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Third order aberrations of the electron deflecting-focusing system

A linear analysis of the deflecting-focusing systems applicable in the scanning microscope has been carried out. The third order aberrations are given while twelve different kinds of deflecting and imaging aberrations of the deflecting-focusing system are discussed.

1. Introduction

The deflecting-focusing systems are applied in the electron-beam devices with the scanning electron beam. The first qualitative analysis of the thirs order aberrations in the deflecting-focusing system was made by Amboss and Wolf [1]. The formulae enabling calculation of third order aberrations for a deflecting-focusing system were given by Munro [2], who employed the trajectory equation for an electron ray in the linear approximations, as well as the coefficients of the expansion into power series of the deflecting and focusing fields.

The analysis of the deflecting-focusing system aberrations may be carried out in another way, i.e. by considering a complex electronooptical system as an ensamble of cascaded electronooptical regions. This method has been employed in the present work. It allows (as compared with Munro's method [2, 3]): a) to analyse a geometrical aberrations of a complex electronooptical system, basing on a well-known theory of aberration of its each subensambles; b) to control the behaviour of aberrations introduced by subensambles of the system. The property b) is useful for minimizing the electronooptical system aberration.

In the section 2 a general description of a deflecting-focusing system will be given together with the simplifing assumptions. In section 3 the linear aproximation of the discussed system is formulated, while the method of aberration coefficient determination is given in section 4, 5 and 6. The geometrical aberrations of deflecting-focusing system are characterized in section 7.

2. Description of the deflecting-focusing system

The deflecting-focusing system is composed of two deflecting units which form a deflecting doublet DD and a focusing lens; the latter being most frequently the end lens of the electronooptic device. The scheme of the deflecting-focusing system is shown in fig. 1.

The following simplifications has been assumed in the analysis of the deflecting-focusing system:

— the focusing field of the end lens and DD coils do not overlap,

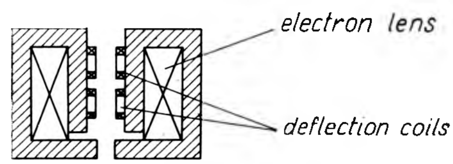

Fig. 1. The scheme of the deflecting-focusing system

— the image plane of the deflecting-focusing system is the Gaussian plane of the lens,

— the deflecting field of DD starts at the object plane of the lens.

3. A linear approximation of the deflecting-focusing system

For linear approximation the parameters u, u' of an electron ray in the image plane are

$$\begin{bmatrix} u \\ u' \end{bmatrix} = S \begin{bmatrix} u_0 \\ u'_0 \end{bmatrix}, \tag{1}$$

where:

$u = x+iy$ are the complex coordinates of an electron ray in the image region of the lens,

$u_0 = x_0+iy_0$ are the tangents of the electron ray inclination angles in the image region of the lens,

$u_0 = x_0+iy_0$ are the complex electron ray coordinates in the object plane z_0 lens,

$u'_0 = x'_0+iy'_0$ are the tangents of the ray inclination angles in the z_0 plane.

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The matric S in the equation of the electron ray trajectory has the form [4]:

$$S(z)=\begin{bmatrix}\mu(z) & \nu(z)\\ \mu'(z) & \nu'(z)\end{bmatrix}. \tag{2}$$

In accordance with the assumptions accepted in section 2 the object and image planes of the deflecting-focusing system are identical with the object and image planes of the lens. The deflecting doublet DD is placed in the object region of the lens (see fig. 2). In the image plane z_i of the deflecting-focusing system the elements of matrix S have the form [4]:

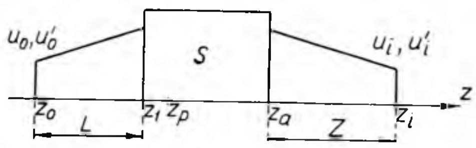

Fig. 2. The simplified scheme of the deflecting--focusing system

$$\begin{aligned}\mu_i&=\mu(z_i)=M,\\ \nu_i&=\nu(z_i)=0,\\ \mu_i'&=\mu_i'(z_i),\\ \nu_i'&=\nu'(z_i),\end{aligned} \tag{3}$$

M — linear magnification of the lens.

The electron beam trajectory in the deflecting-focusing system is characterized by the so-called central ray. The initial conditions for the central ray (in the plane z_0) have the form [5]:

$$x_0=y_0=x_0'=y_0'=0.$$

The central ray which runs initially along the z axis is deflected by the first deflecting unit of DD being next deflected in the opposite direction. The central ray trajectory in DD and in the lens are shown in fig. 3. In accordance with this figure the central ray accordinates in the z_2 plane φ are

$$\begin{aligned}x_c(z_2)&=-dx_c'(z_2),\\ y_c=(z_2)&=-dy_c'(z_2),\\ x_c'(z_2)&=-\tan\gamma_x,\\ y_c'(z_2)&=-\tan\gamma_y,\end{aligned} \tag{4}$$

where

$$\begin{aligned}\tan\gamma_x&=\mu_0 k\int\limits_{z_0}^{z_0}(H_0^{(1)}-H_0^{(2)})\,dz,\\ \tan\gamma_y&=-\mu_0 k\int\limits_{z_0}^{z_2}(V_0^{(1)}-V_0^{(2)})\,dz,\end{aligned} \tag{5}$$

and

$H_0^{(1)}, H_0^{(2)}$ are the y-coordinates of the magnetic strength vector on the axis z for the first and second deflecting system [6],

$V_0^{(1)}, V_0^{(2)}$ are the x-coordinates of the magnetic strength vector on the axis z for the first and second deflecting system;

$$\mu_0=4\pi 10^{-7}\frac{H}{m},\ k=\left(\frac{e}{2m_0U_0^*}\right)^{1/2};$$

$U_0^*=U_0(1+\varepsilon U_0)$ — the accelerating voltage with the relativistic correction

$$\varepsilon=\frac{e}{2m_0c^2}.$$

The coordinates of the electron ray of initial parameters u_0, u_0' in the z_2 plane are [5]

$$\begin{bmatrix}u_2\\ u_2'\end{bmatrix}=L\begin{bmatrix}u_0\\ u_0'\end{bmatrix}+U \tag{6a}$$

where

$$\begin{aligned}U=U_c(z_2)&=\begin{bmatrix}u_c(z_2)\\ u_c'(z_2)\end{bmatrix}\\ &=-(\tan\gamma_x+i\tan\gamma_y)\begin{bmatrix}-d\\ 1\end{bmatrix}.\end{aligned} \tag{6b}$$

The matrix $L=\begin{bmatrix}1 & L\\ 0 & 1\end{bmatrix}$ is a field-free matrix in the region of length L.

