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Examples of operating characteristics and power balance in pump cavity of cw Nd:YAG laser*

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Experimental results are presented on the performance of the cw ND : YAG laser employing two different Nd : YAG crystals (⌀ 5 mm × 80 mm) and two different krypton arc lamps (US made and Polish made). In a separate experiment the pump power absorbed by the gold-plated, single elliptical pump cavity was measured. This was done by calorimetric measurements of the heat removed from the reflector of the laser head. The comparison of laser propertie of both Nd : YAG crystals, calculated from the experimental data and from our simplified model of cw Nd : YAG laser derived in the theoretical Section, has been presented. The power balances in gold-plated and in silver + SiO_2 evaporated pump cavity have been described at the end.

1. Introduction

In the first part of this paper the simplified model of the cw Nd : YAG laser at the steady-state conditions is derived to find the optimum laser operating conditions for high laser system efficiency.

In the second part the LCW-1 laser system has been presented and the experimental results are described on the performance of cw Nd : YAG laser employing different rods, lamps and output mirrors. From the experimental data and from our simplified theoretical model the laser properties of the rods and laser head parameters have been calculated. The results obtained are within the values reported in the references.

The results of the calorimetric measurements of the pump power absorbed by the LCW-1 laser head reflector and calculated power balance in the gold-plated and in the silver evaporated pump cavity have been described in the third part of this paper.

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2. Simplified model of cw Nd: YAG laser in the steady-state conditions

The performance of a steady-state cw Nd : YAG laser is analysed, based on simplified model in which conditions are assumed to be uniform throughout the laser.

The pumping and emission processes in the cw Nd : YAG laser system are shown in Fig. 1. The total Nd ion density is $n_t = n_0+n_1+n_2+n_3$ and the inversion density is $N = n_2-n_1$. As in [1] we will assume that the transition from the pump band into the upper laser level occurs rapidly and that the terminal laser level empties infinitely fast to the ground level. In this case the entire population is divided between the ground level and the upper laser level. With $n_1 = 0$, $n_2 = 0$ the total Nd ion density will be $n_t = n_0+n_2$ and the inversion population $N' = n_2$.

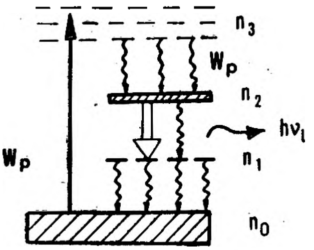

Fig. 1. Optical process in cw ND : YAG laser system

The inversion density in the laser material and the photon density within the laser resonator are described by the following rate equations:

$$\frac{\partial N}{\partial t} = W_p n_0 - N\sigma c\Phi - \frac{N}{\tau_f}, \tag{1}$$

$$\frac{\partial \Phi}{\partial t} = N\sigma c\Phi \frac{l_p}{l_R+l_r(n-1)} - \frac{\Phi}{\tau_c} \tag{2}$$

where N (cm^{-3}) is the inversion density, σ (cm^2) — the laser transition cross-section, c (cms^{-1}) — the light velocity, Φ (ph cm^{-3}) — the photon density, τ_f (s) — the fluorescent life time, W_p (s^{-1}) — the pump rate, τ_c (s) — the decay time for photons within the resonator, i.e., the photons average life time in the resonator. In our case l_p, l_R, l_r are the pumped lengths of laser rod, the resonator length and the laser rod length, respectively.

The photon density Φ is given by the sum of two beams travelling in opposite directions within the laser cavity, as can be seen in Fig. 2. The photon density Φ can be expressed by the power density (J) within the laser. With $J = J_1+J_2$ we obtain

