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Binary holographic lens — a study of the image quality *

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The imaging quality obtainable by using the binary holographic lenses was investigated. A suitable algorithm for determining the amplitude distribution in the image plane of the point object has been elaborated. It makes it possible to estimate the quality of the investigated holographic lenses. A few holographic lenses have been investigated. The case of a lens for which the coma correction is obtained via the relevant modification of the hologram-forming fringes rather than by recording the hologram on a curved surface was also included.

1. Introduction

The methods of construction of holographic lenses, meeting relevant requirements ragarding the correction of particular aberrations, have been known for years [1]–[4]. The lens made on a spherical surface provides some extra possibilities of correction, but its production involves great technological difficulties. There are no such difficulties in the case of computer-generated hologram. Then, the distribution of interferential fringes can be transformed so that a plane hologram will produce an image identical with that produced by a hologram made on a sphere. The assumed working conditions of the lens determine the "recording" and reconstruction geometry. On this basis the distribution of fringes on a hologram is calculated. The distribution is then drawn and photographed. The rectangular transmittance can be treated as a deformation of the sinusoidal transmittance, that is why the binary lens can be expected to produce a poorer imaging than the sinusoidal transmittance lens.

2. Numerical calculations

In order to analyse numerically the quality of the binary holographic lens, an algorithm for calculating the amplitude (and light intensity) distribution in the image of a point object was elaborated. A similar algorithm for the sinusoidal transmittance lens can be found in paper [5]. Based on the light amplitude distribution in the image of a point object, the image of any extended object can be calculated.

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The process of calculating the light amplitude distribution in the image plane can be divided into two stages. Given the positions of the reference and object wave sources, the curvature radius, the position and diameter of the entrance pupil, it is possible to determine the diffraction structure of the binary lens, that is, the positions of the interferential fringes which form the lens. The positions are determined from the following equation:

$$\begin{aligned} &R_p - R_r = n\lambda_1, \\ &R_p = [(X(n)-X_p)^2+(Z(n)-Z_p)^2]^{1/2}, \\ &R_r = [(X(n)-X_r)^2+(Z(n)-Z_r)^2]^{1/2} \end{aligned} \tag{1}$$

where: n – integer,
λ_1 – wavelength used for recording,
(X_p, Z_p) – object wave source coordinates,
(X_r, Z_r) – reference wave source coordinates,
$(X(n), (Z(n))$ – coordinates of the n-th bright fringe of the lens.
In the considered case $X_p = 0$, $X_r = 0$.

At the second stage we calculate the amplitude distribution of the reconstruction wave, which was subject to diffraction at the diffraction structure of the lens determined earlier. This distribution is calculated in the following way: In the image plane the amplitudes of all the rays which passed throught the bright fringes are summed up. Thus, summation is done with respect to all the bright fringes. The resultant amplitude is given by the following equation:

$$U(n, X_3, Z_3) = \sum_n \cos[\varphi_2(n, X_3, Z_3)] + i\sum_n \sin[\varphi_2(n, X_3, Z_3)], \tag{2}$$

$$\begin{aligned} \varphi_2(n, X_3, Z_3) = k_2 \{&[(Z(n)-Z_c)^2+(X(n)-X_c)^2]^{1/2} \\ &+[(Z(n)-Z_3)^2+(X(n)-X_3)^2]^{1/2}\} \end{aligned} \tag{3}$$

where: λ_2 – wavelength used for reconstruction,
(X_c, Z_c) – reconstruction wave source coordinates,
(X_3, Z_3) – investigated point in the image,
$k_2 = 2\pi/\lambda_2$.

The image of extended object was calculated as the convolution of the point spread function and the object transmittance (amplitude transmittance in coherent light and intensity transmittance in incoherent light). The calculations were done for the one-dimensional case. Such a procedure considerably reduces the computation time and the results can be extended to a two-dimensional case. The quality of the lenses was estimated from the light intensity distribution, they produced similarly as in papers [5], [6], for the analytical estimation of this distribution, the following parameters were used: maximum light intensity I_{max}, intensity distribution moments M_1, M_2, M_3 of the 1st, 2nd and 3rd orders, respectively, and spot diameters which contain 80% of energy, $d_{0.8}$. As binary lens may work both in coherent and incoherent light, then, for the sake of a more accurate estimation of its quality, the image of a two point object was estimated both in coherent and incoherent light and the incoherent modulation transfer function MTF was calculated.

