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On the effects at reflection of light signals from cube corner reflectors on satellites as viewed in classical physics

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The anomalies observed at the returning of laser light signals after their reflection from an array of cube corner reflectors mounted on satellites are discussed and explained on the ground of classical physics. The considerations base on the deflection effect at reflection from a moving reflecting surface; this effect corresponds, in some way, to the effect of "aberrational refraction" in special relativity, but it is n times greater when the reflector is a prism (n – refractive index). The substantional diffuse of the signal and its "weakness" at recording depend or may depend on the angle between signal and reflector axis. Broadening of the vertex angle of the back surface in the cube corner reflector causes a diffusion of the returning signal. A terrestrial experiment is proposed either to confirm or to reject the validity of special relativity in question.

1. Introduction

Since the sixties the geodetic measurements have been performed with the help of laser light signals in pulses reflected from arrays of cube corner reflectors placed on satellites. The pulses have internal structure consisted of a number of relaxation oscillations lasting even up to 2 μs [1]. However, the receipt of the returning signals has been very difficult, so that nowadays the one-photon-receipt technique is introduced generally.

To our knowledge, based on the attainable publications, these unexpected and undesirable difficulties in receiving the returning signals have been explained with the help of aberration effect within the framework of special theory of relativity (STR). Therefore, to counteract this effect the area of the returning signals was spread out. The velocity aberration was assumed to displace the centre of this area by a distance of the order of 10^2 m [2] from the laser in the direction of the satellite speed. In order to compensate the aberration effect, the array of retroreflectors was designed to spread the returning signals so that more than 50 per cent [3] (35 per cent in [2]) of their energy was inside a cone of 0.1 mrad. Due to divergent effects and limited effective area of the retroreflecting mirrors, the returning signals were assumed to cover an area, on the Earth, of about 100 m in diameter [3]. To neutralize the aberration effect and keep the receiver in the beam's "footprint", the right angle reflecting back surface of each rectangular tetrahedron (cube corner reflector) was enlarged by 1.5 s (1.5″) of arc in the Lageos satellite [2].

In spite of these efforts to neutralize the aberration effect, the returning signals were not as clear as desired [4]. They were not observed in all the cases when the conditions would have predict a return [1], [5]; only a few from 10^{19} photons transmitted in each pulse were received back [2]. Even the internal structure of a returning pulse did not correspond to what had been emitted at the moment of transmission [1]. SNYDER et al. [5] write that the cause why returning signals were not observed "is known at this time, but the effect is not unreasonable if the nonuniformity of the return beam from the satellite corner reflectors are considered".

The laser and receiver were, as a rule, close to one another. When the receiver was displayed by 20 m [1], what was to compensate the aberration effect, the received signals had a greater intensity than when the receiver was a few meters from the laser. During receiving the signals from the Anna 1–B and Lageos–1 satellites the receiver was 60 m distant from the laser [6]. In the other experiment, to compensate the aberration effect, the receiver was displaced by about 60 m from the laser [3], although for the results presented in [3] that distance amounted to some meters. (The results concerning the case of the receiver distant from the laser by 60 m are not known to us).

In our considerations within the framework of classical physics we discuss only the effects that take place at reflection from an array of cube corner reflectors. These effects diffuse the returning signals. We leave aside, here, all technical nonuniformities and imperfections as well as such elements as, e.g., the influence of our atmosphere or the background noises, or which part of a signal covers the array of retroreflectors. They all add in some way to the effects that arise at reflection and, in general, still diffuse or/and "weaken" the returning signals to be recorded. First, we present reflections from moving mirrors in STR in order to show more distinctly the difference between relativistic and classical predictions in question and to bring into relief the predominance of the classical approach to a subject. Some effects valid in both theories (Sections 6 and 7) but not raised in the publications (to our knowledge) are considered.

2. Reflections in special relativity

The aberration effect, for signals reflected from a moving satellite, in the case of a plane mirror, was considered by PLOTKIN, NUGENT and CONDON [7], DASHCHUK [8], ASHWORTH and DAVIES [9]. CENSOR [10] and ASHWORTH and DAVIES [11] considered the case of a corner reflector. All the authors supported themselves on the "aberrational refraction" somewhere between laser L and satellite S, and the formulas were derived "by stages" (that is, the satellite was treated as "moving" object when the signals travelled from L to S and, next the Earth was "moving" when the signals travelled from S to L). No effects at the moment of reflection were taken into account.

It is not our aim to discuss or to accede to the discussion on relativistic derivations and their differences or mistakes. However, deriving the formulas "by stages" is rather strange from the physical point of view in STR. One should remember that a signal "refracts aberrationally" at passing from the "stationary" into the "moving" frame (to the back) as well as from the "moving" into the "stationary" frame (to the front). The two procedures give, of course, the same result, that is the centre of a returning signal reaches the Earth at K distant by

$$2\beta_s = 2V_s/c = 9.652691'' \tag{1a}$$

angularly, and

$$d_s = 2\beta_s h = 46.67 \text{ m} \tag{1b}$$

linearly from laser L in the direction of the satellite speed V_s; β_s is the angle of aberration, c – light velocity, h – altitude of the satellite ($V_s = 7$ km/s, $h = 1000$ km).

Ashworth and Davies [12] derive the equivalent classical formulas for reflection from a plane mirror. They support themselves the classical formulas of aberration and the specular reflection in the moving mirror frame. But they copy the relativistic procedure [13]. However, when the "aberrational refraction" is a main agent causing the deviation of the returning signal within the framework of STR, the aberration effect in classical physics is useless (immaterial) in question (there is no relative motion between the laser-receiver set and the signal path after transmission as well as after reflection).