$$\Phi = \frac{J}{ch\nu_l}. \tag{3}$$

In the cw Nd : YAG laser conditions we can assume, as in [2], that

$$\frac{dJ}{dl_R} = 0. \tag{4}$$

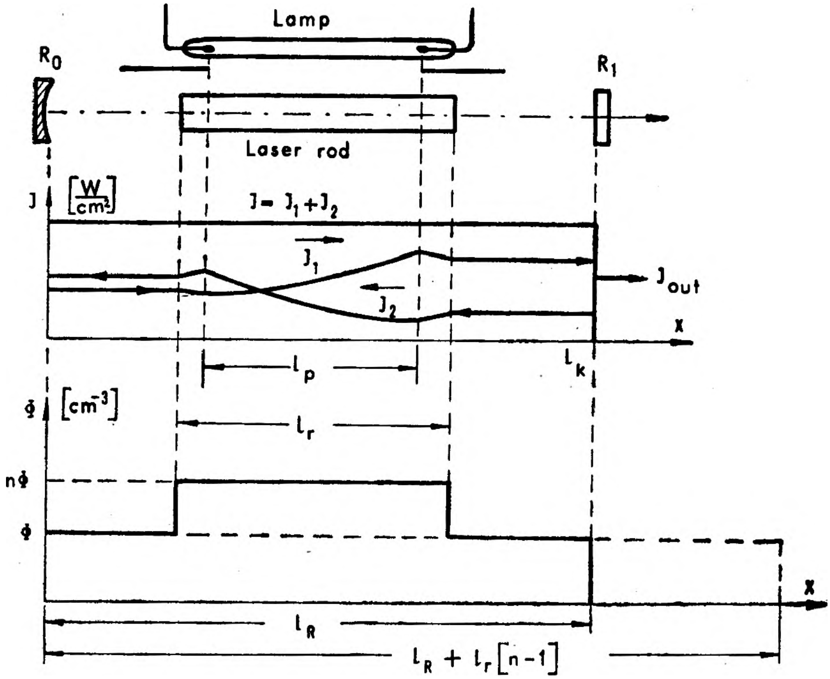

Fig. 2. Steady-state power density and photon distribution within cw Nd : YAG laser resonator

Factor $l_p/[l_R+l_r(n-1)] = V_P/V_R$ represents the ratio of the pumped volume of the laser crystal and the average resonator volume with the photon density Φ.

Introducing the steady-state condition $\partial\Phi/\partial t = 0$ and $\Phi \neq 0$ into the rate Eq. (2) we obtain

$$N = (c\sigma\tau_c)^{-1}\,\frac{l_R+l_r(n-1)}{l_p}. \tag{5}$$

We can express τ_c by the power losses per round trip: $2l_rA$ in the resonator

$$\tau_c = \frac{t_R}{2l_rA} \tag{6}$$

where $t_R = (2l_{R\,\mathrm{opt}})/o$ is the round-trip time of the resonator and A is the total loss coefficient

$$A = a - \frac{1}{2l_r}\ln R_0R_1, \tag{7}$$

a is the loss coefficient and R_0, R_1 are the reflectivities of the resonator mirrors.

Substitution of (6) into (5) gives the steady-state inversion density

$$N = \frac{l_r}{l_p} \frac{A}{\sigma}. \tag{8}$$

As we can see from (8) the inversion density in these conditions depends only on the parameter of the laser crystal (σ), the parameter of the resonator (A) and on the parameter of the laser head (l_r/l_p). For the given laser head with Nd : YAG crystal and for the given resonator the inversion density is constant, i.e., $\partial N/\partial t = 0$.

Rearranging the Eq. (8) we obtain the well-known condition for oscillations

$$g_0 l_p = \left(a - \frac{1}{2l_r} \ln R_0 R_1\right) l_r \tag{9}$$

where $g_0 = \sigma N$ is the small-signal gain coefficient.

Setting (8) in (1) and with $\partial N/\partial t = 0$ we obtain the steady-state photon density within the resonator

$$\Phi = \frac{W_p n_0}{cA} \frac{l_p}{l_r} - \frac{1}{c\sigma\tau_f}. \tag{10}$$

Substituting Φ from (3) into (10) we derive an expression for the steady-state power density within the resonator

$$J = \frac{W_p n_0 h\nu_l}{A} \frac{l_p}{l_r} - J_0 = J_2 \left(\frac{l_p}{l_r} \frac{W_p n_0 \tau_f \sigma}{A} - 1\right) \tag{11}$$

where J_2 is the saturation parameter of the Nd : YAG crystal [1, 5]

$$J_0 = \frac{h\nu_l}{\sigma\tau_f}. \tag{12}$$

From the Eq. (11) we can obtain the threshold pump rate

$$\Phi \geqslant 0 \text{ if } W_p \geqslant W_{p\,\mathrm{th}},\ W_{p\,\mathrm{th}} = \frac{l_r}{l_p} \frac{A}{n_0 \sigma \tau_f}, \tag{13}$$

which is required to maintain the oscillations.