For the correction of binary holographic lens, aberration coefficients determined for the classical holographic lens were used. This was justified inasmuch as the rectangular transmittance in binary lenses can be treated as the deformation of the sinusoidal transmittance in classical lenses. Since classical holographic lens aberration coefficients were used for the correction of binary holographic lenses, the terms "aplanatic lens", "corrected astigmatism", etc., refer to classical lenses corresponding to the binary lenses under consideration.

As it has been mentioned earlier, by a proper transformation of the distribution of interferential fringes (which can be done because the lens is to be drawn), it is possible to obtain a plane lens which produces an image identical with the one given by the spherical lens. This transformation is based on the fact that two lenses will produce identical images if they have the same phase distribution in the image plane. The positions of the transformed fringes can be found by comparing the image plane phase expressions of the spherical lens with those for the plane lens. The distribution produced by the spherical lens being given, we obtain the equation with one unknown which is the position of the transformed fringe. The phase distribution in the image plane produced by binary spherical holographic lens $\varphi_2(n, X_3, Z_3)$ is given by Eq. (3). After the transformation made on a fringe system plane, the distribution must remain the same with the accuracy of $2\pi m$ (where m is an integer)

$$\varphi_2'(n, X_3, Z_3) = \varphi_2(n, X_3, Z_3) + 2\pi m; \tag{4}$$

φ_2' (n, X_3, Z_3) can be presented in form of the following equation:

$$\varphi_2'(n, X_3, Z_3) = k_2\{[Z_c^2 + (X'(n) - X_c)^2]^{1/2} + [Z_3 + (X'(n) - X_3)^2]^{1/2}\}. \tag{5}$$

By solving the system of Eqs. (4) and (5) the coordinates $X'(n)$ of the centres of the bright fringes on a plane hologram can be obtained. Solution of this system is reduced solving the quadratic equation

$$a X'(n) + b X'(n) + c = 0 \tag{6}$$

where:

$$a = (X_c - X_3)^2 - \frac{\varphi_2^2(n, X_3, Z_3)}{k_2^2},$$

$$b = (X_c - X_3)(Z_c^2 - Z_3^2) - (X_c + X_3)\left(\frac{\varphi_2^2(n, X_3, Z_3)}{k_2^2} + X_c X_3\right) + X_c^3 + X_3^3,$$

$$c = \frac{1}{4}(Z_c^2 + X_c^2 - Z_3^2 - X_3^2)^2 - \frac{1}{2}(Z_c^2 + X_c^2 + X_3^2 + Z_3^2)\frac{\varphi_2^2(n, X_3, Z_3)}{k_2^2} + \frac{1}{4}\frac{\varphi_2^4(n, X_3, Z_3)}{k_2^4}.$$

3. Results of numerical calculations. Conclusions

The geometries of "recording" and reconstruction of the lens are presented in Tab. 1 (t – position of the entrance pupil, D – hologram diameter, ϱ – radius of sphere). The parameters of the calculated intensity distributions are given in Tab. 2

Table 1. Investigated geometries of "recording" and reconstruction

No.	X_c	X_c/Z_c	Z_p	Z_r	Z_c	t	D	ϱ
1a	0	0	200	-200	-200	0	20	∞
1b	4	0.02	200	-200	-200	0	20	∞
2a	0	0	150	-300	-300	0	15	∞
2b	6	0.02	150	-300	-300	0	15	∞
3a	–	0	100	$-\infty$	$-\infty$	0	10	∞
3b	–	0.03	100	$-\infty$	$-\infty$	0	10	∞
3b′	–	0.03	100	$-\infty$	$-\infty$	-50	10	∞
3b″	–	0.03	100	$-\infty$	$-\infty$	0	10	100
3b‴	–	0.03	100	$-\infty$	$-\infty$	-100	10	200