Thus, in STR, the deviation effect of the returning signal is exclusively due to the "aberrational refraction" taking place before and after reflection(s). This does not depend on the kind of retroreflecting system. Any effects at the moment of reflection do not exist. Therefore, the centre of the returning signal cannot be divided into parts, and its distance from the laser is defined by Eqs. (1a) and (1b).

3. Deflection effect

In classical physics one must accept the existence of the so-called deflection effect at reflection from a mirror (reflecting surface) moving relative to the light beam path. The formulas were derived by Klinkerfues [14] and Fizeau [15]. Starlight deflects additionally in the plane of incidence-reflection always in the direction of the mirror speed V: from the mirror (to the normal) when reflected from the front side and to the mirror (from the normal) when reflected from the back side. The angular deflection (Fizeau's formula) is

$$\alpha_d = 2\frac{V}{c}\cos\iota\sin\lambda \tag{2}$$

where ι is the angle between the mirror surface and the light beam (direction on a source or star) before reflection, and λ is the angle between the mirror surface and the direction of speed V. Either angles are measured in the mirror frame. Note that angle α_d arises and is also measurable in the mirror frame. For starlight the speed V was related to the Sun. And the formula

$$\beta = \frac{V}{c}\sin\varphi = 2\frac{V}{c}\cos\iota\sin\lambda - \frac{V}{c}\sin\psi \tag{3}$$

was used [16] for the aberration effect (where φ is the angle between speed V and direction on the star, and ψ is the angle between speed V and lunette axis).

The deflection effect of starlight was observed and formula (3) was used and confirmed for different angles ι and λ. LOEWY and PUISEUX [16] and COMSTOCK [17] tested whether a reflected starlight beam behaved in the same manner as a direct beam from the aberrational point of view. LOEWY and PUISEUX fixed a prism with silvered sides in front of the equatorial objective. Two stars as one pair, specially chosen, were observed simultaneously. The difference between angles φ and ψ, for each of these stars, amounted to tens of degrees. The aberration effect as measured directly with their instrument was as if the instrument formed the angle φ with speed V. Therefore, two effects took place: first, the deflection effect at reflection from the prism and, next, the pure aberration effect inside the instrument but for angle ψ. COMSTOCK placed a set of three mirrors in front of the equatorial telescope objective. Two stars distant by about 120° to one another were observed simultaneously. The result was the same as those in the experiments by LOEWY and PUISEUX.

Equation (2) tells us that $\alpha_d = 0°$ independently of what is the angle ι when the reflecting surface is parallel to speed V ($\lambda = 0°$). Similarly, $\alpha_d = 0°$ independently of what is the angle λ when $\iota = 90°$. The latter case takes place in the Reflex Zenith Tube and was already confirmed by AIRY'S [18] and ROSS' [19] observations.

Another confirmation of the deflection effect can be found, for instance, in the first experiment by KLINKERFUES [20]. The transit instrument with broken tube was used to observe passing of the Sun (at noon) and two stars (at midnight) through the meridian on June 12, 1987. Then, the level part of the tube, between prism and focal plane, was parallel or almost parallel to the line of the Earth motion along its orbit; thus the aberration effect could not occur along that part of the tube. This deficiency was fully compensated by the deflection effect ($\lambda = 45° = \iota$). Besides, if the deflection effect did not exist, the aberration effect of starlight would have to be observed when the starlight beam, after perpendicular reflection from a plane mirror, falls into the lunette oriented perpendicularly to speed V.

4. Deflection effects at reflection from a moving cube corner reflector

The signal is perpendicular to the reflector base and the latter is parallel to the satellite speed V_s when the satellite is in zenith. We shall define the resultant deflection effect being an algebraical sum of deflections after three reflections. We use the same symbols and derived relationships as those in Appendix with changed numerical subscripts for the latter ones. We take $V_s = 7$ km/s and $h = 1000$ km for the numerical values. Formulas (2) and (A12) give the formula

$$\alpha_{a,b,c} = 2\frac{V_s}{c}\cos\iota_{a,b,c}\sin\lambda_a\sin\Phi_{a,b,c}, \tag{4}$$

for defining the single deflection effects. To abridge the symbols, these effects will be denoted by *a*, *b*, and *c* only. The orientation of the reflector base is as in Fig. A1**a**. Formula (4) gives:

$$a = 6.417127'', \quad b = 3.208564'', \quad c = 3.208564''. \tag{5}$$

There are six combinations-successions of reflection: *a–b–c*, *a–c–b*, *b–a–c*, *c–a–b*, *b–c–a*, and *c–b–a*. The first two ones are symmetrical to spped V_s (Fig. 1A**b**); similar symmetries are made by the combinations of third with fourth and fifth with sixth. Only the *a–b–c* succession of reflection will be considered in detail.

The *a–b–c* succession of reflection. The standard signal and the signal to be deflected reflect first from the surface S_a. Therefore, the signal is deflected downwards in the incidence-reflection place P_a related to the standard signal by the angular distance equal to *a*. Next, the standard signal pierces the place π_a, perpendicular to it after first reflection (Fig. 1**a**). The deflected signal pierces π_a at

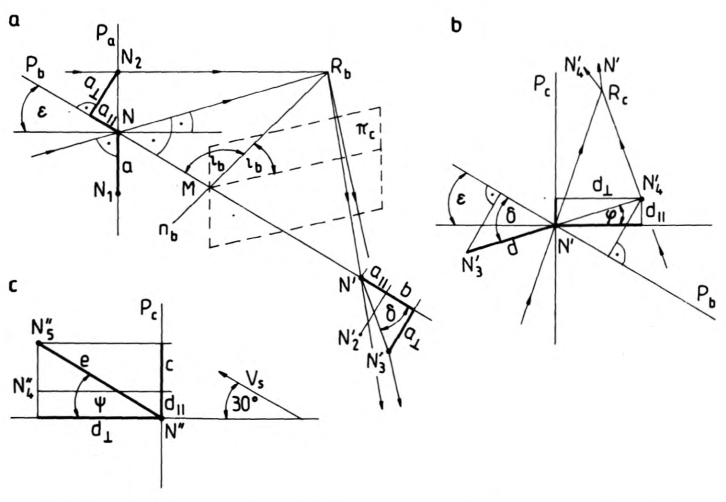

Fig. 1. Signals pierce the plane π_a at N (standard) and at N_1 (deflected and shifted to N_2) after first reflections and at N′ and at N'_3 after second reflections – **a**. In the plane π_c after second reflections – **b**. Signals pierce the reflector base at N″ (standard) and at N''_5 (deflected) – **c**