The deflecting-focusing system is composed of two cascaded four-poles: an active four-pole DD, and a passive lens four-pole (comp. fig. 4). In the further

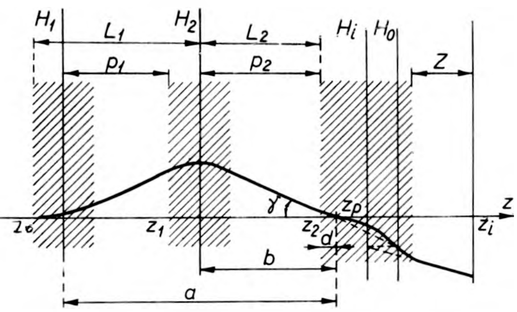

Fig. 3. The central ray trajectory in DD and in the end lens;

H_1, H_2 — principal planes of the deflecting systems, H_i, H_0 — principal planes of the lens

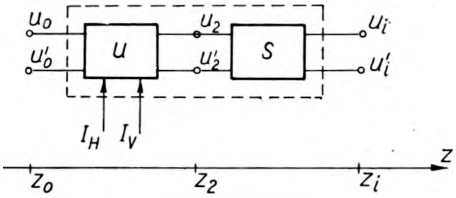

Fig. 4. The scheme of the deflecting-focusing system; I_H, I_V — the deflecting currents in the DD coils

considerations the input coordinates u_0, u_0' of the electron beam are referred to the z_2 plane, the latter being removed from the object plane of the deflecting-focusing system by a distance L. Hence

$$\begin{bmatrix} u_0 \\ u_0' \end{bmatrix} = L^{-1} \begin{bmatrix} u_2 \\ u_2' \end{bmatrix}. \tag{7}$$

By virtue of (1), (6a), and (7) we have

$$\begin{bmatrix} u_i \\ u_i' \end{bmatrix} = S_l \begin{bmatrix} u_0 \\ u_0' \end{bmatrix} + L^{-1} U. \tag{8}$$

This dependence associates the electron beam coordinates in the image plane z_i of the deflecting-focusing system with the ray coordinates in the object plane z_0. In particular on the basis of (6b) and (8) we get

$$u_i = M[u_0 + (\tan\gamma_x + i\tan\gamma_y)(d+L)], \tag{9}$$

where:

$L = L_1 + L_2$ — the length of DD along the z-axis,

M — linear magnification of the lens.

The central ray coordinates ($u_0 = 0$) in the z_i plane have the form

$$x_i = \mu_0 kLM \int_{z_0}^{z_2} (H_0^{(1)} - H_0^{(2)})\, dz,$$
$$y_i = -\mu_0 kLM \int_{z_0}^{z_2} (V_0^{(1)} - V_0^{(2)}\, dz. \tag{10}$$

The lens in the deflecting-focusing system causes the rotation of the coordinate system by angle Θ_i with respect to the z_2 plane [4]

$$u_i = u_2 \exp i\Theta_i,$$

where

$$\Theta_i = \frac{1}{2} k \int_{z_2}^{z_a} B_0(z)\, dz.$$

The function $B_0(z)$ is an axial component of magnetic induction vector in the lens.

4. Third order aberrations of the deflecting-focusing system

The deflection Δx_i of the electron beam caused by the deflecting errors of third order has the form

$$\Delta x_i = d_i^{jk} x_j x_k$$
$$(i, j, k = 1, \ldots, 5) \tag{11}$$

where:

Δx_i — first rank tensor ($\Delta x_1 = 0$, $\Delta x_2 = \Delta x$, $\Delta x_3 = \Delta y$, $\Delta x_4 = \Delta x'$, $\Delta x_5 = \Delta y'$),

x_j — first rank tensor being an element of the column vector: $\mathrm{col}\{1, x_0, y_0, x_0', y_0'\}$.

The coordinates x_0, y_0, x_0', y_0' characterize the properties of the electron ray in the $z = z_0$ plane. The third rank tensor d_i^{jk} in (11) is an element of a quadratic matrix, which for $i = 1$ is a zero matrix, i.e. $D_1 = 0$, while for $i = 2, \ldots, 5$:

$$D_i = \begin{bmatrix} d_i^{11} & d_i^{12} & d_i^{13} & d_i^{14} & d_i^{15} \\ 0 & d_i^{22} & d_i^{23} & d_i^{24} & d_i^{25} \\ 0 & 0 & d_i^{33} & d_i^{34} & d_i^{35} \\ 0 & 0 & 0 & d_i^{44} & d_i^{45} \\ 0 & 0 & 0 & 0 & d_i^{55} \end{bmatrix} \tag{12}$$
$$(i = 2, \ldots, 5).$$

The matrix elements $D_i(i = 0)$ depend upon the value of perfect deviation X_s, Y_s in the screen plane. This deviation depends upon the deflecting field strength H_0, V_0 on the axis z

$$X_s = \mu_0 k \int_{z_0}^{z_s} (z_s - z) H_0(z)\, dz,$$
$$Y_s = -\mu_0 k \int_{z_0}^{z_s} (z_s - z) V_0(z)\, dz, \tag{13a}$$

(copm. [6]).

The tangents of the electron ray slope angles in the screen plane z_s are

$$X_s' = \mu_0 k \int_{z_0}^{z_s} H_0(z)\, dz,$$
$$Y_s' = -\mu_0 k \int_{z_0}^{z_s} V_0(z)\, dz. \tag{13b}$$

The significance of the quantities z_0, z and x_0, y_0 is explained in fig. 5.

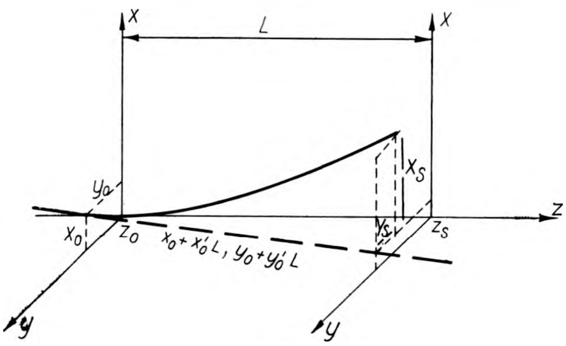

Fig. 5. Electron ray trajectory in the deflecting field

5. Geometrical third order aberrations of the end lens

Deflection of the electron ray in Gaussian image plane of the lens caused by the third order aberrations amounts to

$$(\Delta x_i)_k = s_k^{\alpha\beta\gamma} x_\alpha x_\beta x_\gamma,$$
$$(\alpha, \beta, \gamma, k = 1, \ldots, 5) \tag{14}$$

The fourth order tensor $s_k^{\alpha\beta\gamma}$ is symmetrical with respect to the contravariant indices. For $k = 0$, $s_0^{\alpha\beta\gamma} = 0$, while for $k = 2,3$ the tensor elements $s_k^{\alpha\beta\gamma}$ depend upon the focusing lens coefficients (comp. [7]). Tensor x_α in (14) is a first rank tensor being in fact a column vector element

$$\bar{x} = \mathrm{col}\{1, x_0, y_0, x_a, y_a\}, \tag{15}$$

x_0, y_0 — coordinates of the electron ray in the object plane z_0 of the lens,
x_a, y_a — coordinates of the electron ray in the aperture plane z_a (comp. fig. 3).

6. Third order aberration of the deflecting-focusing system

The ideal and real rays in the deflecting-focusing system are presented diagramatically in fig. 6. The regions (z_0, z_1) and (z_1, z_2) are the regions of the first and second deflecting systems, respectively, while the region (z_2, z_i) is the region of the focusing lens.

