>
<tr>
<td>-58.50</td>
<td>2.0</td>
<td>LaSFN15</td>
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<tr>
<td>-95.90</td>
<td></td>
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</tbody>
</table>

### Table 2d Construction parameters of apochromatic objective No. 4.

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<th>$R$ [mm]</th>
<th>$d$ [mm]</th>
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<td>42.48</td>
<td>0</td>
<td>DOE: $z_\alpha = -19.00$ mm, $z_\beta = -18.978$ mm</td>
<td></td>
</tr>
<tr>
<td>42.48</td>
<td>5.5</td>
<td>fluorite</td>
<td></td>
</tr>
<tr>
<td>-125.00</td>
<td>0.8</td>
<td>air</td>
<td></td>
</tr>
<tr>
<td>-76.50</td>
<td>1.0</td>
<td>SF5</td>
<td></td>
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<tr>
<td>-150.87</td>
<td>39.0</td>
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</table>
Fig. 5. Aberration characteristics of hybrid apochromate No. 1 (DOE+BK3+SF5). Relative aperture 1:4, maximum field angle $w = 5^\circ$ ($\triangle$ for $\lambda = 436$ nm, $\square$ for $\lambda = 486$ nm, $+$ for $\lambda = 588$ nm, $\times$ for $\lambda = 656$ nm, $\bigtriangledown$ for $\lambda = 852$ nm).
Fig. 6. Aberration characteristics of hybrid apochromate No. 2 (DOE + FK54+SF5). Relative aperture 1:3, maximum field angle $w = 5^\circ$ ($\triangle$ for $\lambda = 436$ nm, $\square$ for $\lambda = 486$ nm, $+$ for $\lambda = 588$ nm, $\times$ for $\lambda = 656$ nm, $\triangledown$ for $\lambda = 852$ nm).
Fig. 7. Aberration characteristics of hybrid quasi-superachromate No. 3 (DOE+fluorite+LaSFN15). Relative aperture 1:3, maximum field angle \( w = 5^\circ \) (\( \Delta \) for \( \lambda = 436 \text{ nm} \), \( \square \) for \( \lambda = 486 \text{ nm} \), \( + \) for \( \lambda = 588 \text{ nm} \), \( \times \) for \( \lambda = 656 \text{ nm} \), \( \triangledown \) for \( \lambda = 852 \text{ nm} \)).
Fig. 8. Aberration characteristics of hybrid apochromate No. 4 (DOE+fluorite+SF5). Relative aperture 1:3, maximum field angle \( w = 5^\circ \) (\( \triangle \) for \( \lambda = 436 \) nm, \( \square \) for \( \lambda = 486 \) nm, + for \( \lambda = 588 \) nm, \( \times \) for \( \lambda = 656 \) nm, \( \triangledown \) for \( \lambda = 852 \) nm).
– the secondary spectrum of the objectives designed is similar and very small irrespective of whether the objective was designed according to apochromatic or superachromatic formulas. Full correction of chromatic aberration in the wavelength range 0.45–0.85 µm is obtained;
– in practice, the use of fluorite is not necessary, however it is not possible to find formal solution for superachromatic correction without fluorite.

Concluding we would like to express an opinion that in hybrid triplet objective it is possible to obtain very good correction of chromatic aberration in the wavelength range practically covering the whole visual spectrum. Depending on the glasses used as a starting point for design process two alternative conditions can be used: apochromatic or superachromatic correction. The imaging quality of the final objective depends on the value of secondary spectrum and not on the formal type of correction.

References

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Multiscale contrast image fusion scheme with performance measures

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A new method of image fusion from various sensing modalities is proposed. This method adopts a perceptual fusion operator by using a sequence of multiscale contrast pyramid images. The method is tested by merging parallel registered visible and infrared images. Several performance measures clearly indicate that this method outperforms the other three approaches producing better visual effects.

Keywords: image fusion, multiscale image analysis, ratio of low-pass pyramid, visual perception, image processing.

1. Introduction

For images of the same scene obtained at different bandwidths, such as visible and infrared images, it is nearly impossible to capture all the details and salient features. Image fusion can be used to integrate different input sources into a single image, which is able to assimilate all these individual features potentially to be useful for human observation [1].

