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Analysis of imaging in guided wave holography*

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Imaging properties of the waveguide hologram recorded by using free space waves and reconstructed by a guided wave are analysed. The conditions for stigmatic and non-anamorphic imaging are derived. In the analysis, not only the change of the arrangement geometry but also the difference in wavelengths between the recording and reconstruction are taken into account.

1. Introduction

Holograms have been currently recorded using free space waves, and the reconstruction has been also made by a free space wave. One obtains a holographic recording, and a holographic image is created as a free space wave, too. The free space reconstructing wave can be given also by gliding wave which falls on the hologram under a very big angle of incidence. In this way, the space behind the hologram can be strongly limited, and the transmission hologram situated very close to the wall. In the limiting case, a guided wave can be used as a reconstructing wave. This means that the hologram must be recorded on the waveguiding layer.

The aim of this work is to analyse the imaging properties of the hologram of that kind. The hologram is made using free space waves, but for the reconstruction the guided wave is employed. In the analysis, not only the change of the geometry but also the difference in the wavelengths between the recording and reconstruction are taken into account.

The analysis is made according to the Fresnel diffraction method of description of optical imaging in an infinitely large optical system, adapted to holographic imaging (see, for example, [1]). For large angles between beams, occuring in a real holography, the analysis was published in paper [2], being confined to a meridional problem.

2. Holographic recording by free space waves

The scheme of the recording is plotted in Figure 1. We limit ourselves to the object in the form of transparency, e.g., a plane object. The centre of the object S_0 is connected with the centre of the sensitive layer H_0 by a straight line with length R_S. The

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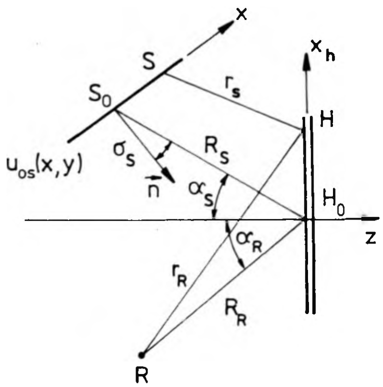

Fig. 1. Recording scheme of the waveguide hologram

sensitive layer is located in the coordinate plane x_h, y_h, and the centre of the layer H_0 is identical with the centre of the coordinate axes. The line R_S makes with the coordinate axis z an angle α_S, and the plane of the object need not be perpendicular to the line R_S. It can be tilted forming an angle σ_S with the perpendicular plane. This angle lies in the meridional plane x_h, z, i.e., no tilt is considered in other directions for the sake of the reasonable simplicity. For the same reason, the reference source $R(x_R, 0, z_R)$ is also situated in the meridional plane, and is connected with the centre of the sensitive layer by the line R_R. We consider an arbitrarily chosen point of the object $S(x, y)$ and an arbitrary point of the layer $H(x_h, y_h)$. The line connecting the two points is denoted by r_s, and the point $H(x_h, y_h)$ is connected with the reference source by the line r_R.

The diffraction pattern formed by the object S on the sensitive layer H is given by the relation

$$u_s(x_h, y_h) = \iint u_{os}(x, y)\exp\{ik_S r_s\}\, dx\, dy \tag{1}$$

where $u_{os}(x, y)$ is the amplitude distribution on the object, and $k_S = 2\pi/\lambda_S$ is the wavenumber of the object wave (λ_S – the wavelength). In the space around the sensitive layer this object wave must interfere with the reference wave in order to obtain the hologram. The reference wave can be written as a simple spherical wave in the form

$$u_R(x_h, y_h) = A_R \exp\{ik_S r_R\}, \tag{2}$$

and it interferes with the object wave (1) giving the intensity

$$I(x_h, y_h) = [u_s(x_h, y_h) + u_R(x_h, y_h)]^2. \tag{3}$$

By introducing (1) and (2) into (3) one obtains

$$\begin{aligned} I(x_h, y_h) = {} & [u_s(x_h, y_h)]^2 + A_R^2 + A_R^* \iint u_{os}(x, y)\exp\{ik_S(r_s - r_R)\}\, dx\, dy \\ & + A_R \iint u_{os}^*(x, y)\exp\{ik_S(r_R - r_s)\}\, dx\, dy. \end{aligned} \tag{4}$$

Let us suppose that the amplitude transmissivity of the hologram is a linear function of the intensity; we limit ourselves to the diffraction of the first orders. For the amplitude transmissivity of the hologram and the primary reconstructed wave it can be written as

$$\tau^{(1)}(x, y, x_h, y_h) = \varkappa A_R^* \iint u_{os}(x, y) \exp\{ik_S(r_s - r_R)\} \, dx \, dy \tag{5}$$

where $\varkappa$ is the coefficient of the linear function.