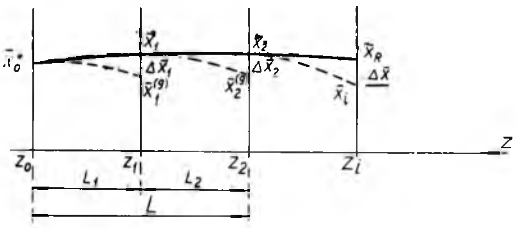

Fig. 6. Electron ray trajectory in the deflecting-focusing system

The electron ray of initial vector

$$x_0^* = \mathrm{col}\{1, x_0, y_0, x_0', y_0'\}$$

intersects the plane z_1 at a point

$$\bar{x}_1 = \{\mathrm{col}\,1, x_1, y_1, x_1', y_1'\}$$

due to the action of the first deflecting field. In absence of the deflecting errors in the first deflecting system the ray coordinates would be

$$\bar{x}_1^{(g)} = \mathrm{col}\{1, x_1^{(g)}, y_1^{(g)}, x_1'^{(g)}, y_1'^{(g)}\}.$$

The difference

$$\Delta\bar{x}_1 = \bar{x}_1 - \bar{x}_1^{(g)}$$

is caused by the deflecting errors of the first system. The vector $\bar{x}_1$ in the z_1 plane is an initial vector of the electron ray for the second deflecting system. Similarly as for the first region the vector $\bar{x}_2$ describes the real ray coordinates, while $\bar{x}_2^{(g)}$ denotes the ideal coordinates in the z_2 plane. The analysis of the trajectory ray in the focusing system is analogous. The deflection $\Delta\bar{x}_1$ in the z_1 plane caused by the deflection errors of the deflecting system in view of (11) amounts to

$$\Delta x_{i1} = d_{i1}^{jk} x_j x_k \tag{16}$$

where:
x_k — element of the vector $\bar{x}_0^*$ in the z_0 plane,
d_{i1}^{jk} — third rank tensor of the first deflecting system (comp. (12)).

The ideal deflection and the electron ray slope in the z_1 plane are

$$\begin{aligned} X_1 &= \mu_0 k \int_{z_0}^{z_1} H_0^{(1)}(z_1 - z)\,dz, \\ Y_1 &= -\mu_0 k \int_{z_0}^{z_1} V_0^{(1)}(z_1 - z)\,dz, \\ X_1' &= \mu_0 k \int_{z_0}^{z_1} H_0^{(1)}\,dz, \\ Y_1' &= -\mu_0 k \int_{z_0}^{z_1} V_0^{(1)}\,dz. \end{aligned} \tag{17}$$

In the further considerations the initial parameters of the electron beam are characterized by the ray coordinates in the object plane z_0 and aperture plane z_a, i.e. by x_0, y_0, and x_a, y_a. By introducing the column-vector

$$\bar{x}_0 = \mathrm{col}\{1, x_0, y_0, x_a, y_a\}$$

relation connecting the variables x_0, y_0, x_0', y_0' with x_0, y_0, x_a, y_a has the form

$$\bar{x}_0^* = W\bar{x}_0 \tag{18a}$$

where

$$W = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & \dfrac{\mu_a M}{Z} & 0 & \dfrac{-M}{Z} & 0 \\ 0 & 0 & \dfrac{\mu_a M}{Z} & 0 & \dfrac{-M}{Z} \end{bmatrix}.$$

The magnitude μ_a in the matrix W depends upon the cardinal elements of the lens and is given by (see the Appendix)

$$\mu_a = M + \frac{Z}{f_i},$$

where
M — linear magnification of the lens,
$Z = z_i - z_a$ — the length of the (image) trajectory region of the lens (see fig. 2),
f_i — image focal length of the lens.

The dependence (18a) in the tensor notation has the form

$$x_i = w_i^\mu x_\mu \quad (i, \mu = 1, \ldots, 15). \tag{18b}$$

The first rank tensor x_μ is an element of the vector x_0. In order to distinguish it form the vector elements x_0^* it has been denoted with a greek index.

By substituting (18b) to (16) a deflection $\Delta\bar{x}_1$ in the z_1 plane become

$$\Delta x_{i1} = \sum_{k,l,m,n} [klmn]_{i1} x_0^k y_0^l x_a^m y_a^n. \tag{19}$$

The four-dimensional matrix $[klmn]_{i1}$ in the relation (19) describes the aberration coefficient of the first deflecting-focusing system

$$
\begin{aligned}
&[0000]_{i1} = d_{i1}^{11},\\
&[1000]_{i1} = d_{i1}^{12}+d_{i1}^{14}\frac{\mu_a M}{Z},\\
&[0100]_{i1} = d_{i1}^{13}+d_{i1}^{15}\frac{\mu_a M}{Z},\\
&[0010]_{i1} = -d_{i1}^{14}\frac{M}{Z},\\
&[0001]_{i1} = -d_{i1}^{15}\frac{M}{Z},\\
&[2000]_{i1} = d_{i1}^{22}+d_{i1}^{24}\frac{\mu_a M}{Z}+d_{i1}^{44}\frac{\mu_a^2 M}{Z},\\
&[0200]_{i1} = d_{i1}^{33}+d_{i1}^{35}\frac{\mu_a M}{Z}+d_{i1}^{55}\frac{\mu_a^2 M}{Z},\\
&[0020]_{i1} = d_{i1}^{44}\frac{M^2}{Z^2},\\
&[0002]_{i1} = d_{i1}^{55}\frac{M^2}{Z^2},\\
&[1100]_{i1} = d_{i1}^{23}+d_{i1}^{25}\frac{\mu_a M}{Z}+d_{i1}^{45}\frac{\mu_a^2 M^2}{Z^2}\\
&\qquad +d_{i1}^{34}\frac{\mu_a M}{Z},\\
&[1010]_{i1} = -d_{i1}^{24}\frac{M}{Z}-2d_{i1}^{44}\frac{\mu_a M^2}{Z^2},\\
&[1001]_{i1} = -d_{i1}^{25}\frac{M}{Z}-d_{i1}^{45}\frac{\mu_a M^2}{Z^2},\\
&[0101]_{i1} = -d_{i1}^{34}\frac{M}{Z}-2d_{i1}^{45}\frac{\mu_a M^2}{Z^2},\\
&[0101]_{i1} = -d_{i1}^{35}\frac{M}{Z}-2d_{i1}^{55}\frac{\mu_a M^2}{Z^2},\\
&[0011]_{i1} = d_{i1}^{45}\frac{M^2}{Z^2}.
\end{aligned} \tag{20}
$$

The deflection $\Delta\bar{x}_2$ in the z_2 plane is caused by the deflecting errors of third order and is equal to

$$\Delta x_{i2} = d_{i2}^{jk} x_{j1} x_{k1}, \tag{21}$$

where x_{i1} is the element of column vector $\bar{x}_1$.