A plethora of algorithms for multiresolution image fusion [2]–[9], such as Laplacian pyramid and wavelet pyramid, have been developed. It is a well-known fact that the human visual system is sensitive to local luminance contrast. Thus the RoLP (ratio of low-pass) pyramid is a better image fusion method compared with other pyramid algorithms in general. But their fusion operators are relatively simple by performing logic or a weighted combination. Therefore, a more elaborate scheme is necessary to improve previous methods.

This paper describes the extension of RoLP pyramidal image decomposition. The proposed algorithm performs a novel fusion operator by judging uniform parameter on the analysis of segmentation techniques. Finally, fusion quality is calculated objectively through three measures that demonstrate the improvements offered by the present scheme over other three fusion approaches. The results all show that this method is perceptually meaningful and explicit.
2. Towards fusion method representation

This section describes an image fusion method based upon a previous successful multiresolution method, with an added ability to tailor the selection criteria to the contrast sensitivity. The basic scheme employs a multiresolution algorithm that uses decomposition, fusion, and reconstruction. Figure 1 shows a schematic diagram of the basic structure of the proposed image fusion scheme.

2.1. Generating kernel

One important component is the generating kernel \( \omega \), which is chosen subject to certain constraints such as separable, normalized, symmetric and equal contribution. Let \( \omega(0) = 0.4 \), \( \omega(1) = \omega(-1) = 0.25 \), \( \omega(2) = \omega(-2) = 0.05 \). Then,

\[
\omega = \frac{1}{400} \begin{bmatrix}
1 & 5 & 8 & 5 & 1 \\
5 & 25 & 40 & 25 & 5 \\
8 & 40 & 64 & 40 & 8 \\
5 & 25 & 40 & 25 & 5 \\
1 & 5 & 8 & 5 & 1 
\end{bmatrix}
\]

2.2. Image decomposition

Suppose the image is represented initially by the array \( G_0 \) which is the bottom level of the Gaussian pyramid. In a similar way each value within level \( l \), representing \( G_l \), is then obtained from values within level \( l - 1 \) by applying the generating kernel. This is performed as follows, for all nodes \( i, j \):
Multiscale contrast image fusion scheme ...

\[ G_l(i, j) = \sum_{m, n = -2}^{2} \omega(m, n) G_{l-1}(2i + m, 2j + n). \]  

(1)

Because of the reduction in the spatial frequency content, each image in the sequence can be represented by an array that has half dimensions of its predecessor. Thus we can expand array \( G_l \) into array \( G_{l-1}^* \) by interpolating new node values between the given values, which is the same size as \( G_{l-1} \)

\[ G_{l-1}^*(i, j) = 4 \sum_{m, n = -2}^{2} \omega(m, n) G_l\left(\frac{i + m}{2}, \frac{j + n}{2}\right). \]  

(2)

Only terms for which \((i + m)/2\) and \((j + n)/2\) are integers are included in this sum.

Then the RoLP pyramid \( C_l \) is the ratio of two successive levels in the Gaussian pyramid. It is then defined as

\[
\begin{cases} 
C_l = \frac{G_l}{G_l^*} & \text{for } 0 \leq l \leq N - 1, \\
C_N = G_N & \text{for } l = N.
\end{cases}
\]  

(3)

2.3. Detail pyramid fusion

Consider a contrast vision system to be the key factor in determining the salient and dominant features of an image \([10]\). Note that this perceptual information association is supported by human visual system studies and is extensively used in image fusion scheme. Thus, we define \( d(B_k) \), namely uniform parameter, the saliency formula for a given level image \( G_l(i, j) \) as follows:

\[
d(B_k) = \frac{1}{m' \times n'} \sum_{(i, j) \in B_k} \frac{|G_l(i, j) - \mu_k|}{\mu_k}
\]  

(4)

where \( \mu_k \) is the mean of block \( B_k \), and \( m' \times n' \) is the block size.