N_1. For these two signals to have a common point of reflection at R_b from the second reflecting surface S_b, N_1 is shifted to N_2 ($N_1N = NN_2$). The angular deflection *a* in P_a is factorized on the parallel $a_{\parallel}$, and perpendicular $a_{\perp}$ components relative to the incidence-reflection plane P_b connected with S_b

$$a_{\|} = a\cos(90° - \varepsilon) = a\sin\varepsilon = 3.260784'', \tag{6a}$$

$$a_{\perp} = a\sin(90° - \varepsilon) = a\cos\varepsilon = 5.526917'' \tag{6b}$$

where $\varepsilon = 30.539858°$ (formula (A7)). After reflections at R_b the signals will be relative to P_b at N′ and N'_2, respectively. But to the component a the second deflection effect equal to b must be added algebraically. The reflection occurs from the back side relative to speed V_s. Therefore, this signal deflects towards S_b (from normal n_b). Thus, after two reflections, the deflected signal at N'_3 is distant by

$$d = N'N'_3 = [a_{\perp}^2 + (a_{\|} + b)^2]^{1/2} = 8.508776'' \tag{7}$$

from N′, and under the angle

$$\delta = \arctan[a_{\perp}/(a_{\|} + b)] = 40.508051° \tag{8}$$

relative to P_b. Next, the right hand side of Fig. 1**a** is carried on plane π_c (the left hand side in Fig. 1**b**). Our sigth always follows the signals. For the two signals to have a common point of reflection at R_c from the third surface S_c, the deflection signal is shifted from N'_3 to N'_4. The deflecting of the signal relative to P_b (d, δ) is next related to the incidence-reflection plane P_c of S_c with parallel $d_{\|}$ and perpendicular $d_{\perp}$ components:

$$d_{\|} = d\sin\varphi = 1.472881'', \tag{9a}$$

$$d_{\perp} = d\cos\varphi = 8.380328'' \tag{9b}$$

where

$$\varphi = \delta - \varepsilon = 9.968193° \tag{10}$$

is the angle between $N'N'_4 = d$ and the line parallel to the bottom edge of S_c (perpendicular to P_c). After reflection at R_c the standard signal pierces the reflector base at N″ (Fig. 1c). The third deflection effect equal to c in P_c added to $d_{\|}$, and the deflected signal pierces this base at N''_5 instead of at N''_4; here, the reflection is from the back side and the signal deflects toward S_c (that is, outside the reflector). The bottom edge of S_c forms an angle of 30° with speed V_s. The deflected signal is angulary distant by

$$e = [d_{\perp}^2 + (d_{\|} + c)^2]^{1/2} = 9.599261'' \tag{11}$$

from N″ and under the angle

$$\psi = \arctan[(d_{\|} + c)/d_{\perp}] = 29.188739° \tag{12}$$

with the line parallel to the bottom edge of S_c. The difference between the direction of speed V_s and the deflected signal is

$$\Delta_{abc} = \psi - 30° = -0.811261°. \tag{13}$$

Thus, the deflected signal is distant by 9.599261″ (≡ 46.54 m) from the laser under the angle of 0.811261° ($\equiv$ 0.60 m from thè line of speed V_s) on the left of the direction of speed V_s. The *a–c–b* succession of reflection is symmetrical to the *a–b–c* one, and the signal reaches the Earth at the same distance from the laser but on the right of the direction of speed V_s.

The *b–c–a* succession of reflection. The procedure is similar. The signal is deflected upwards after first reflection from S_b. At the second reflection, from S_c, the signal deflects towards S_c and it has the component $(c-b_{\|})$ in plane P_c, and $\varphi = \delta + \varepsilon$. After piercing the reflector base, the deflected signal is distant by 9.599261″ ($\equiv$ 46.54 m) from the laser under the angle of 0.268440° ($\equiv$ 0.22 m from the line of speed V_s) on the left of the direction of speed V_s. The *c–b–a* succession of reflection gives the same distance from the laser but on the right of the direction of speed V_s.

The *b–a–c* succession reflection. We have here a left-sided direction of reflections in comparison with *a–b–c* and *b–c–a*. Besides, we have: $(b_{\|}+a)$ and $\varphi = \varepsilon - \delta$. The signal is deflected by 9.677911″ ($\equiv$ 46.92 m) from the laser under the angle of 0.539858° ($\equiv$ 0.44 m from the line of speed V_s) on the right of the direction of speed V_s. The *c–a–b* succession of reflection gives the same distance from the laser but on the left of the direction of speed V_s.

Concluding, the reflected signal (its centre) is divided into six parts that reach the Earth at the vertices, say, generally, of the asymmetrical hexagon relative to point K distant by d_s (see formula (1b)) from the laser and laying in the direction of speed V_s. At other orientations of the base edges, the vertices change their positions but their distances from the laser change within an interval of ± 2 m.

5. Prism cube corner reflector

Our derivations in Section 4 are related to a mirror cube corner reflector. When a prism is used, a new effect occurs at leaving the prism by the signal if the resultant deflection effect is not null. Then, inside the prism, the signal path after third reflection is not parallel to its path before first reflection. Due to the reverse reflection, this difference must be n times greater outside the prism than inside the prism, n is the refractive index. Thus, all the distances from the laser, derived in Sect. 4, must be multiplied by n. Such an effect cannot exist in STR, because the "aberrational reflection" takes place beyond the reflector.

