

# Photoelastic interferometer with amplitude modulation in the reference beam

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The present paper contains a description of the photoelastic interferometer together with an analysis interpretation of the obtained interference images. Due to application of light amplitude modulation in the reference beam, the resultant images contain three families of fringes: isopachics, isochromatics and isoclines. On the basis of these fringes a full description of the stress field in the photoelastic model being in the two-dimensional stress state may be given. There has also been shown a possibility of eliminating the isoclin fringes from these images if a correction of readability of the remaining fringes happens to be necessary.

## 1. Introduction

In the past many investigators introduced the conventional interferometer to the photoelastic examinations in different ways (of the leading researchers we should mention: Favre, Brahtz and Soehrens, Fabry, Frocht, Sinclair, Post, Nisida and Saito, see, for example, [1]–[3], and the references given there). The main purpose has always been to receive information about the distribution of the sum of principal stresses in the examined photoelastic model. A common photoelastic method suffers from known essential restrictions. It enables the determination of two magnitudes only: the difference of the two principal stresses (isochromatics), and the directions of these stresses (isoclin), whereas a complete set of data necessary to describe the two-dimensional stress state must contain three magnitudes.

A subsequent version of the Mach–Zehnder interferometer presented here allows us to determine the third magnitude that we need; moreover, it is possible to read simultaneously all the three magnitudes from the obtained images. In order to describe the changes of the light polarization state in the subsequent phases of its transformation, the double-complex numbers have been employed (this way of description is given in [4]). A Cartesian right-handed coordinate system  $xyz$  has been assumed in which the coordinate  $z$  is associated with the direction of light propagating in all branches of the interferometer.

The light source  $L_s$  (Fig. 1) provides the linearly polarized light. The polarization unity vector in the beam expanded by the beam-expander  $E$  and travelling along the input interferometer path is described in the  $xy$  plane by the following equations (cf. [4], Eq. (10)):

$$\hat{E}_L = e^{j\varphi_L} e^{i\omega t}.$$

When assuming that the linear polarization direction is consistent with the

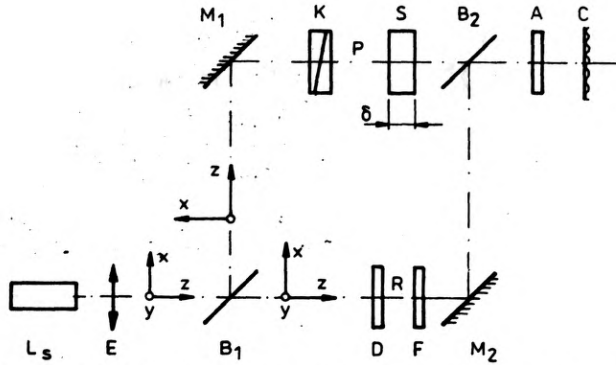


Fig. 1. Optical arrangement of the interferometer,  $x, y, z$  – Cartesian right-handed coordinate system connected with the direction of light propagation in the following way: axis  $z$  denotes the direction of the light rays in all the branches (paths) of the interferometer, axis  $y$  is perpendicular to the base plane of the interferometer identical with the figure plane (in the figure the axis is directed upwards and denoted by the symbol  $\circ$ ), axis  $x$  lies in the base plane of the interferometer

vertical axis  $y$ , i.e.,  $\varphi_L = \pi/2$  (Fig. 2), the above formula in the  $xy$  coordinate system takes the form

$$\hat{E}_L = e^{j\frac{\pi}{2}} e^{i\omega t} = je^{i\omega t}. \tag{1}$$

The beam splitter  $B_1$  divides the input beam into two subbeams travelling along two paths: the object path  $P$  and reference path  $R$ .