In the following, we shall assume that there are two inputs, a visible image (VIS) and an infrared (IR) image. Each level of two RoLP pyramids to be fused is decomposed into sets of smaller blocks and compared using uniform parameter response. The block of fused image is formed here as

\[
BC_{F_k} = \begin{cases} 
BC_{VIS_k} & \text{if } d(BC_{VIS_k}) \geq d(BC_{IR_k}) \\
BC_{IR_k} & \text{otherwise}
\end{cases}
\]  

(5)
where \( d(BC_{\text{VIS}k}) \) and \( d(BC_{\text{IR}k}) \) denote uniform parameters of blocks \( BC_{\text{VIS}k} \) and \( BC_{\text{IR}k} \) of corresponding image pair \( C_{\text{VIS}l} \) and \( C_{\text{IR}l} \), respectively. Therefore, image fusion is performed by selecting the feasible blocks according to Eqs. (4) and (5), so this process yields a sequence of merged images \( C_{F0}, C_{F1}, \ldots, C_{FN} \).

### 2.4. Fused image reconstruction

As the final step the reconstructed composite image \( F_0 \) can be recovered exactly from its RoLP pyramid representation:

\[
\begin{align*}
F_N &= C_{FN} & \text{for} & & l = N, \\
F_l &= C_{Fl}F^*_l & \text{for} & & 0 \leq l \leq N - 1.
\end{align*}
\]

### 3. Measures and results

In order to objectively evaluate the capabilities of different fusion algorithms, three measures of image fusion performance are provided for some quantitative comparison.

#### 3.1. Entropy

Entropy is known to be a measure of the amount of uncertainty about the image. It is then given by

\[
H = - \sum_{i=0}^{L-1} p_i \log_2 p_i
\]

where \( L \) is the number of gray levels; and note that

\[
p_i = \frac{\text{number of pixels} \, D_i \, \text{of each gray level} \, i}{\text{number of pixels} \, D \, \text{in the image}}.
\]

#### 3.2. Cross entropy

Notice that cross entropy measures the difference of two images, so a lesser cross entropy is preferred. It is computed as

\[
H_{ce} = - \sum_{i=0}^{L-1} p_{Ri} \log_2 \left( \frac{p_{Ri}}{p_{Fi}} \right)
\]
where \( p_{Ri} \) and \( p_{Fi} \) are the corresponding probability of input and fused image occurring. Here, we choose the root mean square cross entropy of two inputs to the output image, respectively

\[
\overline{H_{ce}} = \sqrt{\frac{H_{ce1}^2 + H_{ce2}^2}{2}}.
\]  

(8)

### 3.3. Mutual information

Mutual information is a measure that determines how much information is obtained from the fusion of input images. We use this measure as the third evaluation method to assess the performance of different image fusion algorithms;

\[
MI(R, F) = \sum_{i_1=0}^{L-1} \sum_{i_2=0}^{L-1} p_{R,F}(i_1, i_2) \log_2 \left( \frac{p_{R,F}(i_1, i_2)}{p_{R}(i_1) p_{F}(i_2)} \right)
\]

where \( p_{R,F} \) indicates the normalized joint gray level histogram of images \( R \) and \( F \), \( p_R \) and \( p_F \) are the normalized marginal histograms of the two images. We choose the average cross entropy of two inputs to the output image, respectively,

\[
\overline{MI} = \frac{MI_1 + MI_2}{2}.
\]  

(9)

