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Isotropic plane-parallel plate in laser applications: a new approach

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The paper presents a simple method of qualification of the element applied and verification if its dimensions conform to requirements. The matter of the analysis is the possibly simplest optical element, *i.e.*, plane-parallel plate of BK7 glass illuminated with He-Ne laser beam. Analysed examples can be easily generalised to other materials and wavelengths.

1. Introduction

The plane-parallel plates are widely used as beam splitters, optical windows and substrates for thin multilayers. The application of isotropic, plane-parallel plate is a basic technique in optics and it was analysed in detail earlier [1], [2]. The analysis was also made for laser radiation applications [3]. It is continued in [4], [5] due to the new possible applications and new measuring techniques.

The interferencial methods belong to the most precise techniques of measurement of thickness. They afford an accuracy of $\lambda/100$ and even $\lambda/1000$ [6] when interferometer with a discrete change of phase on a special investigation post is applied.

The fabrication of optical elements is closely connected with parameter control at every stage of treatment process. Before fabrication we need to know the characteristics of the raw material and its optical orientation. Then we should plan precisely thickness of manufactured element. In some cases, especially when the elements are illuminated with laser radiation of high power density, it is important for personal safety and element durability. It is why the adequate attest of fabricated optical elements is requested. On the other hand, the comprehensive measurements of elements are very expensive and laborious. To decrease the costs of measurement it is important to define the conditions in which every element will be used and to reduce the scope of measurements to parameters which are fundamental for the selected applications. The test should be made in the same operating conditions as the real application conditions.

The paper presents a simple method of qualification of the element applied and verification if its dimensions conform to requirements. The matter of the analysis is the possibly simplest optical element, *i.e.*, plane-parallel plate of BK7 glass illuminated with He-Ne laser beam. Analysed examples can be easily generalised to other materials and wavelengths. Short wave and long wave absorption bands and dispersion of refractive index of glass have been omitted to simplify the problem.

2. Spectral characteristics of plane-parallel plate

A theoretical analysis of plane-parallel plate of BK7 glass of physical thickness d ($n_O = 1.52$, imaginary part and dispersion of refractive index are omitted) leads to the following dependence, valid for both polarisation factors

$$R_{(i)} = \frac{\left(\frac{n_{S(i)} n_{O(i)}}{n_{P(i)}}\right)^2 \sin^2\varepsilon}{(n_{S(i)} + n_{O(i)})^2 \cos^2\varepsilon + \left(\frac{n_{S(i)} n_{O(i)}}{n_{P(i)}}\right)^2 \sin^2\varepsilon} \tag{1}$$

where: i — one of the linear polarised components, parallel or perpendicular to the plane of incidence, type p or s for:

- plane-parallel plate: $n_{P(p)} = n_P/\cos\beta$, $n_{P(s)} = n_P \cos\beta$,
- surrounding medium (incident side radiation): $n_{O(p)} = n_O = \cos\alpha$, $n_{O(s)} = n_O \cos\alpha$,
- surrounding medium (outlet side radiation): $n_{S(p)} = n_S/\cos\gamma$, $n_{S(s)} = n_S \cos\gamma$,

α — the angle of incident radiation on the plate in surrounding medium with refractive index n_O, β and γ are determined by Snell's law: $n_O \sin\alpha = n_P \sin\beta = n_S \sin\gamma$, $\varepsilon = (4\pi d/\lambda) n_P \cos\beta \pm \pi$ is the phase difference between the two waves (of wavelength λ) at the front and back surfaces.

For each polarization, the reflectivity $R_{(i)}$ and transmissivity $T_{(i)} = 1 - R_{(i)}$ are periodic functions of optical thickness of plane-parallel plate. As a function of the angle of incidence, the periodic function falls within an envelope that is substantially different for each polarization.