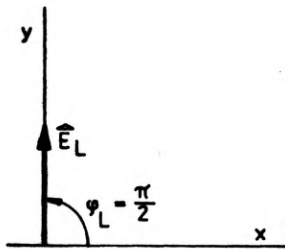


Fig. 2. Linear polarization of laser light

## 2. Object path

The ray reflected from the splitter  $B_1$  and from the mirror  $M_1$  preserves its linear polarization of the direction consistent with the  $y$  axis. It is only the phase that is subject to changes after reflection, while the amplitude is attenuated ([4], Eq. (19))

$$\hat{E}_{PBM} = \hat{R}_{y_B} \hat{R}_{y_M} \hat{E}_L = j\mu_{y_{BM}} e^{i\omega t}.$$

The complex coefficients of reflectance  $\hat{R}_{y_B}$  and  $\hat{R}_{y_M}$  comprise jointly the phase change and the amplitude attenuation. Their product is equal to

$$\hat{\mu}_{y_{BM}} = \hat{R}_{y_B} \hat{R}_{y_M} = R_{y_B} R_{y_M} e^{-i(\psi_{y_B} + \psi_{y_M})} = \mu_{y_{BM}} e^{-i\psi_{y_{BM}}}.$$

The compensator of the optical path  $K$  causes the displacement of the general phase by the value

$$\hat{E}_{PK} = e^{ix} \hat{E}_{PBM} = j\hat{\mu}_{y_{BM}} e^{i(\omega t + \kappa)}. \quad (2)$$

In the field of plane stress of the photoelastic model  $S$ , the mutually perpendicular principal stresses  $\sigma_1$  and  $\sigma_2$  determine the local system of optical axes  $\xi\eta$  (Fig. 3) in an arbitrary point of the model plane. The linear polarization of the ray transillu-

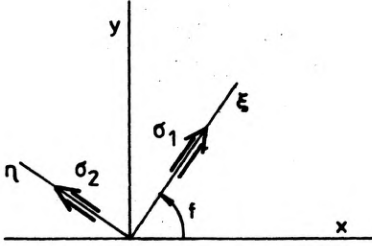


Fig. 3. Photoelastic model

minating the model at such a point is transformed into elliptic polarization. This may be described by a sequence of the following equations.

– Transformation of the vector  $\hat{E}_{PK}$  (2) from the system  $xy$  to the local system  $\xi\eta$  stimulated by the field stresses (cf. [4], Eq. (5))

$$\hat{E}_{PK}(\xi\eta) = e^{-jf} \hat{E}_{PK} = \hat{\mu}_{y_{BM}} (\sin f + j \cos f) e^{i(\omega t + \kappa)}.$$

– Transformation of the linear polarization vector into the vector of elliptic polarization due to transillumination of the birefringent photoelastic model, formula still written down in the  $\xi\eta$  system (cf. [4], Eq. (15a))

$$\hat{E}_{PS}(\xi\eta) = \hat{\mu}_{y_{BM}} (\sin f e^{i\psi_\xi} + j \cos f e^{i\psi_\eta}) e^{i(\omega t + \kappa)}$$

where:  $\psi_\xi = -k\delta n_1$  – phase shift introduced by the model for the first linear component of the vector  $\hat{E}_{PS}(\xi\eta)$  of direction consistent with the  $\xi$  axis;  $\psi_\eta = -k\delta n_2$  – phase shift for the second linear component of the vector  $\hat{E}_{PS}(\xi\eta)$  of direction consistent with the  $\eta$  axis.

– Transformation of the vector  $\hat{E}_{PS}(\xi\eta)$  from the local system  $\xi\eta$  to the  $xy$  system

$$\begin{aligned} \hat{E}_{PS} &= e^{jf} \hat{E}_{PS}(\xi\eta) = \hat{\mu}_{y_{BM}} e^{jf} (\sin f e^{-ik\delta n_1} + j \cos f e^{-ik\delta n_2}) e^{i(\omega t + \kappa)} \\ &= \hat{\mu}_{y_{BM}} \left\{ \frac{1}{2} \sin 2f (e^{-ik\delta n_1} - e^{-ik\delta n_2}) + j (\sin^2 f e^{-ik\delta n_1} + \cos^2 f e^{-ik\delta n_2}) \right\} e^{i(\omega t + \kappa)} \\ &= \hat{\mu}_{y_{BM}} \left\{ -i \sin 2f \sin \frac{k\delta}{2} (n_1 - n_2) + j e^{i\psi_s} \sqrt{1 - \sin^2 2f \sin^2 \frac{k\delta}{2} (n_1 - n_2)} \right\} \\ &\quad \times e^{i(\omega t + \kappa - \frac{k\delta}{2}(n_1 + n_2))}. \end{aligned} \quad (3)$$