The coordinates of the Gaussian ray in the z_1 plane have the form [5]:

$$\bar{x}_1^{(g)} = E_1\bar{x}_0^* + \bar{x}_1, \tag{22}$$

where

$$E_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & L_1 & 0\\ 0 & 0 & 1 & 0 & L_1\\ 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\bar{x}_1 = \mathrm{col}\{0, X_1, Y_1, X_1', Y_1'\},$$

$$\bar{x}_0 = \mathrm{col}\{1, x_0, y_0, x_0', y_0'\},$$

$$\bar{x}_1^{(g)} = \mathrm{col}\{1, x_1^{(g)}, y_1^{(g)}, x_1'^{(g)}, y_1'^{(g)}\}.$$

The elements of the vector $\bar{x}_1$ are given in the formulae (17). The real ray coordinates in the z_1 plane are the sums of Gaussian coordinates and deflection $\Delta\bar{x}_1$, i.e.

$$x_{k1} = e_{k1}^{m} x_m + X_{k1} + d_{k1}^{st} x_s x_t. \tag{23}$$

Substituting (23) into (21) the deflection $\Delta\bar{x}_2$ in the z_2 plane becomes

$$
\begin{aligned}
\Delta x_{i2} = {} & d_{i2}^{jk} d_{j1}^{pq} d_{k1}^{st} x_p x_q x_s x_t +\\
& + d_{j1}^{mn} \varepsilon_{i2}^{jk} e_{k1}^{p} x_m x_n x_p +\\
& (d_{i2}^{jk} e_{j1}^{n} e_{k1}^{m} + d_{j1}^{mn} \varepsilon_{i2}^{jk} X_{k1}) x_m x_n +\\
& + \varepsilon_{i2}^{jk} e_{k1}^{m} X_{j1} x_m + d_{i2}^{jk} X_{j1} X_{k1},
\end{aligned} \tag{24}
$$

where

$$\varepsilon_{i2}^{kt} = \begin{cases} d_{i2}^{lk} \text{ for } k > l,\ i \neq 1\\ d_{i2}^{kl} \text{ for } k < l,\ i \neq 1.\\ 2d_{i2}^{ll} \text{ for } k = l,\ i \neq 1 \end{cases}$$

The expression (24) which has the form of power series takes the form

$$\Delta x_{i2} = \sum_{k,l,m,n} [klmn]_{i2} x_0^k y_0^l x_a^m y_a^n \tag{25}$$

if (18) is taken into account. The elements of the matrix $[klmn]_{i2}$ are

$$
\begin{aligned}
&[0010]_{i2} = -\frac{M}{Z}[d_{i2}^{24}X_1 + d_{i2}^{34}Y_1 + 2d_{i2}^{44}X_1' +\\
&\quad + d_{i2}^{45}Y_1' + 2d_{i2}^{22}L_1X_1 + d_{i2}^{23}L_1Y_1 + d_{i2}^{24}L_1X_1' +\\
&\quad d_{i2}^{25}L_1Y_1' + d_{i2}^{14} + d_{i2}^{12}L_1 + \ldots],\\
&[0001]_{i2} = -\frac{M}{Z}[d_{i2}^{13}L_1 + d_{i2}^{25}X_1 + d_{i2}^{35}Y_1 +\\
&\quad + d_{i2}^{45}X_1' + 2d_{i2}^{55}Y_1' + d_{i2}^{23}L_1X_1 + 2d_{i2}^{33}L_1Y_1 +\\
&\quad + d_{i2}^{34}L_1X_1' + d_{i2}^{35}L_1Y_1' + d_{i2}^{15} + \ldots],\\
&[1010]_{i2} = -\frac{M}{Z}[d_{j1}^{14}\varepsilon_{i2}^{j2} + d_{j1}^{12}(L_1\varepsilon_{i2}^{j2} + \varepsilon_{i2}^{j4}) +\\
&\quad + 2d_{i2}^{22}L_1 + d_{i2}^{25} + \ldots],
\end{aligned}
$$

$$[1001]_{i2} = -\frac{M}{Z}\,[d_{j1}^{15}\varepsilon_{i2}^{j2}+d_{j1}^{12}(\varepsilon_{i2}^{j3}L_1+\varepsilon_{i2}^{j5})+$$
$$+d_{i2}^{23}L_1+d_{i2}^{25}+\ldots],$$
$$[0110]_{i2} = -\frac{M}{Z}\,[d_{j1}^{14}\varepsilon_{i2}^{j3}+d_{j1}^{13}(\varepsilon_{i2}^{j2}L_1+\varepsilon_{i2}^{j4})+$$
$$d_{i2}^{23}L_1+d_{i2}^{34}+\ldots],$$
$$[0101]_{i2} = -\frac{M}{Z}\,[d_{j1}^{15}\varepsilon_{i2}^{j3}+d_{j1}^{13}(\varepsilon_{i2}^{j3}L_1+\varepsilon_{i2}^{j5})+$$
$$2d_{i2}^{33}L_1+d_{i2}^{35}+\ldots]. \quad (26)$$

In the expression (25) there are 35 coefficients $[klmn]_{i2}$. The formulae (26) give only 6 coefficients, at the others concern the fifth and higher orders. In the expressions given above the sum of the exponents in the product $x_0^k y_0^l x_a^m y_a^n \, {}_x^p \, {}_y^q$ being smaller than or qual to 3, some expressions in the parenthesis has been omitted.

The deflection $\Delta\bar{x}$ in the plane z_i — caused by the third order aberrations in accordance with (14) — depends upon the electron ray coefficient in the object plane z_0 and the aperture plane z_a:

$$\Delta\bar{x}_k = s_k^{\alpha\beta\gamma}\tilde{x}_\alpha\tilde{x}_\beta\tilde{x}_\gamma \quad (27)$$

where q

$\tilde{x}_a$ — elements of the column vector:

$$\tilde{x}_0 = \mathrm{col}\{1, \tilde{x}_0, \tilde{y}_0, \tilde{x}_a, \tilde{y}_a\}, \quad (28)$$

$s_k^{\alpha\beta\gamma}$ — fourth rank tensor connected with the lens aberrations in the classical system.

On the basis of (18a)

$$\tilde{\bar{x}}_0^* = W^{-1}x_0^*, \quad (29)$$

where — in accordance with (7) —

$$x_0^* = Lx_2$$

and

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -L & 0 \\ 0 & 0 & 1 & 0 & -L \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, L = L_1+L_2,$$

Hence

$$\tilde{x}_0 = F\bar{x}_2 \quad (30)$$

where

$$F = W^{-1}L.$$

The vector $\bar{x}_2$ in the z_2 plane is a sum of the ideal ray vector and the deflection $\Delta\bar{x}_2$ (see fig. 6)

$$\bar{x}_2 = \bar{x}_2^{(g)}+\Delta\bar{x}_2, \quad (31)$$

and on the other hand

$$\bar{x}_2^{(g)} = E_2\bar{x}_1+\bar{x}_2. \quad (32)$$

The elements of the vector

$$X_2 = \mathrm{col}\{0, X_2, Y_2, X_2', Y_2'\}$$

depend upon the deflecting field in the second deflection system and are

$$X_2 = \mu_0 k \int_{z_1}^{z_2} (z_2-z)H_0^{(2)}(z)dz,$$
$$Y_2 = -\mu_0 k \int_{z_1}^{z_2} (z_2-z)V_0^{(2)}(z)dz,$$
$$X_2' = \mu_0 k \int_{z_1}^{z_2} H_0^{(2)}(z)dz, \quad (33)$$
$$Y_2' = -\mu_0 k \int_{z_1}^{z_2} V_0^{(2)}(z)dz.$$