### 3.4. Results of different schemes

To verify the proposed approach, four fusion algorithms are tested for objective performance evaluation applied to a pair of visible and infrared registered imagery. Obviously, the two images, as presented in Fig. 2a and b, have different spectral characteristics and details in the same depicted scene. Both are optical images and are registered with each other. The visible image has a lower contrast; thus the easily discernable finer features and background are almost indistinguishable in the infrared image. On the other hand, the infrared image also has some unique features, such as the textural patterns with a higher contrast, which are present in the visible image. In scheme 1 (Fig. 2c), Laplacian algorithm is performed with equal weights (i.e., \( w_{VIS} = w_{IR} = 0.5 \)) assigned to a pair of inputs respectively for reconstructing the composite image [3]. Scheme 2 (Fig. 2d) employs Toet algorithm with a maximum absolute contrast node selection technique [5]. In scheme 3 (Fig. 2e), the fusion rule is defined by calculating the wavelet transform modulus maxima using “Daubechies 8” filter [8]. Scheme 4 (Fig. 2f) implements the new algorithm using a decomposition block size of 4×4. All of the four schemes choose the third layer as the fusion level to compare.
these methods for consistency verification. In general, the third layer is a better fusion level proved by more tested cases. From the observer’s viewpoint, by comparing the fused images all four schemes do a good job of preserving visual information from each input image. However, Fig. 2c and 2d do not have the same amount of details compared to other figures. The reduction in intensity then causes the other areas of the image to become less contrasted in Fig. 2e. Obviously, Fig. 2f contains more details
of progressively finer resolution and greater contrast enhancement than other figures, which especially appears to preserve features in the input images that are dominant.

At the same time, this illustrative example is provided here for some quantitative comparison of four fusion schemes in Tab. 1. According to three evaluation criteria, the proposed method achieves the best entropy, cross entropy and mutual information. It is superior to the traditional Laplacian pyramid, ratio of low-pass pyramid and wavelet transform fusion method, with 1.1–3.4% improvement of entropy, 48–78% reduction in cross entropy and 3.5–8.1% enhancement in mutual information through further computations. The evaluation results coincide with the visual effect very well. Very clearly, the result convincingly demonstrates that the image in Fig. 2f contains more perceptual details and features than other figures. Overall the represented
approach explicitly shows better fusion performance, both esthetically and numerically. Thus, we can see that image fusion is not different from singular image enhancement at all, which makes full use of complementary or redundant information of input images to acquire a synergistic combination. To prove the correctness and universality of this algorithm, more cases are tested dealing with visible and infrared images, judged by the above measures, with satisfying results derived.

Here, we discuss the influence of the size of blocks in detail. We choose mainly different decomposition block sizes to test it. Figure 3 shows the fused effects from different decomposition block sizes. At the same time, Tab. 2 shows performance measures for quantitative comparison. Block size 4×4 is the most optimal as a whole. In general, if the block size is too large, a particular block may lead to aberration of image gray level gradients. On the other hand, using a very small block size may lead to the saw-tooth effect. Moreover, a large amount of image experiments suggest that a better fusion result is quite sensitive and suitable to this block size 4×4, and this conclusion is stable and consistent.

<table>
<thead>
<tr>
<th>Block sizes</th>
<th>Entropy</th>
<th>Cross entropy</th>
<th>Mutual information</th>
</tr>
</thead>
<tbody>
<tr>
<td>1×2</td>
<td>7.5787</td>
<td>0.5678</td>
<td>2.6558</td>
</tr>
<tr>
<td>2×2</td>
<td>7.5921</td>
<td>0.5222</td>
<td>2.6078</td>
</tr>
<tr>
<td>4×4</td>
<td>7.6384</td>
<td>0.4909</td>
<td>2.8530</td>
</tr>
<tr>
<td>4×8</td>
<td>7.6336</td>
<td>0.4740</td>
<td>2.6693</td>
</tr>
<tr>
<td>8×8</td>
<td>7.5305</td>
<td>0.6960</td>
<td>3.0559</td>
</tr>
<tr>
<td>16×16</td>
<td>7.4901</td>
<td>0.7666</td>
<td>2.6744</td>
</tr>
</tbody>
</table>

There is an important issue that should be investigated in the future. If one of the images has much smaller local contrast than the other one, the algorithm may have some restrictions because of the uniform parameter \( d \), which is not considered here. Nevertheless, preliminary experiments suggest that the scheme is suitable to the visible and infrared images in general, provided that the local contrast difference between the two images is not too extreme.

4. Conclusions

A visual perception-based multiscale contrast image fusion scheme is presented for multispectral image data. No assumption is made regarding the nature of the relation between the intensities in both input modalities. The desired visual improvements over image fusion agree remarkably well with that obtained from objective results in comparison with other methods. Detection, recognition, and search tasks can therefore benefit considerably from this new image representation.
References


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