6. Inclination of the reflector base to the signal

Let us now assume that the signal is not parallel to the axis of cube corner reflector. We take here two situations: i) the reflector base forms the angle χ with the plane perpendicular to the zenith when the satellite is in zenith and the signal is transmitted from L in Fig. 2a, and ii) the reflector base is parallel to the zenith at L but the signal is transmitted from, say, another laser L′ so that the signal forms the angle σ with the zenith at L. Each of these angles can, of course, lie in any plane (in Fig. 2**a** this plane is parallel to speed V_s). The deflection effect we leave aside, here. Then, the two situations give the effects of the same sort because all possible changes take place inside the reflector and these situations reduce in principle to the angle between the signal and the reflector axis.

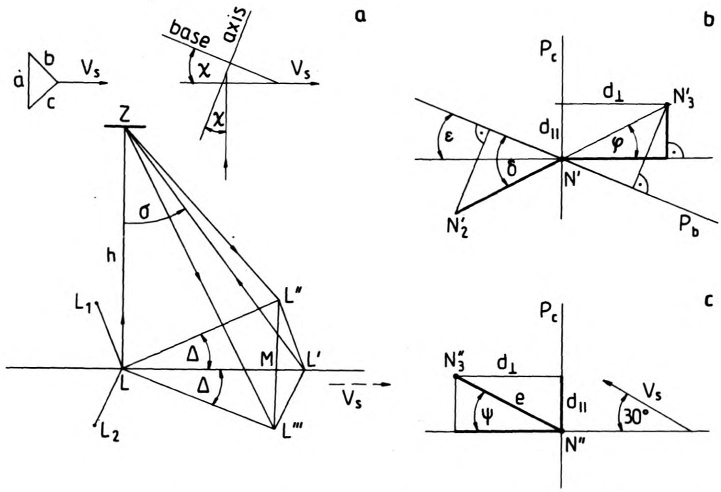

Fig. 2. Laser signal from L′ forms the angle σ with altitude h and as divided returns to L″ and L‴ after reflection at Z, the signal returns to L_1 and L_2 when the reflector base forms the angle χ with the speed V_s and the signal is emitted from L – **a**. Standard and deviated signals pierce the plane π_c at N′ and at N'_2 (shifted to N'_3) after second reflections – **b**. The same signals pierce the base at N′ and at N_3 after third reflections – **c**

The second situation with the *a–b–c* succession of reflection is considered more thoroughly. The standard signal is transmitted from L and forms the angle $\iota_a = 35.26439°$ with the first reflecting surface S_a. The signal from L′ forms the angle σ ($= 5°$ and $20°$) with the satellite altitude h and with the reflector axis; its angle with S_a must be $\iota_a + \sigma$ in Fig. 2**a**. The angles of both signals are kept after the reflection from S_a. From this moment we proceed as at defining the deflection effect in Sect. 4; we assume as if the angle σ was the deflection effect after first reflection ($a = 5°$, $20°$). Only this effect is to be "carried" till piercing the reflector base by the emerging signal. Using the same symbols as in Sect. 4 (see Fig. 2**b**, **c**) we get the distance of the deviated signal equal to

$$e = a = 5° \quad (20°) \tag{14}$$

from the standard signal at N″ (in Fig. 2**c**) and under the angle

$$\psi = \arctan(d_\parallel / d_\perp) = \varphi = \delta - \varepsilon = 28.920284° \quad (28.920284°) \tag{15}$$

with the line parallel to the bottom edge of S_c. The difference between the chosen direction (or speed V_s) and the deviated signal is

$$\Delta = \psi - 30° = -1.079716° \quad (-1.079716°). \tag{16}$$

Thus, the angular distance of the returning signal, from laser L, is equal to the distance of laser L′ from L. However, the signal reaches the Earth not at L′ but at L″ (Fig. 2a) under the angle of 1.079716° on the left of line LL′. When the succession of reflection is *a–c–b*, the signal reaches the Earth at L‴ (LL‴ = LL″) symmetrically to L″ relative to line LL′. When the first reflections are from the other reflecting surfaces, S_b and S_c, the returing signal reaches also points L″ and L‴, respectively.

The received effect is independent of the state of motion of the reflector relative to the signal path. Instead of the direction LL′ parallel to a potential speed V_s we may take any other direction, and the same result is received: division of the signal centre into two parts symmetrically relative to this chosen direction. This deviation angle Δ is always the same and is of a constant value, independent of angle σ.

In these conditions the linear distances LL″ and LL‴ are functions of angle σ and of altitude h. Making use of cosine law for the triangle LL′L‴ we get (LL′ = LL″ = h tan σ)

$$\mathrm{L'L''} = \sqrt{2}h \tan\sigma (1-\cos\Delta)^{1/2}. \tag{17}$$

The values of LL′ and L′L″, L″L‴ and L′M for different angles σ are presented in Tab. 1.

Table 1. Distances of the returning signal centre divided into two parts from a laser for different angles σ when the altitude of a satellite is h = 1000 km; meaning the symbols as in Fig. 2a

σ [min, deg]	LL′ [m]	L′L″ [m]	L″L‴ [m]	L′M [m]
0.1 mrad = 20.626″	100.00	1.88	3.77	0.018
5′	1,454.44	27.41	54.81	0.26
10′	2,908.89	54.82	109.63	0.52
15′	4,363.35	82.22	164.44	0.77
20′	5,817.83	109.63	219.56	1.03
30′	8,726.87	164.45	328.89	1.55
5°	87,488.66	1,648.66	3,297.18	15.53
20°	363,970.23	6,858.74	13,716.93	64.62

Usage of a prism reflector instead of a mirror reflector changes nothing because the angle Δ is the change of the direction of the outlet from the reflector base. It is not the change of the angular distance of the deviated signal from the standard one.