Like in the case of the first deflection system the matrix E_2 has the form

$$E_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & L_2 & 0 \\ 0 & 0 & 1 & 0 & L_2 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

By virtue of formulae (30)-(33)

$$\tilde{x}_0 = FE\bar{x}_0^*+F\bar{X}+FE_2\Delta\bar{x}_1+F\Delta\bar{x}_2 \quad (34)$$

where

$$E = E_2E_1, \; \bar{X} = E_2\bar{X}_1+\bar{X}_2.$$

As for the matrices E and L we have $E = L^{-1}$ the matrix product satisfies

$$FE = W^{-1}LE = W^{-1},$$

and on the basis of (18a)

$$\tilde{x}_a = x_a+F_a^nX_n+F_a^n e_{2n}^i\Delta x_{i1}+F_a^n\Delta x_{n2}. \quad (35)$$

The last two expressions in (35) take account of the higher order aberrations (fifth and seventh), and therefore may be omitted, i.e.

$$\tilde{x}_a = x_a+F_a^nX_n. \quad (36)$$

By substituting the obtained expressions to (27) we get the following relation for Δx_i in the z_i plane

$$\Delta x_k = s_k^{\alpha\beta\gamma}(F_\beta^nX_nx_\alpha x_\gamma+F_\alpha^mX_mx_\beta x_\gamma+F_\gamma^pX_px_\alpha x_\beta+$$
$$+F_\alpha^mF_\beta^nX_mX_nx_\gamma+F_\beta^nF_\gamma^pX_nX_px_\alpha+F_\alpha^mF_\gamma^pX_mX_px_\beta+$$
$$+F_\alpha^mF_\beta^nF_\gamma^pX_mX_nX_p)+s_k^{\alpha\beta\gamma}x_\alpha x_\beta x_\gamma. \quad (37)$$

The term $s_k^{\alpha\beta\gamma}x_\alpha x_\beta x_\gamma$ in (37) gives the aberration value of the lens working in the classical system (an electron beam runs along the electronooptic axis from [7]); the other terms being caused by the screw trajectory of the electron beam in the region of focusing field of the lens. The formulae (37) may be represented in form of a power series

$$\underline{\Delta x_k} = \sum_{k,l,m,n} \underline{[klmn]_k} x_0^k y_0^l x_a^m y_a^n. \tag{38}$$

The relations (18), (25) and (38) enable to determine the coordinates of the intersection point $\bar{x}_R$ of the real ray with the working plane z_i. In accordance with fig. 6 vector $\bar{x}_R$ has the form

$$\bar{x}_R = \bar{x}_i + \underline{\Delta \bar{x}}, \tag{39}$$

where the coordinates of $\bar{x}_i$ vector are Gaussian coordinates of the electron ray in the image plane. This ray in the z_2 plane has the coordinate equal to $\bar{x}_2$. In view of the dependence $\mu_i' = -1/f_i$, $\nu_i' = M^{-1}$ and the vector (1) and (7) we have

$$\bar{x}_i = S_i \bar{x}_2 \tag{40}$$

where

$$\bar{x}_i = \mathrm{col}\{1, x_i, y_i, x_i', y_i'\},$$

$$\bar{x}_2 = \mathrm{col}\{1, x_2, y_2, x_2', y_2'\},$$

$$S_i = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & M & 0 & -ML & 0 \\ 0 & 0 & M & 0 & -ML \\ 0 & \dfrac{-1}{f_i} & 0 & M^{-1}+\dfrac{L}{f_i} & 0 \\ 0 & 0 & \dfrac{-1}{f_i} & 0 & M^{-1}+\dfrac{L}{f_i} \end{bmatrix}.$$

On the other hand, taking account of (22), (31) and (32) we get

$$\bar{x}_2 = E\bar{x}_0^* + \bar{X} + E_2 \Delta\bar{x}_1 + \Delta\bar{x}_2.$$

Substituting the last expression to the (39) we obtain for the vector $\bar{x}_R$

$$\bar{x}_R = S_i E \bar{x}_0^* + S_i \bar{X} + S_i E_2 \Delta\bar{x}_1 + S_i \Delta\bar{x}_2 + \underline{\Delta\bar{x}}. \tag{41}$$

The coordinates of Gaussian ray in the z_i plane determined on the basis of equation (22), (32) and (40) are

$$\bar{x}_g = S_i \bar{x}_2^{(g)} = S_i E \bar{x}_0^* + S_i \bar{X}. \tag{42}$$

The relation obtained allows to determine the aberrations of the deflecting-focusing system in the working plane z_i. By virtue of (41) and (42) we have namely

$$\begin{aligned}\Delta\bar{x} &= \bar{x}_R - \bar{x}_g \\ &= S_i E_2 \Delta\bar{x}_1 + S_i \Delta\bar{x}_2 + \underline{\Delta\bar{x}}.\end{aligned} \tag{43}$$

The deflection $\Delta\bar{x}$ in the working plane z depends on the value X_i, $Y_i (i = 1\ 2)$ of the electron ray deviation in the deflecting systems and on the initial coordinates x_0, y_0, x_a, y_a. The final formulae have a considerably simpler form, if for DD the condition of correct operation is fulfilled [5]:

$$\frac{a_x}{b_x} = \frac{1}{r_x}, \quad \frac{a_y}{b_y} = \frac{1}{r_y}, \tag{44}$$

where

$$\frac{1}{r_x} = \frac{\int_{-\infty}^{\infty} H_0^{(2)} dz}{\int_{-\infty}^{\infty} H_0^{(1)} dz}, \quad \frac{1}{r_y} = \frac{\int_{-\infty}^{\infty} V_0^{(2)} dz}{\int_{-\infty}^{\infty} V_0^{(1)} dz},$$

and $a(b)$ denotes the distance between the deflecting plane of the first (second) deflecting system and the plane z_p (comp. fig. 3). In this case, on the basis of (13) the following relations are true

$$\begin{aligned} X_1 &= \mu_0 k \left(z_1 \int_{z_0}^{z_1} H_0^{(1)} dz - \int_{z_0}^{z_1} z H_0^{(1)} dz \right) \\ &= -r_x p_{1x} \gamma_x, \\ X_1' &= \mu_0 k \int_{z_0}^{z_1} H_0^{(1)} dz = -r_x \gamma_x, \end{aligned} \tag{45}$$

similarly

$$Y_1 = -r_y p_{1y} \gamma_y, \quad Y_1' = -r_y \gamma_y, \quad X_2 = p_{2x} \gamma_x,$$

$$X_2' = \gamma_x, \quad Y_2 = p_{2y} \gamma_y, \quad Y_2' = \gamma_y.$$

Fig. 3. explains the significance of the values p_1, p_2 used in formulae (45). By taking account of (19), (25) and (38) the deviation $\Delta\bar{x}$ in the working plane z_i may be presented in form of the series

$$\Delta x_i = \sum_{k,l,m,n} [klmn]_i x_0^k y_0^l x_a^m y_a^n. \tag{46}$$

In the theory of the electronooptic system aberration only two components of the vector $\Delta\bar{x}$ in the x and y directions are of interest. These are: $\Delta x_2 = \Delta x$ and $\Delta x_3 = \Delta y$. By virtue of the relation (43) and the series (19), (25), and (38) the aberration coefficients of the deflecting-focusing system for the directions x and y have the form

$$\begin{aligned} [klmn]_x &= \overline{[klmn]}_x + \underline{[klmn]}_x \\ [klmn]_y &= \overline{[klmn]}_y + \underline{[klmn]}_y \end{aligned} \tag{47}$$

where:

$$\underline{[klmn]}_x = [klmn]_2, \quad \underline{[klmn]}_y = [klmn]_3,$$

$$\overline{[klmn]}_x = M\{([klmn]_{21} - L_1 [klmn]_{41}) + ([klmn]_{22} - L[klmn]_{42})\},$$

$$\overline{[klmn]}_y = M\{([klmn]_{31} - L_1 [klmn]_{51}) + ([klmn]_{32} - L[klmn]_{52})\}. \tag{48}$$