In deriving Eq. (3) the following auxiliary formulae have been used:

$$e^{i\alpha} - e^{i\beta} = (e^{i\frac{\alpha-\beta}{2}} - e^{-i\frac{\alpha-\beta}{2}}) e^{i\frac{\alpha+\beta}{2}} = 2i \sin\left(\frac{\alpha-\beta}{2}\right) e^{i\frac{\alpha+\beta}{2}},$$

$$A^2 e^{i\alpha} + B^2 e^{i\beta} = (A^2 e^{i\frac{\alpha-\beta}{2}} + B^2 e^{-i\frac{\alpha-\beta}{2}}) e^{i\frac{\alpha+\beta}{2}} = \sqrt{(A^2 + B^2) - 4A^2 B^2 \sin^2\left(\frac{\alpha-\beta}{2}\right)} e^{i\left(\frac{\alpha+\beta}{2} + \psi\right)}$$

where

$$\tan \psi = \frac{A^2 \sin\left(\frac{\alpha-\beta}{2}\right) - B^2 \sin\left(\frac{\alpha-\beta}{2}\right)}{A^2 \cos\left(\frac{\alpha-\beta}{2}\right) + B^2 \cos\left(\frac{\alpha-\beta}{2}\right)} = \frac{A^2 - B^2}{A^2 + B^2} \tan\left(\frac{\alpha-\beta}{2}\right).$$

Thus, the formula (3) describes the elliptic polarization vector in the  $xy$  system which appears after the ray transmission birefringent model at the point  $s$ , in which:  $f$  — local azimuth of the stimulated birefringence which is called isocline parameter in the stress field (Fig. 3),  $n_1, n_2$  — light refractive indices for two mutually perpendicular linear components of the polarization vector of directions consistent with the  $\xi$  and  $\eta$  axes, respectively,  $\delta$  — thickness of the plane model,  $k = 2\pi/\lambda$  — wave number for applied light wavelength  $\lambda$ . The coefficients  $n_1$  and  $n_2$  are, according to the Maxwell–Neumann equations [1], [5], the functions of the principal stresses  $\sigma_1$  and  $\sigma_2$  at the given point:

$$n_1 - n = C_1 \sigma_1 + C_2 \sigma_2$$

$$n_2 - n = C_2 \sigma_1 + C_1 \sigma_2$$

where  $C_1, C_2$  — the material constants of the model material;  $n$  — refractive index of the unloaded model material as indicated by the following implication:

$$\{(\sigma_1 = \sigma_2 = 0) \Rightarrow (n_1 = n_2 = n)\} \Rightarrow (\psi_\xi = \psi_\eta = -k\delta n = \psi_n). \quad (4a)$$

Taking advantage of these equations the following notations are introduced: *isochromatic parameters*

$$q_C = \frac{k\delta}{2}(n_1 - n_2) = \frac{k\delta}{2} K_C (\sigma_1 - \sigma_2) = \pi N_C,$$

*isochromatic order*

$$N_C = \frac{\delta}{\lambda} K_C (\sigma_1 - \sigma_2),$$

*photoelastic constant in the difference field of principal stresses*

$$K_C = C_1 - C_2, \quad (4b)$$

*isopachic parameter*

$$q_P = \frac{k\delta}{4} \{(n_1 + n_2) - 2n\} = \frac{k\delta}{4} K_P (\sigma_1 + \sigma_2) = \pi N_P,$$

isopachic order

$$N_P = \frac{\delta}{\lambda} K_P \left( \frac{\sigma_1 + \sigma_2}{2} \right),$$

photoelastic constant in the summation field of the principal stresses

$$K_P = C_1 + C_2.$$

Substituting (4) into (3) we obtain

$$\begin{aligned} \hat{E}_{PS} &= \hat{E}_{xPS} + j\hat{E}_{yPS} \\ &= \hat{\mu}_{yBM} (-i \sin 2f \sin q_C + j e^{i\psi_S} \sqrt{1 - \sin^2 2f \sin^2 q_C}) e^{i(\omega t + \alpha - \psi_n - 2q_P)}. \end{aligned} \quad (5)$$

It should be noted that the phase of the first component  $\hat{E}_{xPS}$  is shifted with respect to the general phase by  $-\pi/2$  (since  $-i = e^{-i\pi/2}$ ). The phase of the second component  $\hat{E}_{yPS}$  may be calculated from the relation  $\tan \psi_S = \cos 2f \tan q_C$ . (The auxiliary formulae have been used when deriving formula (3)).