3. Holographic reconstruction by guided wave

This hologram can be replayed using reconstructing guided wave of the "planar spherical" form. Such a wave has the spherical wavefront in the plane of the waveguide, in the perpendicular plane it is, however, a guided wave. This reconstructing wave can be written as

$$u_C(x_h, y_h) = A_C \exp\{ik_I r_C\} \tag{6}$$

where $k_I = 2\pi N/\lambda_I$ is the wavenumber of the reconstructing wave (N – effective index of refraction of the guided wave, and λ_I – wavelength).

A diffraction pattern describing the diffraction on the hologram in the arbitrary plane x', y' can be written in the form

$$u_d(x', y') = A_C \iint \tau^{(1)}(x, y, x_h, y_h) \exp\{ik_I(r_C - r_d)\} \, dx_h \, dy_h \tag{7}$$

where, according to Fig. 2, the symbol r_d denotes the length of the straight line

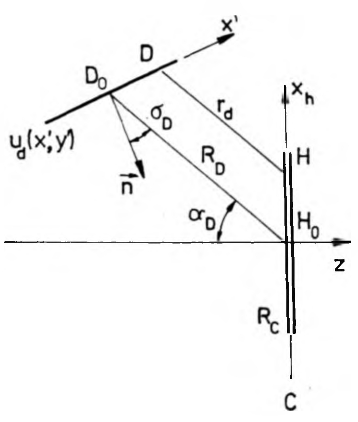

Fig. 2. Reconstruction scheme of the waveguide hologram

connecting an arbitrary point $H(x_h, y_h)$ of the hologram with an arbitrary point $D(x', y')$ in the diffraction pattern $u_d(x', y')$. This diffraction pattern lies in the plane which forms an angle σ_D with the plane perpendicular to the line R_D. This angle lies again in the meridional plane.

If Equation (5) is inserted into the last Eq. (7), one obtains for the diffraction pattern of the primary wave the relation

$$u_d(x', y') = \varkappa A_C A_R^* \iiiint u_{os}(x, y) \exp\{i[k_S(r_s - r_R) + k_I(r_C - r_d)]\} \, dx \, dy \, dx_h \, dy_h. \tag{8}$$

For the further analysis it is necessary to calculate the length r_s of the distance from the arbitrary point $S(x, y)$ of the object to an arbitrary point $H(x_h, y_h)$ of the future hologram. The same quantities should be known for the reference, reconstructing and reconstructed waves. In a general case, we introduce the length r_q and further quantities with index Q. From Figs. 1 and 2 we obtain

$$r_q^2 = \left(R_Q \sin\alpha_Q + x\sin\left\{\frac{\pi}{2}-\alpha_Q-\sigma_Q\right\}-x_h\right)^2+(y-y_h)^2$$
$$+\left(R_Q\cos\alpha_Q - x\cos\left\{\frac{\pi}{2}-\alpha_Q-\sigma_Q\right\}\right)^2, \tag{9}$$

whence, using the terms up to the second order only, we get

$$r_R \approx R_Q+\frac{x^2}{2R_Q}\cos^2\sigma_Q+\frac{x_h^2}{2R_Q}\cos^2\alpha_Q+\frac{y^2}{2R_Q}+\frac{y_h^2}{2R_Q}$$
$$-x\sin\sigma_Q - x_h\sin\alpha_Q-\frac{xx_h}{R_Q}\cos\sigma_Q\cos\alpha_Q-\frac{yy_h}{R_Q}. \tag{10}$$

For the reference wave the condition $x = y = 0$ holds, and Eq. (10) is simplified to

$$r_R \approx R_R+\frac{x_h^2}{2R_R}\cos^2\alpha_R+\frac{y_h^2}{2R_R}-x_h\sin\alpha_R. \tag{11}$$