In the first situation, the angle χ is formed by the reflector base and speed V_s as well as by the reflector axis and the signal. Here, the centres of the returning and divided signal will be at $\mathrm{L_1}$ and $\mathrm{L_2}$ ($\mathrm{L_1 L = L_2 L = L'L''}$).

7. Broadening of the vertex angle of the reflecting back surface

The bottom edge of the reflecting back surface S_a (Fig. A1**a**) is perpendicular to the possible speed V_s. The base of reflector is perpendicular to the signal path. Broadening of the vertex angle of S_a (of surface BCD in Fig. A1**a**) by the angle v (that is, to $90° + v$) causes: dropping of the vertex of S_a from D to D′, diminishing of the angle between the bottom and the side edges of S_a from 45° to $(45° - v/2)$, and increasing of the angle between altitude DE_a of S_a and axis DF from the angle ι (angle $E_a DF$) to the angle ι' (angle $E_a D'F$). The angles ι and ι' are defined by:

$$\sin \iota = E_a F / E_a D = \tan 30° / \tan 45°, \tag{18}$$

$$\sin \iota' = E_a F / E_a D' = \tan 30° / \tan (45° - v/2), \tag{19}$$

and hence

$$\Delta\iota = \iota' - \iota = \arcsin\left[\tan 30° \left(1 + \tan\frac{v}{2}\right)^2\right] - \arcsin(\tan 30°). \tag{20}$$

In the case of the Lageos satellite [2], $v = 1.5''$ so that $\Delta\iota = 1.0606''$. And now, we must differentiate three situations. The deflection effect we leave aside here.

First, when the third reflection is just from S_a (the *b–c–a* or *c–b–a* succession of reflection) the change of the inclination of S_a by $\Delta\iota$ causes deviation of the signal centre to the back by the angular distance equal to ($h = 5.900$ km)

$$2\Delta\iota = 2.121'' \ (\equiv 60.68 \text{ m}). \tag{21}$$

(Thus, in this case, the deflection effect will be diminished by $2\Delta\iota$ if it is not left aside).

Second, when the first reflection is from S_a (the *a–b–c* or *a–c–b* successions of reflection), after this reflection, we have a situation as that considered in Sect. 6 with $\sigma = 2\Delta\iota$. In this case, the centre of returing signal must be deviated to the front by $2\Delta\iota$ and divided into two parts (angle Δ, formula (16)). Thus, in this case, the deflection effect will be enlarged by $2\Delta\iota$ if not left aside.

Third, it is still otherwise when the succession of reflection are *c–a–b* (Fig. 3**abc**) or *b–a–c*. Now, the incidence-reflection plane P_{av} (precisely, the plane denoted by points Z, R_a, Z′ and M'_1, in which the line P_a lies parallelly to k_a which, in turn, is parallel to the bottom edge of S_a) forms the angle $\Delta\iota$ with the standard plane P_a (precisely, the plane denoted by points N, R_a and N′, in which the line P_a lies parallelly to k_a), Fig. 3**a**. Therefore, plane $NN'_1 R_a$ is a new incidence-reflection plane for the (standard) signal at N. The signal from N reflects at R_a and at N'_1 is distant by $N'N'_1 = 2M'M_1 = M'M_2$ linearly and by $2\Delta\iota$ angularly from the standard signal at N′. The points N′ and N'_1 are carried on plane π_b (Fig. 3**b**), point N'_1 is shifted to N'_2 for the two signals to have a common point of reflection (at R_b from S_b). The distance $d = N'_1 N'$ is perpendicular to P_a, so we have

$$\delta = 90° - \varepsilon + 59.46142° = \psi. \tag{22}$$

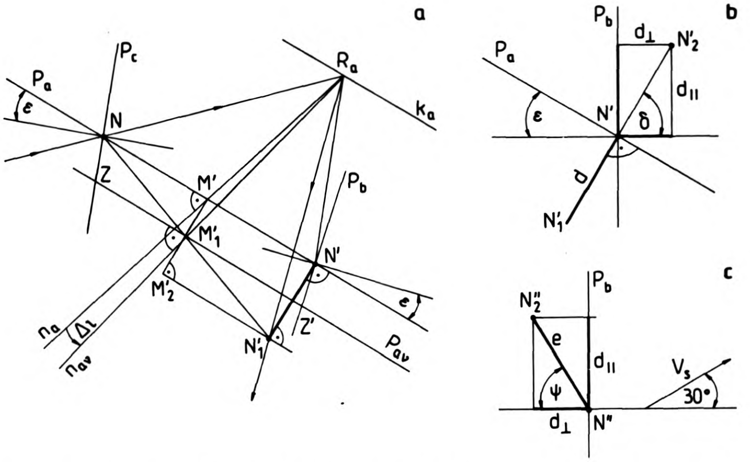

Fig. 3. Second reflection of the signal is at R_a in plane $NR_aN'_1$ from the back surface S_a whose angle with the reflector axis is broadened by $\Delta\iota$ – **a**. The same signal pierces: the plane π'_b at N'_1 (shifted to N_2) – **b**, and the base at N''_2 – **c**

Thus, the deviated signal from N'_2 pierces the reflector base at N''_2 (Fig. 3**c**) distant by

$$e = d = 2\Delta\iota = 2.121'' \ (\equiv 60.68 \text{ m}) \tag{23}$$

from the standard signal, but under the angle

$$\Delta_v = 180° - (30° + \psi) = 90.539508° \tag{24}$$

relative to speed V_s. Even the succession of reflection is *b–a–c*, the signal reaches the Earth symmetrically on the other side of the direction of speed V_s.