Table 2. Parameters of the calculated intensity distributions

No.	I_{max}	M_1-X_g	M_2	M_3	$d_{0.8}$	δ_{nc}	δ_c
1a	1	0	62.85×10^{-6}	0	0.009	0.014	0.021
1b	0.458	29.74×10^{-6}	126.89×10^{-6}	6.59×10^{-10}	0.014	0.02	0.03
2b	0.750	41.61×10^{-4}	80.43×10^{-6}	9.89×10^{-9}	0.013	0.019	0.025
3b	0.709	54.53×10^{-4}	79.49×10^{-6}	5.41×10^{-8}	0.013	0.019	0.026
3b′	0.921	41.72×10^{-4}	65.9×10^{-6}	1.06×10^{-7}	0.011	0.017	0.023
3b″	0.794	13.1×10^{-6}	74.48×10^{-6}	1.93×10^{-9}	0.011	0.015	0.022
3b‴	0.907	37.92×10^{-4}	64.79×10^{-6}	1.53×10^{-8}	0.011	0.014	0.022

(X_g – position of the "centre of gravity" of the aberration spot, δ_{nc} and δ_c values of the resolution in incoherent and coherent light, respectively). All the values are given in millimetres. In aberration free case $X_c/Z_c = 0$, the results of numerical calculations being identical for all lenses. Therefore they are presented only for the lens No. 1. The analysis was started with lens No. 1, because this type of lens is most frequently considered in publications. It is an aplanatic but highly astigmatic lens. In such lenses the maximum field angle, for which the results of calculations are presented, is 0.02 rd, for greater angles the imaging quality is markedly deteriorated because of astigmatism. The intensity distribution in the aberration spot of this lens is presented in Figs. 1**a** and 1**b**, and the incoherent modulation transfer function – in Fig. 1**c** and Fig. 1**d**.

In order to improve the imaging produced by the lens under consideration, the coma correction condition was rejected and replaced by the reduction of astigmatism (lens No. 2). The intensity distribution in the aberration spot as well as the incoherent modulation transfer function (MTF) of this lens are presented in Figs. 2**a** and 2**b**, respectively. The intensity distribution shows that the neglecting of the coma correction condition has caused a disarrangement of the spot symmetry, but the decrease of the maximum intensity in the spot with the increase of the field angle is much slower. All the other analysed lenses are of collimator type. Lenses of this type were analysed for field angles of 0.03 rd, because only then there occurred any

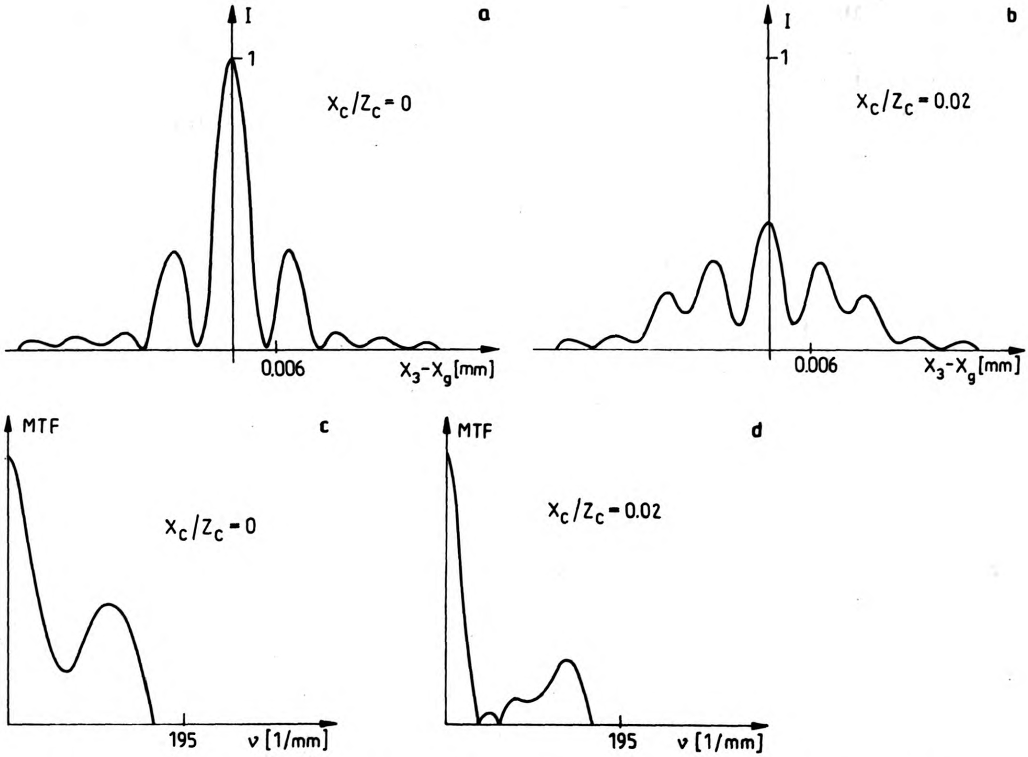

Fig. 1. Light intensity distribution (I) in the aberration spot and incoherent modulation transfer function (MTF). For example No. 1