As formula (5) shows, the loaded model introduces some differentiation of the beam polarization within the beam cross-section. Its distribution carries information concerning the stress field which is contained in the parameters  $q_C$ ,  $q_P$  and  $f$ .

The light ray goes from the object path to the output path, transilluminating the splitter  $B_2$  on its way. By using the following notations for the mirror transmittance coefficients: i)  $T_{rx}$ ,  $T_{ry}$  – transmittance coefficients attributed to the front surface for two mutually perpendicular components of the polarization vector lying in the incidence plane and in the one perpendicular to it, respectively, ii)  $T_{lx}$ ,  $T_{ly}$  – transmittance coefficients attributed to the back surface for the components of the polarization vector lying in the planes as above, the formula describing the vector of the object beam polarization along the output path takes the form (cf. [4], Eq. (20))

$$\begin{aligned} \hat{E}_P &= T_{lx} T_{rx} \hat{E}_{xPS} + j T_{ly} T_{ry} \hat{E}_{yPS} = \mu_{xB} \hat{E}_{xPS} + j \mu_{yB} \hat{E}_{yPS} \\ &= \hat{\mu}_{yBM} (-i \mu_{xB} \sin 2f \sin q_C + j e^{i\psi_S} \mu_{yB} \sqrt{1 - \sin^2 2f \sin^2 q_C}) e^{i(\omega t + \alpha - \psi_n - 2q_P)} \\ &= (\hat{\mu}_{xP} \sin 2f \sin q_C + j \hat{\mu}_{yP} \sqrt{1 - \sin^2 2f \sin^2 q_C}) e^{i(\omega t + \alpha - 2q_C)} \end{aligned} \quad (6)$$

where:

$$\hat{\mu}_{xP} = -i \hat{\mu}_{yBM} \mu_{xB} e^{-i\psi_n} = \mu_{xP} e^{-i\psi_{xP}},$$

$$\mu_{xP} = R_{yB} R_{yM} T_{lx} T_{rx},$$

$$\psi_{xP} = \psi_{yB} + \psi_{yM} + \psi_n + \frac{\pi}{2},$$

$$\hat{\mu}_{yP} = \hat{\mu}_{yBM} \mu_{yB} e^{-i(\psi_n - \psi_S)} = \mu_{yP} e^{-i\psi_{yP}},$$

$$\mu_{yP} = R_{yB} R_{yM} T_{ly} T_{ry},$$

$$\psi_{yP} = \psi_{yB} + \psi_{yM} + \psi_n - \psi_S,$$

and its linear components are:

$$\hat{E}_{xP} = \hat{\mu}_{xP} \sin 2f \sin q_C e^{i(\omega t + \kappa - 2q_P)}, \quad (6a)$$

$$\hat{E}_{yP} = \hat{\mu}_{yP} \sqrt{1 - \sin^2 2f \sin^2 q_C} e^{i(\omega t + \kappa - 2q_P)}. \quad (6b)$$

In the interferometer discussed, the final result is obtained on the basis of the component  $\hat{E}_{xP}$ . Therefore, the analyser  $A$  (Fig. 1) is adjusted so that the component  $\hat{E}_{yP}$  be extinguished (which means that the analyzer azimuth  $\varphi_A = 0$ ). Thus, by illuminating the screen  $C$  exclusively with an object beam (reference beam being shaded) an image is obtained behind the analyser, the intensity of which is (cf. [4], Eq. (2a))

$$J_{xP} = \hat{E}_{xP} \hat{E}_{xP}^* = \mu_{xP}^2 \sin^2 2f \sin^2 q_C = \mu_{xP}^2 \sin^2 2f \sin^2 (\pi N_C). \quad (7)$$

This is a typical image obtained in a linear polariscope during the so-called dark field observation. It contains two families of fringes: isoclins ( $\sin^2 2f$ ) and isochromatics ( $\sin^2 \pi N_C$ ).