Concluding, the centre of returning signal is divided into five parts which reach the Earth at the vertices of symmetrical pentagon. It would be a hexagon [21] if all three reflecting surfaces were inclined relatively to the reflector base.

8. Discussion

The deflection effect in classical physics corresponds to the "aberrational refraction" in STR which shifts the signal centre to K from the laser (Eqs. (1a), (1b)). In the latter case, the signal centre is divided into six parts laying at the vertices of asymmetrical hexagon with K inside. Moreover, when the reflectors are prisms, all the calculated distances must be multiplied by the refractive index *n*. Thus, in the same technical and observational conditions the returning signal is more diffused and the signal falling into the receiver is more "weakened" in classical physics than in STR.

A substantional influence on diffusing, but first of all on the "weakening" the signal falling into the receiver, has the inclination of the signal relative to the reflector axis (angles σ and χ in Sect. 6), see Tab. 1. These angles can lie in separate planes, have different values, and be changing in time. The above effect influences the final effect (being an algebraical sum of all the effects), and it enlarges or diminishes or even neutralizes the other effects in a given situation. In practice, one may say, the satellites were in most cases beyond the zenith, or the bases of not all arrays of reflectors or/and the bases of single reflectors were not perpendicular to the signals. Such situations must exist in the case of the Lageos satellite on which the single prism reflectors were placed on two hemispheres with 60 cm in diameter [2]; here, angle σ can amount even to tens of degrees, in all directions (from 0° to 360°) relative to speed V_s. Also, on the Explorer XXII satellite the reflectors were placed on nine planes of an octagonal transcated piramid [4].

"To keep the receiving station in the beam's "footprint" for receipt of the signal, the rightangle corner at the back of each reflector had to be broadened by 1.5 seconds of arc" in the Lageos satellite [2]. Then, as it is shown in Sect. 7, the returning signal is divided into five parts. The diminishing of the deflection effect (or the aberration effect in STR), by the angle $2\Delta\iota$ ($\equiv 60.68$ m) is given only by this part of the reflected signal the third reflection of which is from the surface S_a (!). The remaining reflections enlarge the deflection effect or/and diffuse the returning signal. Note, displacement of the signal defined by $2V_s/c$ is equal to $7.563043''$ ($\equiv 216.33$ m) when $V_s = 5.5$ km/s ($h = 5.900$ km), ($V_s = [kM/(R+h)]^{1/2} = 5.706$ km/s for the circular orbit, k, M and R are, in turn, the gravitation constant, the Earth's mass and radius in the equatorial plane, respectively).

Changing of the orientation of the reflector axis relative to both the signal and speed V_s causes the changes of the deflection effects as all the angles ι and λ are changed then. The resultant deflection effect for three values (0°, 5°, 20°) of angles σ and χ are shown in Table 2 when the successions of reflection are *a–b–c* and *c–a–b*.

The photons of this part of a signal that covers the array of reflectors are distributed on all the single reflectors. In every reflector the photons are distributed in turn on the reflecting surfaces (at the first reflection). And on every such a surface one must differentiate the left and right directions of reflections. Each pulse has a definite time of duration, and in this time the satellite is moving. All these above influence the distribution of the photons of a pulse among the separate parts (having own cross-sections and centres) of the divided signal after leaving the array of retroreflectors. The quantitative effects (divisions), into how many parts will be divided the signal and with what intensities, depend on: how the reflectors are packed, what the surfaces of packing (plane, spherical) are, and what the inclinations of the surfaces of packing relative to the signal (angles σ and χ) are. Besides, the photons distribution among the reflecting surfaces and their halves of a given retroreflector comes under the law of probability; it is similar with the distribution of the photons of a whole signal, which covers the retroreflectors, among the individual retroreflectors. Summing up all the effects considered herein, it is not or cannot be strange that the return of a signal is not recorded in some

Table 2. Resultant deflection effect for three values (0°, 5°, 20°) of angles σ and χ and two successions of reflection (*a–b–c* and *c–a–b*) when the satellite altitude is $h = 1000$ km

Succession of reflection	σ [deg]	χ [deg]	Deflection effect [m]	Deviation from direction of speed V_s
a–b–c	0	0	46.54	−48′40.54″
	5	0	44.98	−1°04′46.49″
	20	0	38.96	−1°04′46.87″
	0	5	46.73	−46′25.41″
	0	20	45.98	−39′08.11″
	5	5	45.38	−1°04′46.42″
	20	20	42.29	−1°04′46.79″
	5	20	45.31	−1°04′46.21″
	20	5	40.03	−1°04′46.84″
c–a–b	0	0	46.92	−32′23.49″
	5	0	45.38	+1°04′46.01″
	20	0	39.41	+1°04′46.79″
	0	5	47.17	+32′23.49″
	0	20	46.58	+32′23.49″
	5	5	45.83	+1°04′46.01″
	20	20	42.98	+1°04′46.78″
	5	20	45.94	+1°04′46.04″
	20	5	40.53	+1°04′46.78″

situations, that its internal structure is or can be changed, that, generally, the further from the zenith the less intensity of the returning signal falling into the receiver.

9. Conclusions

The "aberrational refraction" has been used to explain many phenomena and effects. However, a detailed analysis [22], [23] of AIRY'S and KLINKERFUES' experiments with filled instruments, denies existing such an "aberrational refraction" for starlight; in classical physics there must exist the independence in AIRY'S and the dependence in KLINKERFUES' experiment of/on filling the instrument [24].