The coefficients $\overline{[klmn]}_{x,y}$ introduced above describe the lens aberration with electron beam running obliquely in the focusing region, while the coefficients $\overline{[klmn]}_{x,y}$ denote aberrations of DD referred to the working surface for an aberrationless end lens.

7. An analysis of the results

In the up to now considerations the aberration coefficients $\overline{[klmn]}$ of DD, and $\underline{[klmn]}$ of lens have been analysed separately. A detailed analysis of the aberration coefficients for deflecting-focusing system has been performed in paper [8]. In the present paper only the final conclusions are formulated.

7.1. Astigmatism and field curvature of the image DD$\overline{[klmn]}$ ($k = l = 0, m+n = 1$)

This error is caused by the deflection astigmatism of the deflection systems. The isotropic and anisotropic astigmatisms depend respectively on isotropic and anisotropic astigmatism of the corresponding deflecting systems. Moreover, the astigmatism is enlarged additionally by coma of the second deflection system (positioned near the lens). For identical vertical and horizontal coils of DD the astigmatism coefficients have the form

$$
\begin{aligned}
\overline{[0010]}_x &= \frac{M^2}{Z}(A\gamma_x^2+B\gamma_y^2),\\
\overline{[0001]}_y &= \frac{M^2}{Z}(A\gamma_x^2+B\gamma_y^2),\\
\overline{[0001]}_x = \overline{[0010]}_y &= \frac{M^2}{Z}C\gamma_x\gamma_y,
\end{aligned}
\tag{49}
$$

where

A, B, C — constants depending upon the deflecting coils shape,

γ_x, γ_y — tangents of beam deflection angles by DD,

M — linear magnification of lens,

Z — length of image trajectory region of the lens.

The rotation angle of the ellipse with respect to the x-axis, and the length of the astigmatic ellipse semiaxis caused by the deflection astigmatism DD are

$$\tan 2\delta = \frac{C}{A-B}\tan 2\Phi_i,$$

$$\frac{R^2}{2L^2}\left[A+B\pm(A-B)\frac{\cos 2\Phi_i}{\cos 2\delta}\right]\omega.$$

The sign "+" in the above formula is assigned to the large semiaxis, while "−" to the small one. The magnitudes

$$R_i^2 = x_i^2+y_i^2$$

depend upon the central ray coordinates of the Gaussian beam in the z_i plane (comp. formula (10)), L is the length DD along the electronooptic axis

$$\tan\Phi_i = y_i/x_i,$$

while ω is an aperture half-angle of the electron beam.

7.2. Astigmatism and field curvature of the image $\underline{[klmn]}$ ($k = l = 0, m+n = 1$)

For identical vertical and horizontal coils the astigmatism coefficients have the form

$$
\begin{aligned}
\underline{[0010]}_x &= K\gamma_x^2+G\gamma_y^2+R\gamma_x\gamma_y,\\
\underline{[0001]}_y &= K\gamma_x^2+G\gamma_y^2-R\gamma_x\gamma_y,\\
\underline{[0010]}_y &= -N\gamma_x^2-P\gamma_y^2+Q\gamma_x\gamma_y,\\
\underline{[0001]}_x &= N\gamma_x^2+P\gamma_y^2+Q\gamma_x\gamma_y,
\end{aligned}
\tag{50}
$$

where

$$
\begin{aligned}
K &= Bh_3^2+2Bh_2^3+2Ch_1^2+Dh_1^2-6Fh_1h_2,\\
G &= Bh_4^2+Dh_2^2-2Fh_2h_4,\\
R &= -2G_2h_1h_2+4G_3h_2h_3,\\
N &= -2Fh_1^2-2G_3h_1h_3,\\
P &= 2Fh_2^2+2G_3h_2h_4,\\
Q &= 2Bh_3h_4+2Ch_1h_2-2Fh_1h_4+h_2h_3.
\end{aligned}
\tag{51}
$$

Constants $B, C, \ldots, G_3$ in the formulae (51) are third order aberration coefficients of the lens [7] (B-spherical aberration coefficient, C-isotropic astigmatism, D-image field curvature, F-isotropic coma, G_2 and G_3-anisotropic astigmatism and coma). The constants $h_i (i = 1, \ldots, 4)$ depend upon the linear DD and amount to

$$
\begin{aligned}
h_1 &= p_{2x}-L-r_x(p_{1x}-L_1),\\
h_2 &= p_{2y}-L-r_y(p_{1y}-L_1),\\
h_3 &= \mu_a h_1-r_x\frac{Z}{M},\\
h_4 &= \mu_a h_2-r_y\frac{Z}{M}.
\end{aligned}
\tag{52}
$$

For identical vertical and horizontal coils ($h_1 = h_2$, $h_3 = h_4$) the rotation angle δ of the astigmatic ellipse is

$$\tan 2\delta = \frac{(N-P)\cos 2\Phi_i+Q\sin 2\Phi_i}{Q\cos 2\Phi_i+R\sin 2\Phi_i}.$$

If the lens does not introduce coma or anisotropic astigmatism ($F = G_2 = G_3 = 0$) then $N-P = R = 0$ and the rotation angle of the ellipse is equal to the angle Φ_i, while the semiaxis amounts to

$$\frac{1}{2} Z\left(\frac{R_i}{ML}\right)^2 |K+G\pm Q|\,\omega.$$

7.3. Astigmatism of imaging by DD $\overline{[klmn]}$ ($k+l = 1, m+n = 1$)

The error is caused by astigmatism and coma of the deflection of both the deflection systems and is partially compensated (the astigmatism coefficients for the first and second deflecting systems have opposite signs). In the case of identical horizontal and vertical lenses the coefficients $\overline{[klmn]}$ have the form

$$\overline{[0110]}_x = \frac{M^2}{Z} A\gamma_y,$$

$$\overline{[1001]}_y = \frac{M^2}{Z} A\gamma_x,$$

$$\overline{[0110]}_y = \frac{M^2}{Z} B\gamma_x,$$

$$\overline{[1001]}_x = \frac{M^2}{Z} B\gamma_y,$$

$$\overline{[1010]}_x = \frac{M^2}{Z} G\gamma_x,$$

$$\overline{[0101]}_y = \frac{M^2}{Z} C\gamma_y,$$

$$\overline{[1010]}_y = \frac{M^2}{Z} D\gamma_y,$$

$$\overline{[0101]}_x = \frac{M^2}{Z} D\gamma_x.$$

where the constants $A, \ldots, D$ depend upon the shape of the deflecting coils. In the case when the deflecting coils length $2l_1, 2l_2$ are small the following approximate relation holds