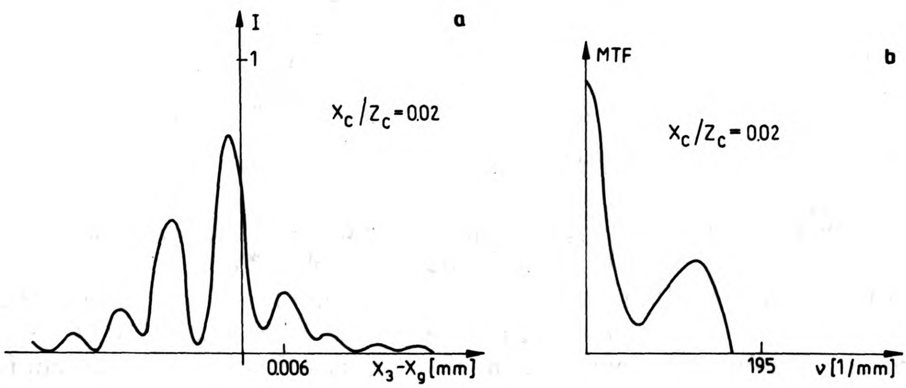

Fig. 2. The same as in Fig. 1, but for example No. 2b

distinct lowering of the imaging quality. First of these lenses, 3b, like the two discussed above, is planar and the entrance pupil coincides with the plane of the lens. The intensity distribution in the aberration spot as well as the incoherent modulation transfer function of this lens are presented in Figs. 3**a** and 3**b**, respectively. The lens

3b′ is also planar, but it is the entrance pupil that is shifted in order to correct the astigmatism. The intensity distribution calculated for this lens and the MTF are presented in Figs. 4**a** and 4**b**, respectively. The intensity distribution shows that by correcting the astigmatism the imaging quality is considerably improved; this is