Assuming that the laser crystal is pumped uniformly, the pump power which is absorbed in the pump bands of the laser crystal (M_r) is given by

$$M_r = W_p n_0 V_p h\nu_p \tag{14}$$

where $h\nu_p$ is the photon energy in the pump bands of Nd : YAG, n_0 — the population of the ground level, V_p — pumped volume of the laser rod and W_p — the pump rate from the ground to the pumping level. Since almost all ions are pumped to the excited level end up to the upper laser level, W_p is also the pump rate to the upper laser level.

Taking the above into account and assuming that the pump rate W_p is a linear function of lamp power we obtain from (14)

$$W_p = \frac{\eta_1 P_{\text{in}}}{n_0 V_p h\nu_p} \tag{15}$$

where $\eta_1 = M_r/P_{\text{in}}$ – the total pumping efficiency. (16)

Introducing (15) into (14) and inserting (14) into (13) we obtain the expression for the threshold power

$$P_{\text{th}} = l_r BA \tag{17}$$

where

$$B = \frac{J_0 S}{\eta_1 \eta_2} \tag{18}$$

is the pumping parameter (S is the cross-section area of the laser rod and η_2 represents the ratio of the laser photon energy $h\nu_l$ and the pump-band energy $h\nu_p$).

For $P_{\text{in}} = P_{\text{th}}$, $J = 0$. If we plot output laser power as a function of input lamp power, the extrapolation of this curve to $P_{\text{out}} = 0$ will give the P_{th} value.

By measuring the threshold power as a function of the reflectivity of the mirrors (R_0, R_1) or additional power losses in the resonator for at least two points the unknown $A(a)$ and B parameters can be roughly determined from (17).

Figure 2 shows schematically the power distribution within a laser cavity. J_1 and J_2 are the power densities of the incident and the reflected beam, respectively. P_{out} is the laser output power

$$J_{\text{out}} = J_1 - J_2, \tag{19}$$

and

$$P_{\text{out}} = SJ_{\text{out}}. \tag{20}$$

Combining the equations (19), (20) and with $J = J_1 + J_2$ we obtain

$$P_{\text{out}} = S\frac{1-R}{1+R}J. \tag{21}$$

Substitution of J from (11) into (19) yields

$$P_{\text{out}} = S\frac{1-R}{1+R}J_0\left(\frac{l_p}{l_r}\,\frac{W_p\, n_0 \tau_f \sigma}{A} - 1\right). \tag{22}$$

Combining the Eqs. (21), (16) and (17) we obtain the expression for the output laser power as a function of the input lamp power

$$P_{\text{out}} = S\frac{1-R}{1+R}J_0\left(\frac{P_{\text{in}}}{P_{\text{th}}}-1\right), \tag{23}$$

or

$$P_{\text{out}} = \eta(P_{\text{in}}-P_{\text{th}}) \tag{24}$$

where

$$\eta = \frac{1-R}{1+R}\frac{1}{Al_r}\eta_1\eta_2 \tag{25}$$

is the slope efficiency of the cw laser.

The saturation parameter J_0 can be calculated from the Eq. (23) if the slope efficiency of laser power characteristics is measured

$$J_0 = \frac{1+R}{1-R}\eta\frac{P_{\text{th}}}{S}. \tag{26}$$

If the material parameter J_0 and the B factor of laser head are known the total pump efficiency η_1 can be calculated from the expression (19)

$$\eta_1 = \frac{SJ_0}{\eta_2 B} \tag{27}$$

where η_2 is equal to 0.73 [3] for Nd : YAG crystal and a krypton pumping lamp.