$$A = B = -C = D$$

and the semiaxes of ellipse are

$$\sqrt{2}MA\frac{r_0 R_i}{L}\begin{cases} \cdot\left|\sin\left(\frac{\pi}{4}+\Phi_i+\varphi_0\right)\right| \\ \cdot\left|\cos\left(\frac{\pi}{4}+\Phi_i+\varphi_0\right)\right| \end{cases}$$

where

$$r_0^2 = x_0^2+y_0^2,\ \tan\varphi_0 = \frac{y_0}{x_0}.$$

7.4. Astigmatism of imaging by lens $\underline{[klmn]}$ ($k+l = 1, m+n = 1$)

The coefficient of astigmatism for equal vertical and horizontal coils are

$$\begin{aligned} \underline{[1001]}_x &= P\gamma_x+R\gamma_y, \\ \underline{[0101]}_x &= Q\gamma_x+S\gamma_y, \\ \underline{[1010]}_y &= Q\gamma_y-S\gamma_x, \\ \underline{[0110]}_y &= R\gamma_x-P\gamma_y, \\ \underline{[1001]}_y &= N\gamma_x-T\gamma_y, \\ \underline{[0101]}_y &= K\gamma_y-M\gamma_x \end{aligned} \tag{55}$$

where:

$$\begin{aligned} K &= 4Ch_1+2Dh_1-6Fh_3, \\ R &= 2(Ch_1-Fh_3), \\ N &= 2(Dh_1-Fh_3), \\ T &= -2G_2h_1+6G_3h_3, \\ M &= -2(G_2h_1+G_3h_3), \\ S &= -2(G_2h_1-G_3h_3),\ P = -S. \end{aligned} \tag{56}$$

The constants $C, \ldots, G_3$ in the formulae (56) are the third order aberrations of the end lens. The axes of the astigmatism ellipse have the form

$$\frac{Z}{ML} r_0 R_i \left|\sqrt{R^2+S^2} \pm \frac{K+N}{2}\cos(\Phi_i-\varphi_0) - \frac{T-M}{2}\sin(\Phi_i-\varphi_0)\right|\omega$$

while the rotation angle δ of ellipse is

$$\tan 2\delta = \frac{R\sin(\Phi_i+\varphi_0)-S\cos(\Phi_i+\varphi_0)}{S\sin(\Phi_i+\varphi_0)+R\cos(\Phi_i+\varphi_0)}.$$

If the end lens does not introduce any anisotropic aberrations of defocusing ($G_2 = G_3 = 0$), then the rotation angle of the astigmatic ellipse is

$$\delta = \frac{(\Phi_i+\varphi_0)}{2}.$$

7.5. Coma and anticoma of DD $\overline{[klmn]}$ ($m+n = 2,\ k = l = 0$)

The deviation caused by coma and anticoma are partially compensated by each other. For identical horizontal and vertical coils the coma coefficient have the form

$$
\begin{aligned}
\overline{[0020]}_x &= \frac{M^3}{Z^2}\lambda\gamma_x, \\
\overline{[0020]}_y &= \frac{M^3}{Z^2}\varepsilon\gamma_y, \\
\overline{[0011]}_x &= \frac{M^3}{Z^2}\eta x_y, \\
\overline{[0002]}_y &= \frac{M^3}{Z^2}\lambda\gamma_y, \\
\overline{[0002]}_x &= \frac{M^3}{Z^2}\varepsilon\gamma_x, \\
\overline{[0011]}_y &= \frac{M^3}{Z^2}\eta\gamma_x.
\end{aligned}
\tag{57}
$$

The constants λ, ε, η depend upon the shape of the deflecting coils. The centre of the ellipse family is positioned on the straight line inclined to the x-axis by an angle Φ_i, and is located from the (x_i, y_i) point by a distance

$$\frac{R_i M^2}{2L}(\lambda+\varepsilon)\omega^2.$$

The ellipse semiaxes are:

— long semiaxis — $\frac{R_i M^2}{2L}|\lambda-\varepsilon|\omega^2$,

— short semiaxis — $\frac{R_i M}{2L}|\eta|\omega^2$.

7.6. Deflection coma of the deflecting-focusing system $\overline{[klmn]}$ ($m+n = 2$, $k = l = 0$)

The coefficients of coma for identical vertical and horizontal coils are

$$
\begin{aligned}
\underline{[0020]}_x &= K\gamma_x+T\gamma_y, \\
\underline{[0002]}_x &= K\gamma_y-T\gamma_x, \\
\underline{[0011]}_x &= M\gamma_x+N\gamma_y, \\
\underline{[0011]}_y &= M\gamma_y-N\gamma_x, \\
\underline{[0002]}_y &= P\gamma_x+Q\gamma_y, \\
\underline{[0020]}_y &= P\gamma_y-Q\gamma_x,
\end{aligned}
\tag{58}
$$

where

$$P = Bh_3-Fh_1,\ Q = G_3h_1,$$

$$K = 3P,\ M = 2P,\ L = 3Q,\ N = -2Q.$$

Constants $B, \ldots, G_3$ are the third order aberrations of the lens. Coma of the deflecting lens results in a family of circles of radius

$$\frac{R_i Z^2}{ML}\sqrt{P^2+Q^2}\,\omega^2,$$

the centre of which lies on the straight line inclined to the x-axis by the angle β:

$$\tan\beta = \frac{P\gamma_y-Q\gamma_x}{P\gamma_x+Q\gamma_x}.$$

This centre is shifted from the point (x_i, y_i) by the value

$$\frac{2R_i Z^2}{ML}\sqrt{P^2+Q^2}\,\omega^2.$$

If the lens does not introduce the third order coma, i.e. $F = G_3 = 0$, the deflection coma of lens depends upon the expression

$$\frac{Z^2 h_3}{ML}R_i B\omega^2,$$

where B is a spherical aberration coefficient of the lens.

7.7. Coma of the deflect ion-focusing system $[klmn]$ ($k = 1$ or $l = 1$, $m+n = 2$)

The kind of aberration is caused only by coma of the end lens and does not depend upon the deflection angle γ_x, γ_y by DD.

7.8. Spherical aberration of the deflecting-focusing system $[klmn]$ ($k+l = 3$, $m = n = 0$)

It is caused by the spherical aberration of the end lens and does not depend upon the deflecting angle γ_x, γ_y due to DD. The diameter of the confusion circle referred to the object plane of the lens is

$$\delta_r = C_s\omega^3$$

where

$$C_s = \frac{Bz^3}{M}.$$

7.9. Deflection distortion of DD $[klmn]$ ($k = l = m = n = 0$)

It is caused by the deflection distortion of the first and second deflecting systems, as well as by astigmatism and coma of the second system. A partial compensation takes place. The distortion coefficients for equal vertical and horizontal coils have the form

$$
\begin{aligned}
\overline{[0000]}_x &= M(A\gamma_x^3+B\gamma_x\gamma_y^2),\\
\overline{[0000]}_y &= M(A\gamma_y^3+B\gamma_y^2\gamma_x^2),
\end{aligned}
\tag{59}
$$

The constant A in (59) is responsible for linear errors (ladder distortion), while B causes either pillow--type distortion ($B<0$) or barrel-type distortion ($B>0$).