### 3. Reference path

Passing through the splitter  $B_1$  the ray preserves the linear polarization of direction consistent with the  $y$  axis, and simultaneously the vector amplitude is attenuated. Taking advantage of the notation of transmittance coefficients introduced earlier ( $T_{rx}$ ,  $T_{ry}$ , and  $T_{ix}$ ,  $T_{iy}$ ), the vector of the linear polarization behind the splitter  $B_1$  may be expressed by the relation

$$\hat{E}_{RB} = T_{ry} T_{iy} \hat{E}_L = j\mu_{yB} e^{i\omega t}.$$

The subsequent element, which is transilluminated by the reference beam, is the field amplitude modulator. In its simplest version this may be a photographic plate with a positive image of fringes recorded on it due to its exposure by the object beam exclusively obtained in accordance with formula (7). The amplitude of the beam transilluminating the modulator is subject to attenuation to the degree corresponding to the local blackening. The degree of attenuation is differentiated within the beam cross-section and is defined by the modulation coefficient

$$\hat{m} = m e^{-i\psi_D} = |\sin 2f \sin q_C| e^{-i\psi_D} \quad (8)$$

where:  $m = |\sin 2f \sin q_C|$  — real coefficient of modulation,

$\psi_D = \text{const}$  — total phase introduced by the modulator.

Behind the modulator the polarization vector is

$$\hat{E}_{RD} = \hat{m} \hat{E}_{RB} = j\mu_{yB} m e^{i(\omega t - \psi_D)}.$$

The task of the half-wave plate  $F$  (Fig. 1) is to perform the transformation of the rotation of the linear polarization vector  $\hat{E}_{RD}$  so that it takes a quasi-horizontal position. If the azimuth of the optical axes of the half-wave plate  $\xi_F \eta_F$  amounts to  $\varphi_F$  (Fig. 4), the rotated vector is described by the equation (cf. [4], Eq. (18))

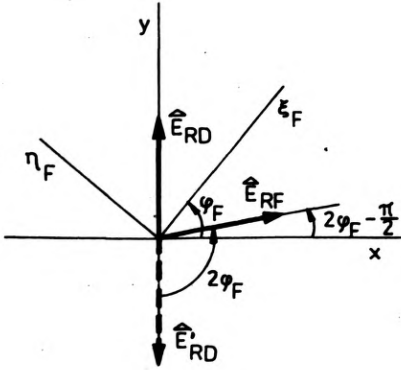


Fig. 4. Half-wave plate

$$\begin{aligned} \hat{E}_{RF} &= e^{j2\varphi_F} \hat{E}'_{RD} e^{-i\psi_F} = -j e^{j2\varphi_F} \mu_{y_B} m e^{i(\omega t - \psi_D - \psi_F)} \\ &= \mu_{y_B} m (\sin 2\varphi_F - j \cos 2\varphi_F) e^{i(\omega t - \psi_D - \psi_F)} \end{aligned}$$

where  $\psi_F$  – total phase introduced by the half-wave plate.

The light ray gets to the output path as a result of its reflection from the mirror  $M_2$  and beam splitter  $B_2$ . The notations of the reflectance coefficients take the following forms:

$\hat{R}_{x_M} = R_{x_M} e^{-i\psi_{x_M}}$ ,  $\hat{R}_{y_M} = R_{y_M} e^{-i\psi_{y_M}}$  – complex reflectance coefficients of the mirror for two mutually perpendicular components of the polarization vector lying in the incidence plane and in the one perpendicular to it, respectively;

$\hat{R}_{x_B} = R_{x_B} e^{-i\psi_{x_B}}$ ,  $\hat{R}_{y_B} = R_{y_B} e^{-i\psi_{y_B}}$  – complex reflectance coefficients of the beam splitter for the polarization vector components lying in the planes mentioned above.