Optimum output mirror reflectivity R_{opt} can be calculated from Eq. (23). Differentiation of this equation with $\delta P_{\text{out}}/\delta R_1 = 0$ gives an expression for the optimum output mirror

$$R_{\text{opt}} = \exp 2\left(al_r - \sqrt{\frac{P_{\text{in}}}{B}al_r}\right). \tag{28}$$

3. Experimental results

The LCW-1 cw Nd : YAG laser developed and built at the Institute of Optoelectronics of the Military Technical Academy, Warsaw (Poland) generates a beam monochromatic IR radiation ($\lambda = 1.06$ μm) with power up to 150 W (6 kW input). It finds application in cutting, drilling and welding or in R-D works. The LCW-1 laser system (Fig. 3) comprises: the laser head mounted with the mirror holders on an optical rail (ZHL-ZU1 PZO), the solid-state power supply providing 6 kW – 50 A dc power with a continuously variable output to the lamp and the cooler which maintains the temperature of the deionized water in the closed loop at 28–32 °C. The cooler contains a deionizer and a microparticle filter.

The laser rod and the krypton lamp are placed at the both foci of a single elliptical dry reflector. The major and the minor axes of the ellipse are 45 mm and 41.3 mm, respectively.

The rod and the lamp are surrounded by water-cooling jackets, the diameter of which is 10 mm and the flow rate 11.5 l/min. The reflector in the laser head is cooled by independent water loop. The reflector length is equal to that of the arc in the krypton lamp — 75 mm. Reflecting surface of the pumping cavity (side and face surfaces) are gold-plated and polished or silver and SiO_2 evaporated.

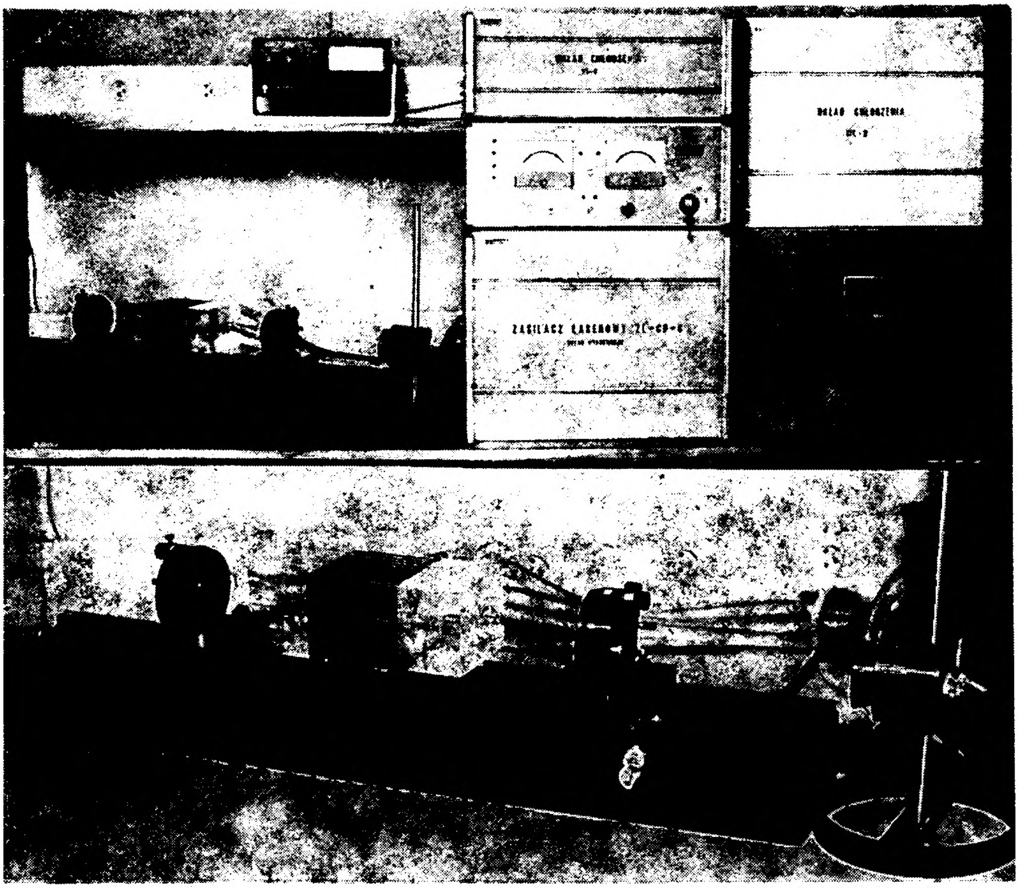

Fig. 3. Cw Nd : YAG laser LWC-1 with power supply and cooling systems

The experimental measurements were performed with two different Nd : YAG crystals ⌀ 5 mm × 80 mm with AR layers on their flat surfaces. The difference between the multimode output power for the silver evaporated reflector and for the gold-plated reflector in LCW-1 can be seen in Fig. 4. The silver evaporated pump cavity is more efficient. The ratio of the slope efficiencies of silver and golden reflectors is 1.4. All the next experimental measurements of output laser power were performed with the silver evaporated reflector.

