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Determination of the parameters of a single mode optical fibre of step-index type from the measurements made in both far and near zones

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Two methods of determination of parameters of single mode optical fibre of step-index type are presented. The first one (measurement in far field) is characterized by the fact that it allows us to find the parameters of the examined optical fibre by measuring two angular widths in the far field. The other one — measurement in the near field — is based on the proposed in this work approximation of the near field distribution in the fibre clad by an exponential function with an argument linearly depending on the radial coordinate. If the suitable measurement conditions are assured it is possible to achieve the accuracies of the fibre parameters determination better than 2%.

1. Introduction

In the course of the last years a number of works describing the methods of determination of single mode optical fibre parameters have appeared. A part of them is devoted to the methods exploiting the measurement in the near and far field (see [1]–[5], for example). The far field distribution for optical fibre of step-index type has been described by Gambling [1]. Pask and Sammut [2] showed how the core diameter and the reduced frequency of the optical fibre could be calculated when knowing both the half-width θ_h and the width θ_x of the first minimum of the far field distribution. In the work [3], a method has been reported in which the half-width of this distribution was measured as a function of the wavelength. The calculated mutual dependence of these parameters allows us also to determine the optical fibre parameters. The latter may be determined immediately from the refractive index distribution. The refractive index profile is obtained by suitable transformations of the near field distribution in the optical fibre [5]. High scanning accuracy is required in this method since single and double differentiation of the experimental curve is employed here.

A shortcoming of the method presented in [2] is that the measurement of the first minimum requires the measurement dynamics of order of 40–50 dB, as well as the fact that a significant influence of the position of this minimum is due to all kinds of disturbances of the refractive index distribution (see remarks in [6], p. 317). In the second part of this work, a modification of this method is proposed. It consists in measuring the angular width θ_m for which the intensity in the field distribution drops down to 0.1 of the maximum value (alternatively the angle θ_n is considered, for which the intensity drops down to 0.01 of the maximum value) instead of the width θ_x of the first minimum of the far field distribution. Thanks to that, the said shortcomings of the method reported in [2] become lesser. In the third part of this paper the approximation of the field in the single mode optical fibre clad by the suitable function of exponential type is presented together with some graphical procedure of determining the fibre parameters from the near field measurements. Part 4 shows the results of measurements which were obtained by using the methods described above.

2. Far field

GAMBLING [1] presented the formula for the far field distribution in the optical single-mode fibre of step-index type which took the form

$$|\varphi(\theta)|^2 = \begin{cases} \left\{ \dfrac{u^2 w^2}{(u^2-t^2)(w^2+t^2)} \left[J_0(t) - tJ_1(t) \dfrac{J_0(u)}{uJ_1(u)} \right] \right\}^2, & t \neq u, \\ \left\{ \dfrac{u^2 w^2}{2V^2 uJ_1(u)} [J_0^2(t) + J_1^2(t)] \right\}^2, & t = u \end{cases} \tag{1}$$

$$\varphi|(\theta)|^2 = I(V, t) \tag{1a}$$

where: $t = ka\sin\theta$, $V = ka\,\mathrm{NA}$ – reduced frequency, θ – angle of detection, $k = 2\pi/\lambda$, a – core radius, NA – numerical aperture, $u^2+w^2 = V^2$, u, w – transversal parameters of the fundamental mode.

By using the numerical analysis from Eq. (1a) the relation $t = t(V)$ has been calculated under the condition $I = c = \mathrm{const}$ for $c = 0.5$ and 0.1. This means that there are considered here the angles θ_h and θ_m, for which the intensity I drops down to the values 0.5 and 0.1 of the maximal value, respectively, as well as two following equations:

$$I(V, t_h) = 0.5, \tag{2}$$

$$I(V, t_m) = 0.1, \tag{3}$$

which resulted in determination of the following functions:

$$t_h = ka\sin\theta_h = t_h(V), \tag{4}$$

$$t_m = ka\sin\theta_m = t_m(V). \tag{5}$$

The quantity

$$\frac{t_h}{t_m} = \frac{\sin\theta_h}{\sin\theta_m}, \tag{6}$$

which is given by experiment, is the starting point to the calculation of the fibre parameters. The following dependences of the reduced frequency V and the parameter t_h on the quotient given above have been found:

$$V = 1.073 + 2.093x - 1.512x^2 + 0.9714x^3, \tag{7}$$

$$t_h = 0.1829 + 1.251x - 0.8259x^2 + 0.4268x^3 \tag{8}$$

where:

$$x = \frac{\dfrac{\sin\theta_h}{\sin\theta_m} - 0.44}{0.07}, \qquad 0.44 \leqslant \frac{\sin\theta_h}{\sin\theta_m} \leqslant 0.501,$$

$$1.24 \leqslant V \leqslant 2.4, \qquad 0.28 \leqslant t_h \leqslant 0.93.$$

When t_n and V are known, the core diameter $2a$ can be calculated from the definition of the parameter t

$$2a = \frac{\lambda t_h}{\pi \sin\theta_h}, \tag{9}$$

while the numerical aperture NA is estimated from the formula

$$\mathrm{NA} = V\frac{\sin\theta_h}{t_h}. \tag{10}$$

The analogic course is taken by the measurement-calculation procedure, the starting point of which is the measurement of analoges θ_h and θ_n, where θ_n — angle at which the intensity I drops down to the value 0.01 of the maximum value. The respective expressions for t_n and V take the forms:

$$V = 1.194 + 1.413y - 0.6871y^2 + 0.5093y^3, \tag{11}$$

$$t_h = 0.2584 + 0.8161y - 0.2767y^2 + 0.1436y^3 \tag{12}$$

where: $y = 10\left(\dfrac{\sin\theta_h}{\sin\theta_n} - 0.23\right)$, $\quad 0.23 \leqslant \dfrac{\sin\theta_h}{\sin\theta_h} \leqslant 0.33.$

The accuracies of the optical waveguide parameters determination are

$$\frac{\Delta(2a)}{2a} \approx 3\frac{\Delta\theta_h}{\theta_h} + 0.004, \tag{9a}$$

$$\frac{\Delta(\mathrm{NA})}{\mathrm{NA}} \approx 4\frac{\Delta\theta_h}{\theta_h} + 0.006 \tag{10a}$$

in the first case (θ_h, θ_m), and

$$\frac{\Delta(2a)}{2a} \approx 2\frac{\Delta\theta_h}{\theta_h} + 0.002, \tag{9b}$$

$$\frac{\Delta\mathrm{NA}}{\mathrm{NA}} \approx 2.5\frac{\Delta\theta_h}{\theta_h} + 0.003 \tag{10b}$$

in the other one (θ_h, θ_n). The values of the constant terms in the above expressions follow from the accuracies of the approximations (7), (8), (11) and (12), while those of the coefficients at $\Delta\theta_h/\theta_h$ – from Eqs. (9) and (10). When relative error of measurement $\Delta\theta_h/\theta_h$ amounts to 1%, the determination accuracies of the optical fibre parameters are 3.4% and 4% and 2.2% and 2.8%, respectively. However, when the relative error is equal to 0.2%, the accuracies amount to 1% and 1.4% as well as 0.6% and 0.8%, respectively.

3. Near field

The normalized field distribution at the front of the fibre may be put in the form

$$E(r) = \begin{cases} J_0(u\varrho) & \text{for } \varrho < 1, \\ \dfrac{J_0(u)}{K_0(w)} K_0(w\varrho) & \text{for } \varrho > 1 \end{cases} \tag{13}$$

where $\varrho = r/a$, r – radial coordinate. The amplitude of the field at the clad-core boundary is equal to $J_0(u)$ and changes from 0.41 for $V = 2.4$ to 0.71 for $V = 1.2$. This distribution of the field has the property that all its points, for which the amplitude $E(r)$ is less than 0.41, belong to the region of the fibre clad. The field in the clad described by the formula (13) has been approximated as follows:

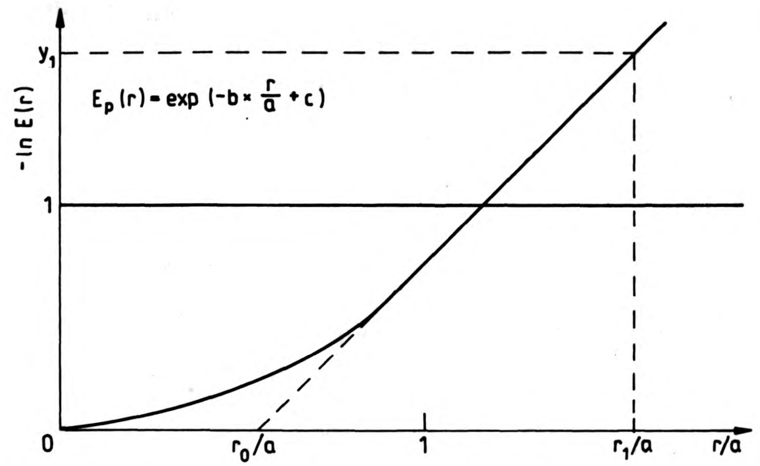
$$E_p(r) = J_0(u)\exp[-b(\varrho-1)] = \exp(-b\varrho+c) \tag{14}$$

for $1.5 \leqslant V \leqslant 2.4$, where the coefficients b and c depend on the reduced frequency V only:

$$b = 0.0364V^2+1.008V-0.6214, \tag{15}$$

$$c = 0.1154V^2+0.2847V-0.2116. \tag{16}$$

Hence, it follows that the natural logarithm of the field distribution within the clad described by Eq. (14) is a linear function of the coordinate r, for such values of r, at least, for which $-\ln E(r) \geqslant 1$. In the Figure an exemplified dependence of the natural logarithm of the normed field distribution versus the coordinate r has been shown. The determination of the coefficients of the said linear function $y(r) = br/a - c$ renders it possible to calculate the optical fibre parameters. These coefficients may be found by extending the linear part of the logarithm of field distribution from the values r for which $-\ln E(r) \geqslant 1$ to the point of intersection of this straight line with the abscissa. This intersection determines the coordinate r_0 (Fig.) as well as the slope coefficient of the straight line $\dfrac{y_1}{r_1-r_0} = b/a$, where $y_1 = -\ln E(r_1)$, and r_1 – chosen coordinate of r convenient to determine this coefficient. Strictly speaking, the approximation by the linear function $y(r)$ is not suitable for great r. The accuracy of



Distribution of the near field logarithm for the reduced frequency $V = 2$. The broken lines determine the characteristic points of straight line approximating the logarithm of the field distribution in the clad

this approximation, understood as the greatest difference between the field value in the clad calculated from (13), and the value calculated from (14), amounts to about 5% for such r for which $1 \leqslant -\ln E(r) \leqslant 3$. Just from this interval the coordinate r_1 should be chosen. In the experiment this limitation means that it suffices to measure the curve of the near field intensity distribution I in the range from $I_{\max}$ to $e^{-6} I_{\max} \approx 0.0025 I_{\max}$. The sequence of transformations specified below allows us to calculate the core diameter and the numerical aperture of the optical fibre when the values of r_0, r_1 and y_1 are determined:

$$c = r_0 \frac{y_1}{\Delta r}, \quad \Delta r = r_1 - r_0, \tag{17}$$

$$V = -0.3253c^2 + 1.8925c + 0.669, \tag{18}$$

$$b = -0.3023c^2 + 2.06c + 0.057, \tag{19}$$

$$2a = 2b \frac{\Delta r}{y_1}, \tag{20}$$

$$\mathrm{NA} = \frac{V\lambda}{2\pi a}. \tag{21}$$

Equations (18) and (19) were obtained by suitable transforming the Eqs. (15) and (16). The critical point in this method is the way the straight line $y(r)$ is determined. The correctness of this procedure may be verified by comparing the reduced frequency V with the frequency which follows from the value of the field amplitude $E(a)$ (where $E(a) = J_0(u) = f(V)$) at the core-clad boundary. The expression