Different results of deviating of the signal, being an outcome of either "aberrational refraction" or deflection effect, give physical differentiation between STR and classical physics when the retroreflector is a prism. Therefore, some experimental possibility ought to exist for testing the two theories and confirming only one of them in the on-the-ground laboratory conditions with the help of laser light. One may assume a modified experimental arrangement based on the arrangements used by JONES [25], and DAVIES and JENNISON [26]: on the edge of a rotor there are two right corner reflectors, one as a set of two mirrors and the other one as a prism, with planes of normals parallel to rotation.

It would be useful to test with what phenomenon the deflection effect is connected: either a real change of the direction of reflection or "carrying" of the path of a signal (in some way similar, e.g., to an arrow shot perpendicularly to the bowman's speed). Then, in the empty lunette, in both cases, a starlight beam (or laser light beam) parallel to the Earth's motion along its orbit give no aberration effect after perpendicular reflection when the lunette is oriented perpendicularly to this motion direction. But, when the lunette is filled, this effect (its part) ought to be observed if there is "carrying" [24]; the transverse drag coefficient was $k_2 = 1 - 1/n$ in AIRY'S and KLINKERFUES' experiments [22], [23].

Appendix

Geometrical relationships in the cube corner reflector

Let us take an unmoving cube corner reflector (trihedral reflector) with the right vertex angles of reflection surfaces. The signal is perpendicular to the reflector base (plane made by the bottom edges of reflecting surfaces). The reflector is in zenith, and the normal from the vertex of D piercing the centre of base ABC at F (Fig. A1**a**) is the axis or/and altitude. In these conditions the angle ι_1 between entering signal and the first reflecting surface ($S_1 \equiv$ BCD) must be equal to the angle ι_3 between emerging signal and third reflecting surface ($S_3 \equiv$ ABD or ACD); the planes of incidence-reflection are perpendicular to these surfaces, respectively. The

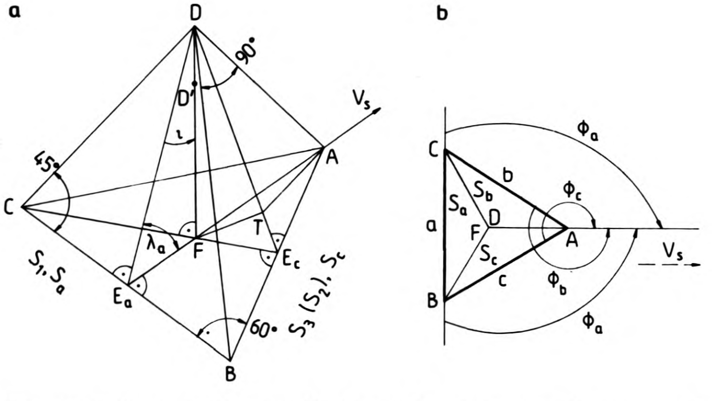

Fig. A1. Trihedral reflector with base ABC and vertex D, geometrical configuration to derive angular relationships — **a**. Assumed orientation of the reflector base relative to a chosen direction (to speed V_s) in derivations — **b** signal falling on S_1 is parallel to the axis DF, so $\iota_1 = \iota_3 = \iota$ and

$$\sin\iota = \mathrm{E}_a\mathrm{F}/\mathrm{E}_a\mathrm{D} = \mathrm{CE}_a\tan 30°/\mathrm{CE}_a\tan 45° = \tan 30°/\tan 45°, \tag{A1}$$

hence

$$\iota_1 = \iota_3 = \iota = 35.26439°. \tag{A2}$$

Let us find the angle ι_2 of the signal falling on and reflecting from the surface S_2 ($\equiv$ ACD or ABD). In vertical (top) view (Fig. A2**a**) the signal reflects at G from S_1, at K from S_2 and at H from S_3. This projection forms the angle of 30° with

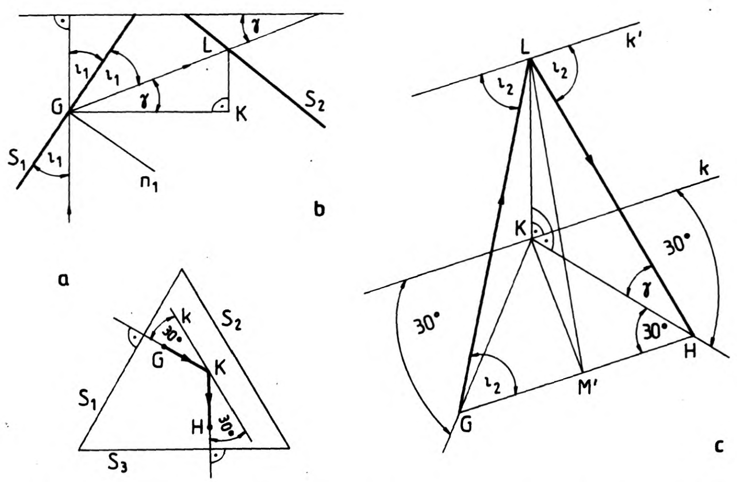

Fig. 2A. Geometrical configurations to find the angle ι_2 at second reflection from S_2: **a** – projection of signal path GKH in vertical view, **b** – side view to find the angle γ, **c** – connection of **a** and **b** to find the angle ι_2

the line k parallel to the bottom edge of S_2. In the side view (Fig. A2**b**) the signal forms the angle

$$\gamma = 90° - 2\iota_1 = 19.47122° \tag{A3}$$

with its vertical projection GK (or HK) and reflects from S_2. In Fig. A2**c** lines k and k' are parallel to GH and to the bottom edge of S_2, and the signal forms the to-be-defined angle ι_2 with S_2. From triangles GLM′ and GKM′ (GM′ = HM′) we get

$$\cos\iota_2 = \mathrm{GM'}/\mathrm{GL} = \mathrm{GM'}\cos\gamma/\mathrm{GK} = \cos\gamma\cos 30° \tag{A4}$$

and hence

$$\iota_2 = 35.26439°. \tag{A5}$$

Let us take a plane π_1 (π_3) perpendicular to the path of a signal reflected from S_1 (falling on S_3), Fig. A3**a**. The signal pierces π_1 (π_3) at N. The incidence-reflection planes P_2 (connected with the surface S_2) and P_1 (P_3), (connected with the surface S_1 (S_3)) and the plane perpendicular to P_1 (P_3), the three planes have common line NN^1 of intersection being the signal path. In Fig. A3**a** the projection of this path on each of the lines being the intersections of these three planes with the plane π_1 (π_3) is equal to zero. It is like this when the signal is perpendicular to the reflector base. The line NN^1 corresponds to the lines GL and HL in Fig. A2**c**, and the point N corresponds to the point G (or H). Note, the planes π_1 and π_3 are in some way "symmetrical" to the signal falling on and reflecting from S_2.