Figure 5 shows the performance of the cw Nd : YAG laser with $l_R = 300$ for two different pumping lamps. Both lamps had the same length – 75 mm. One of them was 4Kr3 ILC krypton (1–2 Atm) arc lamp the maximum input power of which was 3 kW. The other was the new Polish krypton (2–3 Atm) arc lamp model 5Kr75 COBR-POLAM with the maximum input power being 4 kW (short-time rating power to 6 kW). The 5Kr75 lamp is a more efficient pumping source for LCW-1 laser, as can be seen in Fig. 5 and in Table.

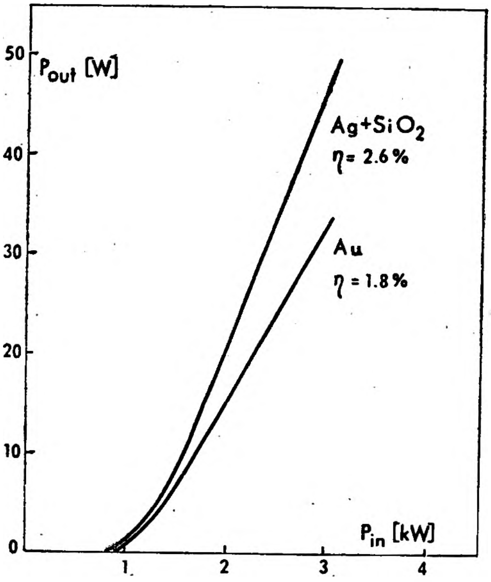

Fig. 4. LWC-1 laser output power vs. input power for two different reflecting surfaces on pump cavity walls

Laser properties of the I and II Nd:YAG crystals ⌀ 5×80 AR, calculated from the extrapolated threshold powers and the slope efficiencies

Nd:YAG crystal	Property					Pumping lamp
	α 10^{-3}(cm^{-1})	B (W)	η_1 (%)	J_0 (Wcm^{-1})	σ 10^{-19}(cm^2)	
I	4.5	10520	6.9	2700	3	5Kr75 COBR-POLAM
II	2.6	11240	7.1	2990	2.7	
	2.3	13840	5.7	2940	2.75	4Kr3 ILC

Experimental measurements of the maximum multimode output power as a function of input power were performed using this lamp. The highest output – 150 W, from 7.5 cm long pumped crystal was achieved with the front mirror of 90% reflectivity (Fig. 6). All the measurements of output laser power have been conducted by using Optical-Coherent Radiation Model 213 Power Meter.