$$V = -0.4545g^3 + 1.273g^2 - 2.082g + 2.454 \tag{22}$$

(where $g = 3.33(E(a)-0.41)$, $0.41 \leqslant E(a) \leqslant 0.71$, $1.2 \leqslant V \leqslant 2.4$) describes the last dependence with the accuracy better than 0.5%. If the reduced frequencies calculated from Eqs. (18) and (22) are the same it means that the determination procedure of the straight line $y(r)$ was correct.

4. Results of measurement

An optical fibre produced in the Maria Skłodowska–Curie University in Lublin, Poland *, was measured by using the said methods. The He-Ne laser ($\lambda = 632.8$ nm) has been used as the source of light. The obtained results are shown in the Table. This table contains such parameters as the core diameter $2a$, numerical aperture NA

Results of measurements of optical fibre of Maria Skłodowska–Curie University (Lublin, Poland) production

Parameter	Method		
	Far field		Near field
	(θ_h, θ_m)	(θ_h, θ_n)	
$2a$ (µm)	3.72 ± 0.13	3.81 ± 0.08	3.76 ± 0.15
NA	0.108 ± 0.004	0.107 ± 0.003	0.109 ± 0.004
λ_c (nm)	524 ± 10	532 ± 7	534 ± 15

and the cut-off wavelength λ_c of the higher order modes. From this table it follows that the spread of the obtained parameters with respect to their averages is not greater than 1.3%, which is a magnitude much lesser than the accuracies of determination of those parameters (about 5%). For the near field method the correctness of the procedure of the straight line $y(r)$ determination was verified. From Eq. (18) V is estimated to be 2.028, and from Eq. (22) – 2.032. Thus, the difference between those two values is negligibly small.

5. Conclusions

The methods of optical fibre parameters extraction from the measurement of far field and near field distribution have been presented. From the formal viewpoint these methods are based on the measurement of two characteristic points of the field distribution: in the far field the points $(\theta_h, 0.5)$ and $(\theta_m, 0.1)$ or $(\theta_h, 0.5)$ and $(\theta_n, 0.01)$, and in the near field the points: $(r_0, y = 0)$ and (r_1, y_1), respectively, with the difference that for the near field the approximating procedure should be applied and then the characteristic points determined. It seems that this method is better than

* Optical waveguide produced within PR I.02 Programme.

that based on the measurement of two characteristic points in the near field distribution (the width of the field distribution for which the field amplitude drops down to the values e^{-1} and e^{-2}), since it takes essential account of the course of the field distribution in certain region and not only of the values of the field at two points.

From Equations (9a) and (9b) as well as (10a) and (10b) it follows that for the far field the accuracy of calculations is slightly better when the pair of angles (θ_h, θ_n) is measured. However, the differences are not substantial and either one or the other pair of angles may be exploited equivalently in the calculations.

In principle, the methods described in this paper determine the equivalent step-index parameters for the fibre since it is very difficult to obtain the exact step-index profile of the refractive index in the single mode fibre. The methods are suitable for determining the refractive index profile of gradient-index type as well.

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Определение параметров одномодового волновода со ступенчатым профилем показателя преломления из измерений в близком и далеком поле

Представлено два метода измерения параметров одномодового волновода со ступенчатым профилем показателя преломления. Первый из них — измерение в далеком поле — характеризуется тем, что позволяет найти параметры исследуемого волновода путем измерения двух угловых ширин разложения далекого поля. Второй — измерение в близком поле — опирается на предложенном в той работе приближении разложения близкого поля в оболочке волновода экспотенциальной функцией с аргументом линейно зависимым от радиальной координаты. При обеспечении соответствующих условий измерения можно, применяя каждый из этих методов, получить точности определения параметров не хуже чем 2%.