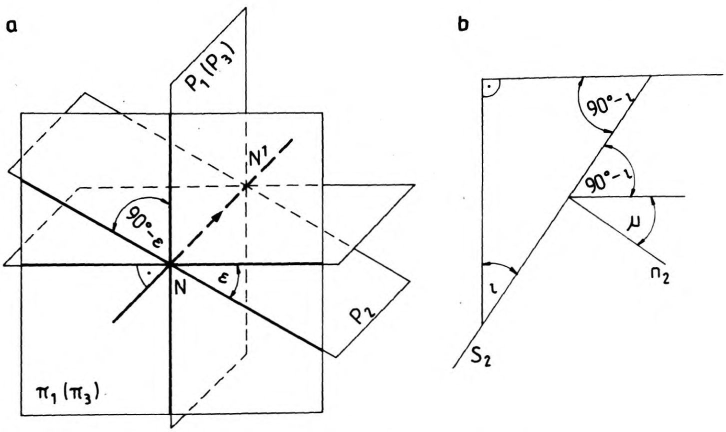

Fig. 3A. Incidence-reflection planes P_2 and P_1 (P_3) from the angle $(90°-\varepsilon)$ in the plane π_1 (π_3) perpendicular to the signal path NN^1 – **a**. Configuration for finding the angle μ between normal n_2 to surface S_2 and the line parallel to the reflector base – **b**

It is necessary to find the angle ε between P_2 and plane perpendicular to P_1 (P_3), lying in the plane π_1 (π_3). The plane P_2 forms the angle (see Fig. A3**b**)

$$\mu = 90° - (90° - \iota) = \iota = 35.26439° \tag{A6}$$

with the plane parallel to the reflector base. If the bottom edges of S_1 and S_3 were perpendicular to the bottom edge of S_2, we would have $\varepsilon = \mu$; if those edges were parallel, $\varepsilon = 0°$. Since these edges form angles of 60°, we have

$$\varepsilon = \mu \sin 60° = 30.359858°. \tag{A7}$$

All the relationships derived above and the signal itself (perpendicular to the base) are called <u>the standard</u> ones. Then, we have an ideal (pure) retroreflection. These relationships will be a basis for finding deviations from the standard retroreflection when: i) the deflection effect must be taken into account (Sect. 5), ii) the signal is not parallel to the reflector axis (Sect. 6), iii) the vertex angle of the reflecting back surface is not right (Sect. 7), and iv) we search for a change of the deflection effect when the signal is not parallel to the reflector axis.

To calculate the deflection effect we must know the angles (λ_a, λ_b, λ_c) of all three reflecting surfaces with speed V_s. We select one of the infinitely many possible orientations of the reflector base edges relative to the direction of the satellite speed V_s. One edge, edge a, is perpendicular to this direction, $\Phi_a = \pm 90°$ (Fig. A1**b**). Then, the other two edges, b and c, form the angles $\Phi_b = 210°$ and $\Phi_c = -210°$. The angle λ_a, angle DE_aF in Fig. A1**a**, is equal to

$$\lambda_a = 90° - \iota_a = 54.73561° \tag{A8}$$

($\iota = \iota_1 = \iota_a$) when the chosen direction is FA in Fig. A1**a**. The angles λ_b and λ_c are symmetrically inclined to this direction. The angle λ_c, angle FAT, lies in the plane denoted by lines FA and FT (perpendicular to the latitude DE_c of S_c). The triangle AFT (with the help of triangles AE_cF, AE_cT and E_cFR) and the law of cosines give

$$\cos\lambda_c = \left(\frac{1}{\cos^2\Phi_c} + \frac{1}{\cos^2\varrho} - \tan^2\Phi_c \sin^2\lambda_c\right)\cos\Phi_c \cos\varrho/2 \tag{A9}$$

where the angle ϱ (angle E_cAT) is defined by

$$\tan\varrho = \tan\Phi_c \cos\lambda_a \tag{A10}$$

and is equal to 18.434949°. Formula (A9) gives:

$$\lambda_c = -(24.094843° + 180°) = -204.094843°, \tag{A11a}$$

$$\lambda_b = (24.094843° + 180°) = 204.094843°. \tag{A11b}$$

The same values one gets from the formula

$$\sin\lambda_{c,b} = \frac{FT}{FA} = \frac{\tan\Phi_{c,b}\sin\lambda_a}{1/\cos\Phi_{c,b}} = \sin\lambda_a \sin\Phi_{c,b}. \tag{A12}$$

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Об эффектах при отражении световых сигналов от трехгранных рефлекторов на спутниках когда рассмотрены в классической физике

Аномалии наблюдаемые при повороте лазерных сигналов после отражения от трехгранных рефлекторов на спутниках дискутируются и объясняются на почве классической физики. Рассуждеия опираются на эффект дефлекции при отражении от движущейся отражающей поверхности; этот эффект некоторым способом корреспондирует с „эффектом аберрационного преломления" в теории относительности, однако он в *n* раз больше если рефлектором является призма.