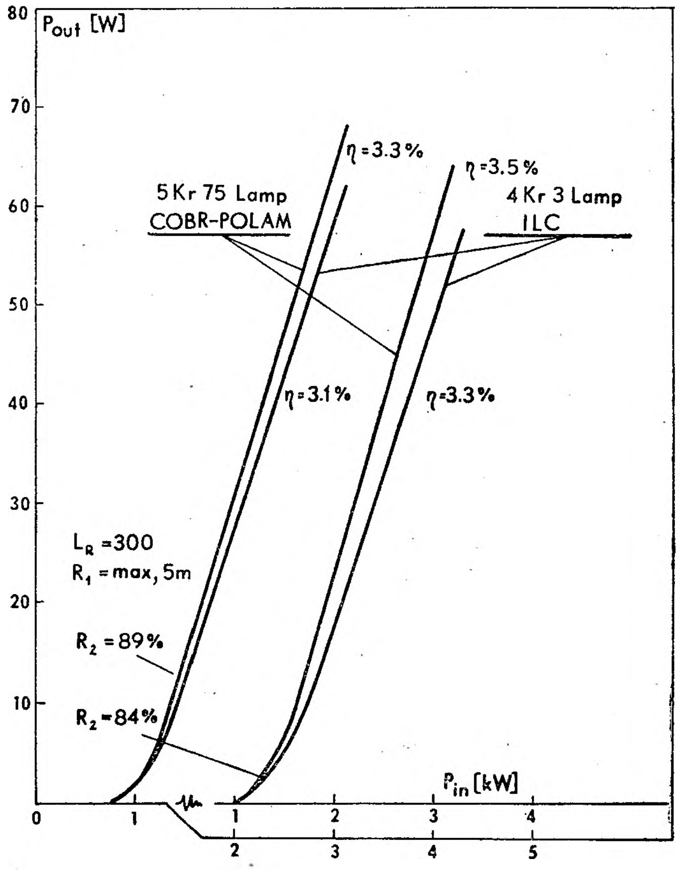

Fig. 5. Performance of LCW-1 laser employing two different lamps and two different output mirrors. Output power (W) as a function of input power (kW)

The measured laser output power as a function of the lamp input power is plotted in Fig. 7, for different Nd : YAG crystals and front-mirror reflectivity. By introducing the experimental data into Eqs. (17), (26) and (27) we can calculate the loss coefficient a, the pump parameter B, the total pump efficiency η_1 and the saturation parameter J_0 of both Nd : YAG crystals. All the calculated parameters of laser rods for both pumping lamp are listed in Table.

We can see that the 5Kr75 lamp is more efficient than the 4Kr3 one for LCW-1 pumping ($\eta_{1(\mathrm{COBR})} > \eta_{1(\mathrm{ICL})}$). The II laser rod has a greater loss coefficient than the I rod, but its pump parameter is smaller. Therefore, for the same output mirrors reflectivity at high input power the performance characteristics of both rods in LCW-1 do not indicate real differences.

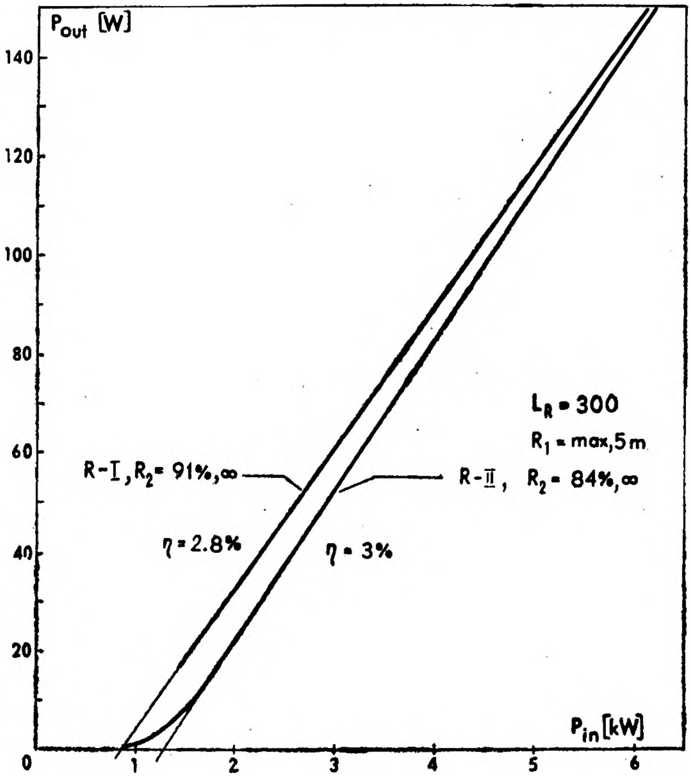

Fig. 6. Maximum multimode output power of LWC-1 employing 5Kr75 lamps vs. power for two different Nd : YAG crystals