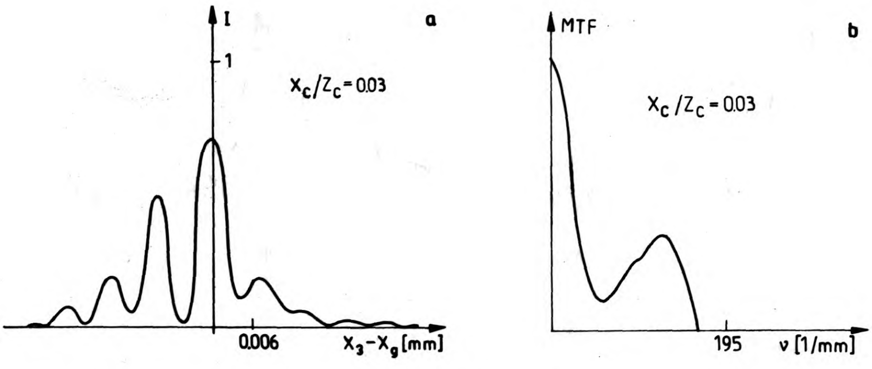

Fig. 3. The same as in Fig. 1, but for example No. 3b

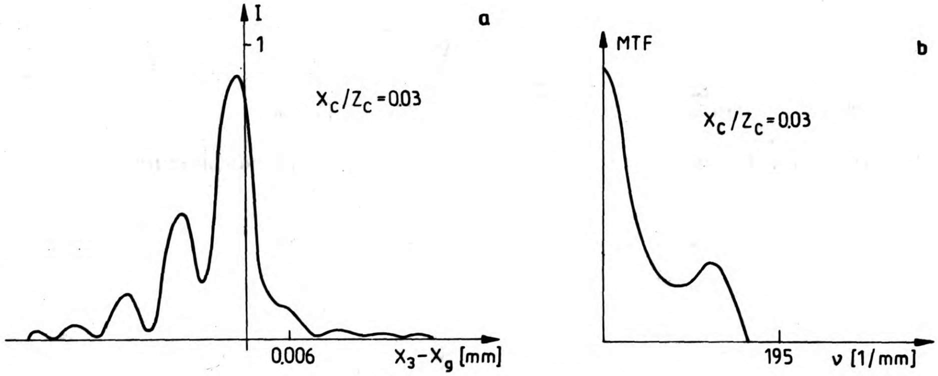

Fig. 4. The same as in Fig. 1, but for example No. 3b′

manifested in the increase of I_{max} and the improved resolution. The next step was to estimate the quality of the collimator lens made on "a sphere" of radius correcting the coma. In fact, the lens with transformed fringe distribution was investigated. The intensity distribution for such a lens (3b″), is presented in Fig. 5**a**, and the MTF – in Fig. 5**c**. The distribution shows that the coma was considerably corrected (the spot is symmetrical) and that the I_{max} increased. A partial correction was done by reducing the coma and the astigmatism by half. For such a partially corrected lens, the intensity distribution in the aberration spot and the MTF are presented in Fig. 6**a** and Fig. 6**b**, respectively. The quality of this lens is intermediated between that of the lens with corrected astigmatism and the quality of the lens with corrected coma. For the binary holographic lens the intensity distribution in the aberration spot has two lateral peaks, which are much greater than the analogical ones for sinusoidal

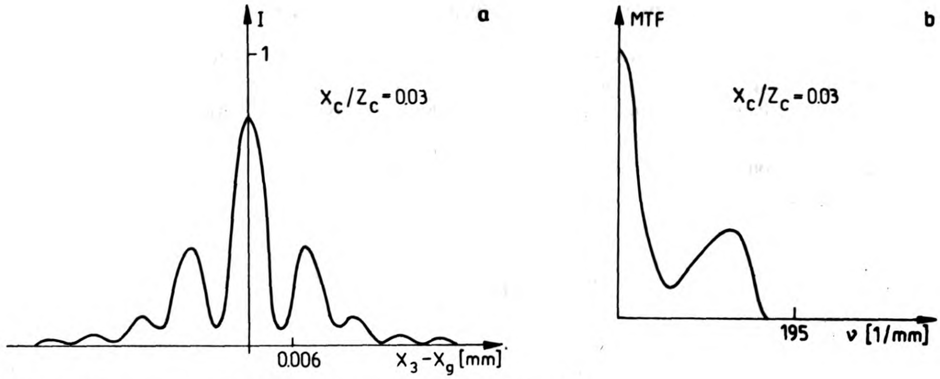

Fig. 5. The same as in Fig. 1, but for example No. 3b″

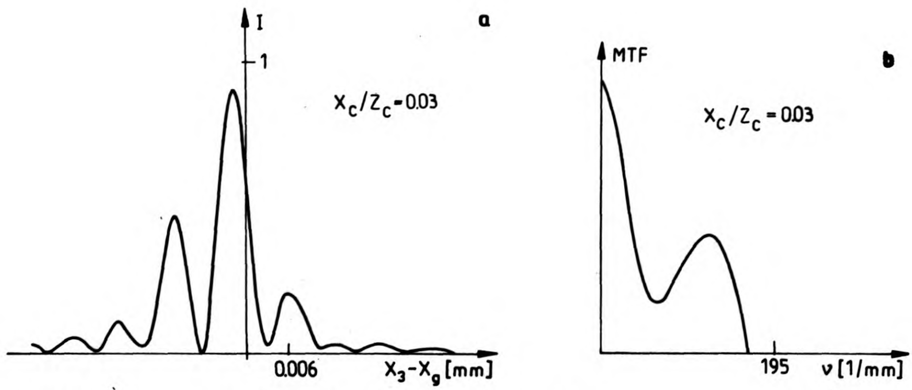

Fig. 6. The same as in Fig. 1, but for example No. 3b‴

transmittance lens. For the binary lens the incoherent modulation transfer function has a large minimum in middle frequence space. The binary lenses presented in this paper are just being made, thus an experimental verification of the calculations will soon be presented.

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Бинарные голографические линзы — исследования качества отображения

Исследовано качество отображения, какого можно достичь, применяя бинарные голографические линзы. Разработан алгоритм определения распределения амплитуды в плоскости образа точечного объекта, который даёт возможность оценить качество отображения, получаемого при помощи бинарных голографических линз. Исследованы некоторые выбранные голографические линзы, м.др. исследуя влияние расположения входного зрачка на качество отображения и проводя модификацию распределения линий в голограмме, которое отвечает кривизне поверхности голограммы.