For the reconstruction wave, the condition $x = y = 0$ holds too, but moreover $\alpha_Q = 90°$. Then (10) becomes

$$r_C \approx R_C+\frac{y_h^2}{2R_C}-x_h. \tag{12}$$

From the relation (10), for the object and diffraction patterns, and from (11) and (12) together with relation (8) it follows that

$$u_d(x', y') = \varkappa A_C A_R^* \exp\{i[k_S(R_S-R_R)+k_I(R_C-R_D)]\}$$
$$\times \iint u_{os}(x, y)\exp\left\{i\pi\left(\frac{\cos^2\sigma_S}{\lambda_S R_S}x^2-\frac{\cos^2\sigma_D}{\lambda_I R_D}x'^2\right)\right\}$$
$$\times\exp\left\{-2\pi i\left(\frac{\sin\sigma_S}{\lambda_S}x-\frac{\sin\sigma_D}{\lambda_I}x'\right)\right\}$$
$$\times\exp\left\{i\pi\left(\frac{1}{\lambda_S R_S}y^2-\frac{1}{\lambda_I R_D}y'^2\right)\right\}dxdy$$
$$\times\iint\exp\left\{i\pi\left(\frac{\cos^2\alpha_S}{\lambda_S R_S}-\frac{\cos^2\alpha_R}{\lambda_S R_R}-\frac{\cos^2\alpha_D}{\lambda_I R_D}\right)x_h^2\right\}$$
$$\times\exp\left\{2\pi i\left(\frac{\sin\alpha_S}{\lambda_S}-\frac{\sin\alpha_R}{\lambda_S}+\frac{N}{\lambda_I}-\frac{\sin\alpha_D}{\lambda_I}\right)x_h\right\}$$

$$\times \exp\left\{i\pi\left(\frac{1}{\lambda_S R_S}-\frac{1}{\lambda_S R_R}-\frac{N}{\lambda_I R_C}-\frac{1}{\lambda_I R_D}\right)y_h^2\right\}$$

$$\times \exp\left\{-2\pi i\left(\frac{\cos\sigma_S\cos\alpha_S}{\lambda_S R_S}x-\frac{\cos\sigma_D\cos\alpha_D}{\lambda_I R_D}x'\right)x_h\right\}$$

$$\times \exp\left\{-2\pi i\left(\frac{1}{\lambda_S R_S}y-\frac{1}{\lambda_I R_D}y'\right)y_h\right\}dx_h\,dy_h. \tag{13}$$

If we wish to obtain the imaging of the object, it is necessary to transform the last relation into the form of the convolution

$$\int U(x)\,\delta(x-c)\,dx = U(c). \tag{14}$$

This can be realized only in the case when the coefficients of the members of the second order x_h^2, y_h^2 and of the linear member x_h are equal to zero. In such a case $R_D \to R_I$, $\sigma_D \to \sigma_I$, and $\alpha_D \to \alpha_I$, but then it is necessary to resolve $R_I^{\|}$ in meridional plane from $R_I^{\perp}$ in the perpendicular plane. The above conditions give the equations for the location of the image:

$$\frac{\cos^2\alpha_I}{R_I^{\|}} = \mu\left(\frac{\cos^2\alpha_S}{R_S}-\frac{\cos^2\alpha_R}{R_R}\right), \tag{15a}$$

$$\frac{1}{R_I} = \frac{N}{R_C}+\mu\left(\frac{1}{R_S}-\frac{1}{R_R}\right), \tag{15b}$$

$$\sin\alpha_I = N+\mu(\sin\alpha_S-\sin\alpha_R) \tag{15c}$$

where $\mu = \lambda_I/\lambda_S$ is the ratio of the wavelengths. From the relations (15) it follows that in a general case the imaging will have an astigmatism of the second order, because $R_I^{\|} \neq R_I$.