By using the above-mentioned reflectance coefficients, the formula describing the polarization vector of the reference beam along the output takes the form

$$\begin{aligned} \hat{E}_R &= \hat{R}_{x_M} \hat{R}_{x_B} \hat{E}_{x_{RF}} + j \hat{R}_{y_M} \hat{R}_{y_B} \hat{E}_{y_{RF}} = \hat{\mu}_{x_{MB}} \hat{E}_{x_{RF}} + j \hat{\mu}_{y_{MB}} \hat{E}_{y_{RF}} \\ &= \mu_{y_B} m (\hat{\mu}_{x_{MB}} \sin 2\varphi_F - j \hat{\mu}_{y_{MB}} \cos 2\varphi_F) e^{i(\omega t - \psi_D - \psi_F)} \\ &= m (\hat{\mu}_{x_R} \sin 2\varphi_F - j \hat{\mu}_{y_R} \cos 2\varphi_F) e^{i\omega t} \end{aligned} \tag{9}$$

where:  $\hat{\mu}_{x_R} = \mu_{y_B} \hat{\mu}_{x_{MB}} e^{-i(\psi_D + \psi_F)} = \mu_{x_R} e^{-i\psi_{x_R}}$ ,

$$\mu_{x_R} = T_{ry} T_{iy} R_{x_M} R_{x_B},$$

$$\psi_{x_R} = \psi_{x_M} + \psi_{x_B} + \psi_D + \psi_F,$$

$$\hat{\mu}_{y_R} = \mu_{y_B} \hat{\mu}_{y_{MB}} e^{-i(\psi_D + \psi_F)} = \mu_{y_R} e^{-i\psi_{y_R}},$$

$$\mu_{y_R} = T_{ry} T_{iy} R_{y_M} R_{y_B},$$

$$\psi_{y_R} = \psi_{y_M} + \psi_{y_B} + \psi_D + \psi_F.$$

The linear components of the vector  $\hat{E}_R$  describe the following formulae derivable from (9):

$$\hat{E}_{xR} = \hat{\mu}_{xR} \sin 2\varphi_F m e^{i\omega t}, \quad (9a)$$

$$\hat{E}_{yR} = -\hat{\mu}_{yR} \cos 2\varphi_F m e^{i\omega t}. \quad (9b)$$

Only the  $\hat{E}_{xR}$  component passes through the analyser along the output path.

#### 4. Output path

As already mentioned above, the components  $\hat{E}_{xP}$  (6a) from the object path and  $\hat{E}_{xR}$  (9a) from the reference path pass through the analyser. Due to the interference a resulting linear polarization vector appears of direction consistent with the  $x$  axis

$$\hat{E}_A = \hat{E}_{xP} + \hat{E}_{xR} = (\hat{\mu}_{xP} \sin 2f \sin q_C e^{-i(2q_P - \kappa)} + \hat{\mu}_{xR} \sin 2\varphi_F m) e^{i\omega t}.$$

Substituting formula (8) to the above equation and taking the expression  $\sin 2f \sin q_C$  out of the bracket the interference equation is put to the form

$$\hat{E}_A = (\pm \hat{\mu}_{xP} e^{-i(2q_P - \kappa)} + \hat{\mu}_{xR} \sin 2\varphi_F) |\sin 2f \sin q_C| e^{i\omega t}.$$

The adjustment of the interferometer by equalizing both the optical paths (by using the compensator  $K$ ) and the intensities (by using the half-wave plate  $F$ ) of the two interfering beams is reduced to fulfilling the condition

$$(\hat{\mu}_{xP} e^{i\kappa} = \hat{\mu}_{xR} \sin 2\varphi_F = \hat{\mu}_x) \Leftrightarrow (\mu_{xP} e^{-i(\psi_{xP} - \kappa)} = \mu_{xR} \sin 2\varphi_F e^{-\psi_{xR}} = \mu_x e^{-i\psi_x}), \quad (10)$$

from which two subconditions follow:

$$(\psi_{xP} - \kappa = \psi_{xR} = \psi_x) \rightarrow (\kappa = \psi_{xP} - \psi_{xR}),$$

$$(\mu_{xP} = \mu_{xR} \sin 2\varphi_F = \mu_x) \rightarrow \left( \varphi_F = \frac{1}{2} \arcsin \frac{\mu_{xP}}{\mu_{xR}} \right)$$

where:  $\kappa$  — phase of regulation,

$\varphi_F$  — azimuth of regulation.