7.10. Deflection distortion of the deflecting-focusing system $\underline{[klmn]}$ $(k=l=m=n=0)$

This is caused by third order aberrations of the lens. For equal vertical and horizontal coils the distortion coefficients amount to

$$
\begin{aligned}
\underline{[0000]}_x &= -K(\gamma_x^2+\gamma_y^2)\gamma_x+L(\gamma_x^2+\gamma_y^2)\gamma_y,\\
\underline{[0000]}_y &= -K(\gamma_x^2+\gamma_y^2)\gamma_y-L(\gamma_x^2+\gamma_y^2)\gamma_x.
\end{aligned}
\tag{60}
$$

The constant K is responsible for the isotropic distortion. The expression $-K\gamma_x^2$ in the first formula and $-K\gamma_y^3$ in the second one, determine the magnitude of the ladder distortion, while the constant L specifies the anisotropic distortion. From the above consideration it follows that the deflection distortion of the deflecting-focusing system is anisotropic. The anisotropy is introduced by the anisotropic aberrations of the end lens, i.e. by G_1 — the ~~an~~isotropic distortion, G_2 — astigmatism, and G_3 — anisotropic coma:

$$L = G_1h_1^3-G_2h_1^2h_3+G_3h_1h_2^3.$$

7.11. Distortion of imaging of DD $[klmn]$

The coefficient of the first order distortion ($k+l=1$, $m=n=0$) for equal vertical and horizontal coils have the form

$$
\begin{aligned}
\overline{[1000]}_x &= M(A\gamma_x^2+B\gamma_y^2),\\
\overline{[0100]}_y &= M(A\gamma_y^2+B\gamma_x^2),\\
\overline{[0100]}_x &= \overline{[1000]}_y^2 = MC\gamma_x\gamma_y.
\end{aligned}
\tag{61}
$$

The constants A, B, and C depend upon the shape of the deflecting coils. The coefficients of second order distortion ($k+l=2$, $m=n=0$) for equal vertical and horizontal coils are

$$
\begin{aligned}
&\overline{[2000]}_x = MD\gamma_x,\ \overline{[0200]}_y = MD\gamma_y,\\
&\overline{[2000]}_y = ME\gamma_y,\ \overline{[0200]}_x = ME\gamma_x,\\
&\overline{[1100]}_y = MF\gamma_y,\ \overline{[1100]}_y = MF\gamma_x.
\end{aligned}
\tag{62}
$$

Similarly as above the constants D, E, and F depend upon the shape of the deflecting coils. In the limiting case when the lengths $2l_1$, $2l_2$ of the deflecting coils are small than for the constants D, E, and F the following relation holds

$$2F = E = -D.$$

7.12. Distortion of imaging lens

The first order coefficients of the imaging lens distortion for equal horizontal and vertical coils have the form

$$
\begin{aligned}
\underline{[2000]}_x &= G\gamma_x^2+T\gamma_x\gamma_y+V\gamma_z^2,\\
\underline{[0200]}_x &= H\gamma_x^2+S\gamma_x\gamma_y+Z\gamma_y^2,\\
\underline{[2000]}_y &= -H\gamma_y^2+S\gamma_x\gamma_y-Z\gamma_x^2,\\
\underline{[0200]}_y &= G\gamma_y^2-T\gamma_x\gamma_y+V\gamma_x^2,
\end{aligned}
\tag{63}
$$

while for the second order we have

$$
\begin{aligned}
\underline{[2000]}_x &= K\gamma_x+N\gamma_y,\\
\underline{[1100]}_x &= S\gamma_x+Q\gamma_y,\\
\underline{[0200]}_x &= M\gamma_x+P\gamma_y,\\
\underline{[2000]}_y &= M\gamma_y-P\gamma_x,\\
\underline{[0100]}_y &= -S\gamma_y+Q\gamma_x,\\
\underline{[0200]}_y &= K\gamma_y-N\gamma_x.
\end{aligned}
\tag{64}
$$

The constants G, T, V, and Z as well as K, S, M, N, Q, and P depend upon the third order aberrations of the lens.

8. Final remarks

The analysis carried above allows to discuss the influence of the single aberrations on the electron beam cross-section in the z_i plane [8]. The formulae allow to determine the aberration values, when third order aberrations of the lens and deflecting errors of the deflecting units are known.

Appendix

By virtue of (1)

$$u_a = u_0\mu+u_0'\nu,$$

where $u_a = u(z_a)$, $\mu_a = \mu(z_a)$, $\nu_a = \nu(z_a)$. Hence

$$u_0' = \frac{u_a}{\nu_a}-\frac{\mu_a}{\nu_a}u_0. \tag{I}$$

In view of fig. I

$$Z = z_i - z_a, \mathrm{t}\ \tan \mathrm{B}_i = \frac{\nu_a}{Z},$$

$$\nu_i' = \tan(\pi - \beta_i) = -\tan\beta = \frac{-\nu_a}{Z} = \frac{1}{M}.$$

The last dependence follows from the Wronskian in the image plane z_i

$$W(z_i) = \begin{vmatrix} u & 0 \\ \mu_i' & \nu_i' \end{vmatrix} = 1.$$

Hence the expression (I) takes the form

$$u_0' = \frac{\mu_a M}{Zu} u_0 - \frac{M}{Z} u. \tag{II}$$

From fig. I it follows

$$\mu_a' = \mu_i' = \frac{-1}{f_i},$$

$$\nu_a' = \nu_i' = \frac{1}{M}.$$

As

$$W(z_a) = \begin{vmatrix} \mu_a & \nu_a \\ \mu_a' & \nu_a' \end{vmatrix} = 1,$$

in the plane z_a then

$$\frac{\mu_a}{M} - \frac{Z}{f_i M} = 1. \tag{III}$$

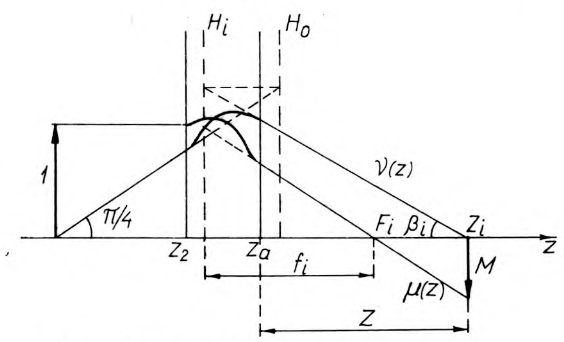

Fig. I. Course of the principal rays in the lens

*

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Аберрации третьего порядка отклоняюще: фокусирующей схемы

Произведен линейный анализ отклоняюще-фокусирующей схемы, применяемой в анализирующих микроскопах. Приведены коэффициенты аберрации третьего порядка и обсуждены двенадцать видов аберрации отклонения и отображения отклоняюще-фокусирующей схемы. Введенные формулы позволяют определить величины аберрации, когда известны аберрации третьего порядка линзы и погрешности отклонения отклоняющих схем DD.

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