In Equation (13), the second integral can be transformed into the form

$$\iint \exp\left\{-2\pi i\left(\frac{\cos\sigma_S\cos\alpha_S}{\lambda_S R_S}x-\frac{\cos\sigma_I\cos\sigma_I}{\lambda_I R_I^{\|}}x'\right)x_h\right\}$$

$$\times \exp\left\{-2\pi i\left(\frac{1}{\lambda_S R_S}y-\frac{1}{\lambda_I R_I}y'\right)y_h\right\}dx_h\,dy_h$$

$$= \frac{\lambda_S\lambda_I R_S R_L}{\cos\sigma_S\cos\alpha_S}\,\delta\left\{x-\frac{\cos\alpha_I\cos\sigma_I}{\cos\alpha_S\cos\sigma_S}\frac{\lambda_S}{\lambda_I}\frac{R_S}{R_I^{\|}}x',\ y-\frac{\lambda_S R_S}{\lambda_I R_I}y'\right\}, \tag{16}$$

and using (14) it can be obtained from (13)

$$u_I(x',y') = A_{oI}\,u_s\left\{\frac{\cos\alpha_I\cos\sigma_I}{\cos\alpha_S\cos\sigma_S}\frac{\lambda_S}{\lambda_I}\frac{R_S}{R_I^{\|}}x',\ \frac{\lambda_S}{\lambda_I}\frac{R_S}{R_I^{\perp}}y'\right\}$$

$$\times \exp\left\{ i\pi\left(\frac{\cos^2\alpha_I}{\cos^2\alpha_S}\frac{\lambda_S}{\lambda_I}\frac{R_S}{R_I^{\parallel}} - 1\right)\frac{\cos^2\sigma_I}{\lambda_I R_I^{\parallel}}x'^2 \right\}$$

$$\times \exp\left\{ -2\pi i\left(\frac{\cos\alpha_I}{\cos\alpha_S}\tan\sigma_S\frac{\cos\sigma_I}{\lambda_I}\frac{R_S}{R_I^{\parallel}} - \frac{\sin\sigma_I}{\lambda_I}\right)x' \right\}$$

$$\times \exp\left\{ i\pi\left(\frac{\lambda_S}{\lambda_I}\frac{R_S}{R_I} - 1\right)\frac{1}{\lambda_I R_I}y'^2 \right\}. \tag{17}$$

In the last relation, the amplitude distribution of the object u_s appears with coefficients in the argument describing the change of the imaging scale. The linear member x' represents the tilt of the image plane, and the quadratic members the curvature of the image surface.

The tilt of the image plane can be obtained when the coefficient of x' equals zero, whence

$$\tan\sigma_I = \frac{\cos\alpha_I}{\cos\alpha_S}\frac{R_S}{R_I^{\parallel}}\tan\sigma_S. \tag{18}$$

In every case, the natural angle of the object occurs when its plane is perpendicular to the line connecting its centre with the centre of the hologram. Then $\sigma_S = 0$, and from (18) it follows that $\sigma_I = 0$.

The curvature of the image surface in the meridional plane is given by the expressions:

$$\frac{1}{r_I^{\parallel}} = \left(\frac{\cos^2\alpha_I}{\cos^2\alpha_S}\frac{\lambda_S}{\lambda_I}\frac{R_S}{R_I^{\parallel}} - 1\right)\frac{1}{2R_I^{\parallel}}, \tag{19a}$$

$$\frac{1}{r_I^{\perp}} = \left(\frac{\lambda_S}{\lambda_I}\frac{R_S}{R_I} - 1\right)\frac{1}{2R_I}. \tag{19b}$$

In special cases both curvatures can be equal to zero. However, this curvature of the image surface cannot be a critical point.

From Equation (17) it follows that the lateral scales of imaging in meridional and perpendicular planes are given by the relations

$$M^{\parallel} = \mu\frac{\cos\alpha_S\cos\sigma_S}{\cos\alpha_I\cos\sigma_I}\frac{R_I^{\parallel}}{R_S}, \tag{20a}$$

$$M^{\perp} = \mu\frac{R_I^{\perp}}{R_S}, \tag{20b}$$

and, in a general case, the imaging is anamorphic.