Finally, the interference equation, with condition (10) being taken into account, takes the form

$$\begin{aligned} \hat{E}_A &= \hat{E}_{x_A} = \hat{\mu}_x (\pm e^{-i2q_P} + 1) |\sin 2f \sin q_C| e^{i\omega t} \\ &= \hat{\mu}_x |\sin 2f \sin q_C| (\pm e^{-iq_P} + e^{iq_P}) e^{i(\omega t - q_P)} \\ &= [2\hat{\mu}_x |\sin 2f \sin q_C| \cos q_P e^{i(\omega t - q_P)}] \sqrt{[2\hat{\mu}_x |\sin 2f \sin q_C| \sin q_P e^{i(\omega t - q_P)}]}. \end{aligned} \quad (11)$$

The light intensity in the appearing image is

$$J_A = \hat{E}_{x_A} \hat{E}_{x_A}^* = [4\mu_x^2 \sin^2 2f \sin^2 q_C \cos^2 q_P] \sqrt{[4\mu_x^2 \sin^2 2f \sin^2 q_C \sin^2 q_P]}. \quad (12)$$

This image contains three families of fringes superimposed one on another: isoclins (described by the factor  $\sin^2 2f$ ), isochromatics (described by the factor  $\sin^2 q_C$ ), and isopachics (described by either  $\cos^2 q_P$  or  $\sin^2 q_P$ ).



## 5. Concluding remarks

A number of both advantages and shortcomings may be attributed to the interferometric method of examining photoelastic models. The most important advantage offered is the possibility of receiving the field image of the isopachic fringes across the whole surface of the examined object. In this report, the proposed method may be positively distinguished from the so-called point interference methods described in [1] and [6], in which the object surface is penetrated point by point which is a highly labourious procedure. It is characterized by the fact that the images of all three families of fringes can be obtained simultaneously and in this respect it may be compared with the field method elaborated by NISIDA and SAITO [2]. This feature has also its negative sides, since it may worsen the readability of the image in some of its parts. The regions of the isoclines are especially disadvantageous in this respect, since isoclines may sometimes take the form of very diffused fringes.

A removal of the isoclines from the image is possible with the help of double exposure on a single photographic film located on the screen *C*. This photographic trick must be performed for two values of parameter *f* differing by  $\pi/4$  for each exposure, because if the image obtained from the first exposure described by the (12) is

$$J_1 = 4\mu_x^2 \sin^2 2f \sin^2 q_C \cos^2 q_P,$$

while that given by the second exposure is

$$J_2 = 4\mu_x^2 \sin^2 2\left(f + \frac{\pi}{4}\right) \sin^2 q_C \cos^2 q_P = 4\mu_x^2 \cos^2 2f \sin^2 q_C \cos^2 q_P,$$

then the summed-up image recorded on the photographic film is

$$J = J_1 + J_2 = 4\mu_x^2 (\sin^2 2f + \cos^2 2f) \sin^2 q_C \cos^2 q_P = 4\mu_x^2 \sin^2 q_C \cos^2 q_P.$$

In a similar way it is possible to remove the isochromatic fringes from the image, but this requires a modification of the system presented in Fig. 1 (inserting of the quarter-wave plates in the object beam).

The most important novelty of the interference method presented seems to be the application of the amplitude modulation in the reference beam in such a way that its distribution across the beam cross-section corresponds to the respective amplitude distribution in the object beam. The presented simple way of performing such a modulation with the help of the photographic plate with the recorded image produced on the basis of the amplitude distribution in the object beam does not, however, ensure a high degree of imaging accuracy. Therefore, the relation (8) is an approximation of the real distribution which is created in the reference beam transilluminating such a modulator. It is also necessary to solve the problem of modulation in such a way that the appearing errors would be minimized and the modulation be performed in the real time. However, the modulator satisfying such demands must be based on different operation principles. It would enable a

significant simplification of the examinations (e.g., by eliminating the necessity of producing separate modulators for each examined model), and will be applicable to the examinations of the models subject to variable loads.

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*Received December 10, 1986,  
in revised form July 13, 1987*

## **Фотоупругий интерферометр с амплитудной модуляцией в пучке отношения**

В работе представлен фотоупругий интерферометр и аналитическая интерпретация полученных интерференционных образов. Благодаря применению модуляции света в пучке отношения результирующие образы содержат совместно три семейства линий: изопахтики, изоклины и изохромы. На базе этих образов возможно полное описание напряжённого поля фотоупругого образца в плоском напряжённом состоянии. Указана возможность исключения из этих образов линий изоклины если происходит необходимость улучшения чёткости изображения остальных линий.