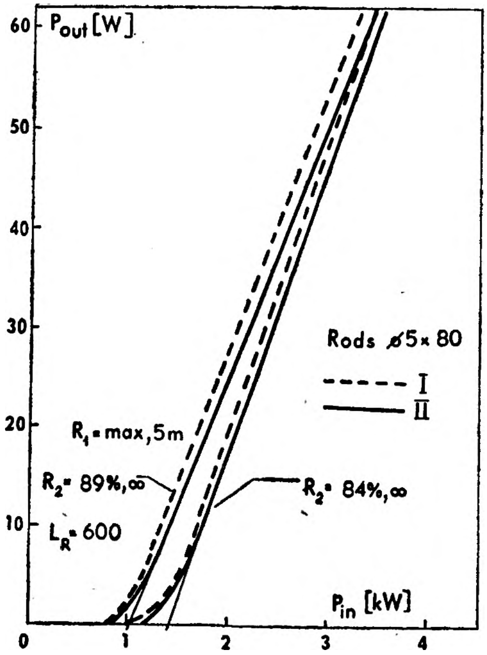

Fig. 7. Laser output power vs. lamp input power for different Nd : YAG rods and front-mirror reflectivities

4. Power balance in pump cavity

The calorimetric measurements were carried out in order to determine the efficiency of the pump cavity of LCW-1 laser.

The power supplied to the lamp is either dissipated as heat by the lamp envelope and electrodes or emitted as radiation. A portion of radiation will be absorbed by the metal walls of the pump cavity or will flow out due to the openings in the reflector into which flow-tubes of rod and lamp are to be inserted. The radiation reflected from the walls will be either absorbed by the lasing medium or by the flow-tubes and coolant.

The power absorbed by the reflector was determined by measuring the heat extracted only from the reflector cooling loop. The rod, lamp and flow-tubes were cooled separately.

From the experiment the following conclusion can be drawn: the fraction of electrical input power of the 4Kr3 ILC lamp absorbed by the gold-plated reflector walls is 15.5 ± 0.15%. Assuming that the lamp radiation efficiency is 45% [1, 4], 34.5% of the radiation power is absorbed by the walls of the gold-plated reflector.

The total pumping efficiency, η_1, of the II laser crystal in the silver evaporated reflector, for the 4Kr3 lamp, is given in Table. Dividing this value by the factor 1.4, which results from the comparison of slope efficiency in Fig. 4, we find that 4.1% of the total electric input power is absorbed by the Nd : YAG crystal in golden reflector LCW-1 laser.

Now, assuming that almost all rays leaving the lamp reach the laser material after only one reflection from the cavity walls, and that the power reabsorbed by the lamp, (M_{Ra}), is 5% of the lamp input, we can calculate the power balance in the pump cavity having the gold-plated walls.

The power absorbed by the flow-tubes, M_T, as a function of their transmission, T, is given by

$$M_T = M_L(1-T) + M_L TR(1-T) \tag{29}$$

where M_L is the power emitted by the lamp, R is the effective reflectivity coefficient of the reflector (Fig. 8).

The total losses of radiation in the reflection are

$$(1-R)M_1 = 0.047 M_1 + 0.155\, M_p \tag{30}$$

where M_1 is the power emitted by the lamp in flow-tube.

In this case 0.047 is the ratio of the area of the holes in the cavity to its total inner area, and 0.155 is the fraction of input power absorbed by the cavity walls.