4. Conditions for stigmatic and non-anamorphic imaging

If the condition for stigmatic imaging is to be found, the distances $R_{\mathrm{I}}^{\parallel}$ and R_{I} in Eqs. (15a) and (15b) must be equal, and using (15c), we get

$$\left[\frac{N}{R_{\mathrm{C}}}+\mu\left(\frac{1}{R_{\mathrm{C}}}-\frac{1}{R_{\mathrm{R}}}\right)\right][N+\mu(\sin\alpha_{\mathrm{S}}-\sin\alpha_{\mathrm{R}})]^2 - \left[\frac{N}{R_{\mathrm{C}}}+\mu\left(\frac{\sin^2\alpha_{\mathrm{S}}}{R_{\mathrm{S}}}-\frac{\sin^2\alpha_{\mathrm{R}}}{R_{\mathrm{R}}}\right)\right]=0 \tag{21}$$

whence, by choosing the other values, the last one can be calculated.

It is usually desired to obtain a stigmatic imaging without anamorphism with a given scale of imaging. In the case of $\sigma_{\mathrm{S}}=0$, $\sigma_{\mathrm{I}}=0$, the distortion does not exist if the condition $\alpha_{\mathrm{I}}=\alpha_{\mathrm{S}}$ is satisfied, and using Eq. (15c) we can obtain for the angle α_{S} the relation

$$\sin\alpha_{\mathrm{S}}=\frac{N-\mu\sin\alpha_{\mathrm{R}}}{1-\mu}. \tag{22}$$

In this case, the scale of imaging in both directions will be the same, and will be given by the relation

$$M=\mu\frac{R_{\mathrm{I}}}{R_{\mathrm{S}}}, \tag{23}$$

provided that the condition (21) is satisfied. The scale of imaging can be expressed with the given quantities of the holographic process by the relation

$$M^{-1}=\frac{N}{\mu}\frac{R_{\mathrm{S}}}{R_{\mathrm{C}}}+\left(1-\frac{R_{\mathrm{S}}}{R_{\mathrm{R}}}\right), \tag{24}$$

from which the expression N/R_{C} can be calculated and substituted into (21). After some algebra we can obtain a relatively simple condition for stigmatic imaging, which is without anamorphism and has a given scale

$$\frac{R_{\mathrm{S}}}{R_{\mathrm{R}}}=\frac{1-M^{-1}}{\cos^2\alpha_{\mathrm{R}}}\left[1-\left(\frac{N-\mu\sin\alpha_{\mathrm{R}}}{1-\mu}\right)^2\right]. \tag{25}$$

It is necessary to calculate the angle α_{S} from Eq. (22).

A special case of the relation (25) which occurs when the scale $M=1$ gives the condition

$$\left(\frac{R_{\mathrm{S}}}{R_{\mathrm{R}}}\right)_{M=1}=0. \tag{26}$$

This condition can be satisfied if either $R_{\mathrm{S}}=0$ or $R_{\mathrm{R}}=\infty$, and the result does not depend on the angle α_{R}. In the first case, the image (focused) holography takes place, in the second case, the reference beam as well as the reconstructing guided beam are collimated.

Another special case is when $\mu = 1$, then we obtain

$$\left(\frac{R_S}{R_R}\right)_{\mu=1} = 1 - M^{-1}, \tag{27}$$

and the angle α_R does not play any role, either.

These calculations show that by making a special choice of the respective quantities during the hologram recording, the astigmatism of the second order and at the same time the anamorphism of the image may be suppresed. Various scales of the image can be obtained.

5. Conclusions

The function of the waveguide hologram has been analysed by using the Fresnel diffraction method, and the conditions for imaging have been obtained. Generally, the image has astigmatism of the second order as well as an anamorphous distortion. The conditions for suppressing these aberrations have been derived. From this analysis, the conditions for suppressing aberrations of higher orders cannot be obtained. For their reduction, only two further quantities are available: the angle of the reference beam α_R, and a distance R_S or R_R. It is obvious that an appropriate compensation of aberrations must be chosen, depending on the properties of the object.

Waveguide holography can be utilized particularly for holographic processing of information, but also the elements produced in such a way can well serve in hybrid optical instruments where one part of the instrument can be designed as a planar guided wave optical component.

References

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Анализ отображения в голографии ведущих пучков

Проанализированы отображающие свойства световодовых голограмм, когда голограмма регистрирована при употреблении волн в свободном распространении и реконструирована при помощи ведущих волн. Выведены условия для стигматечиского и неаморфного отображений. Для анализа учтено не только изменение геометрии устройства, но также разные длины волны, употребленные для регистрации и реконструкции.