Equation (30) can be rewritten as

$$R = 0.953 - \frac{0.155 M_p}{M_1 T}. \tag{31}$$

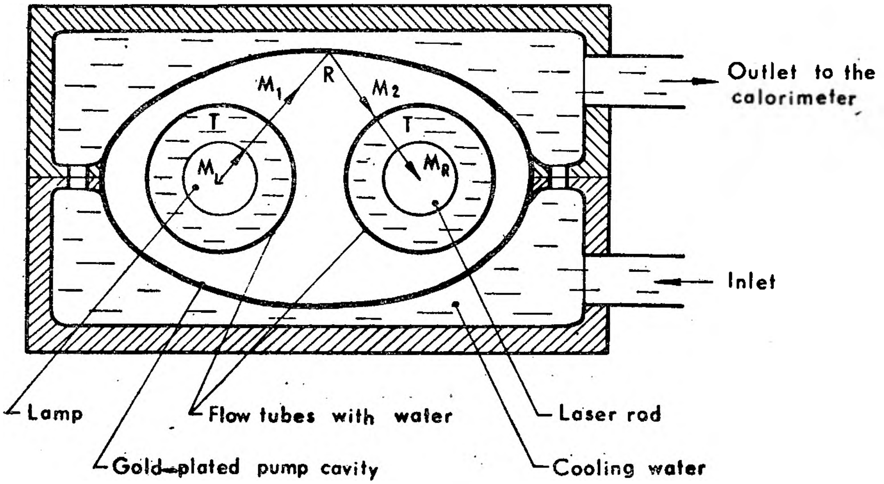

Fig. 8. Cross-section of the elliptical pump cavity in the laser head of LWC-1

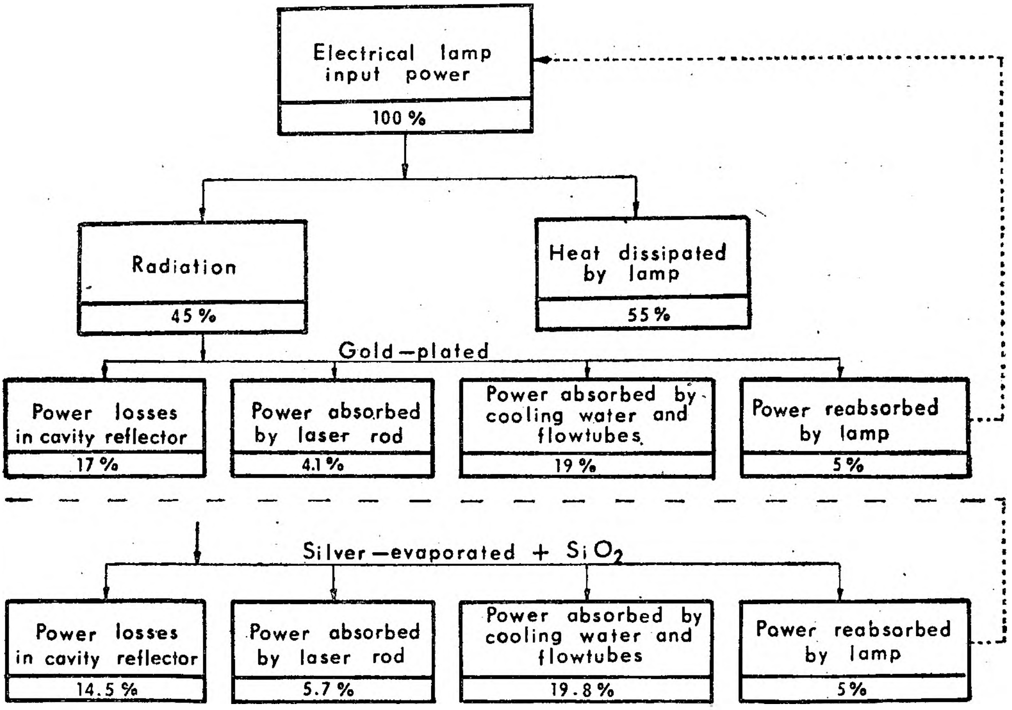

Fig. 9. Balance of the pumping power in the gold-plated and in the silver $+SiO_2$ evaporated pump cavity of LCW-1 laser

From the energy balance we can see that the sum of power losses in the cavity, the power absorbed by the lamp, the rod and by the flow-tubes must be equal to the power emitted by the lamp, i.e.

$$M_L = M_T+(1-R)M_1+M_r+M_{Ra}. \tag{32}$$

Introducing the Eq. (29) and (31) into (32) and setting $M_L = 0.45\ M_p$, $M_r = 0.041 M_p$, $M_{Ra} = 0.05 M_p$ we get

$$T = 0.675, \tag{33}$$

i.e., the transmission of flow-tube with water.

Now, substituting this value into (29) and (31) we obtain the value of power absorbed by the flow-tubes $M_T = 0.19 M_p$ and the effective reflectivity of the gold-plated reflector $R_{\mathrm{Au}} = 0.443$.

From (31) all the radiation losses in the gold-plated reflector (absorption and leakage) are 17°/₀ of M_p, i.e., 1.5°/₀ M_p flow-out through the holes in the reflector.

Setting in (32) $M_r = 0.057\ M_p$ and $T = 0.675$ for the silver evaporated pump cavity of the LCW-1 laser, we obtain

$$R_{\mathrm{Ag}} = 0.522.$$

The balances of the pumping power in the gold-plated and the silver + SiO evaporated pump cavities are presented in Fig. 9.

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