In accordance with Eq. (1), the transmissivity of thin plane-parallel plate of BK7 glass of physical thickness $d = 0.1$ mm has been calculated for radiation of wavelength 632.8 nm as a function of the angle of incidence. Functions of the transmissivity are presented in Fig. 1. For incidence angles between 20° and 80° only envelopes are presented (for bottom values env-dp and env-ds for p and s polarisation, respectively, and for top value $T = 1.0$) The changes of both linear components of non-coherence radiation usually obtained from spectrophotometer measurements are presented for comparison. The results of spectrophotometer measurements are independent of the thickness of a plane-parallel plate in a wide range of the tens of millimetres (when glass used as a substrate is of high quality without bubbles or other elements destroying homogeneity and isotropy of the substrate). Such values typical of plane-parallel plates are applied in optical systems.

Figures 2–6 present the transmissivity of plane-parallel plates of BK7 (starting from ultra-thin plates and increasing thickness ten times in each consecutive figure;

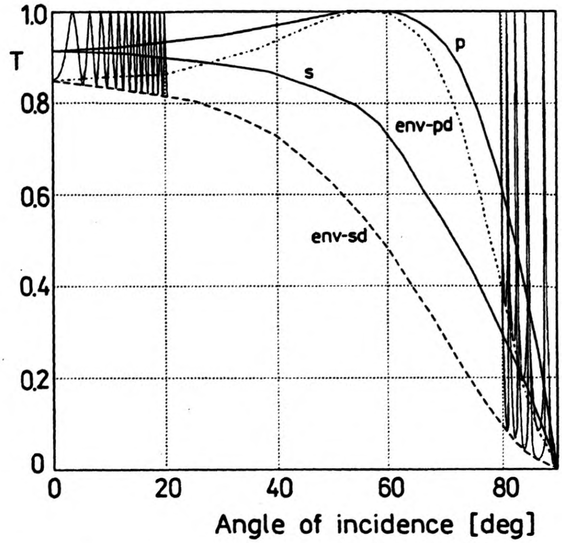

Fig. 1. Dependence of the transmissivity of a plane-parallel plate of BK7 glass for radiation wavelength 632.8 nm on the angle of incidence. The plate thickness amounts to 960.8 and is expressed in quarter-wave optical thickness for 632.8 nm. *p* and *s* — components of polarization for non-coherent radiation, env-ds and env-dp — the bottom envelopes for *s*- and *p*-type polarization of coherent radiation, respectively. Envelopes only for the incidence angles from 20° do 80° are presented

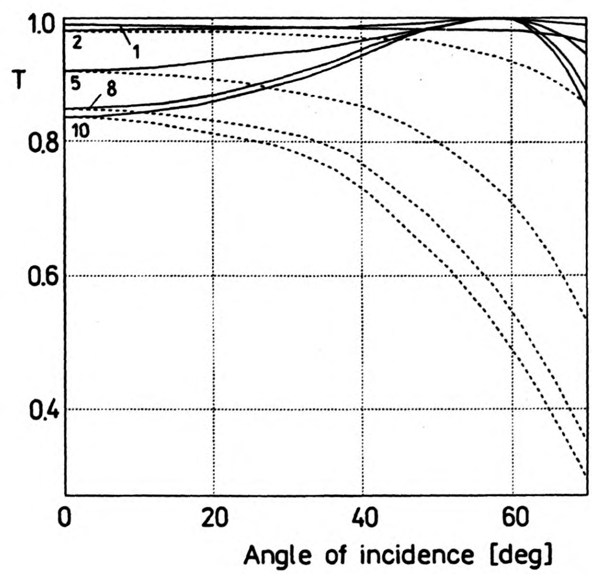

Fig. 2 Dependence of the transmissivity of a plane-parallel plate of BK7 glass for coherent radiation wavelength 632.8 nm on the angle of incidence. The thicknesses are expressed in quartet-wave optical thickness for 632.8 nm: 1 — 0.1, 2 — 0.2, 5 — 0.5, 8 — 0.8, 10 — 1.0 (—— *p*-type polarization, – – – – *s*-type polarization)

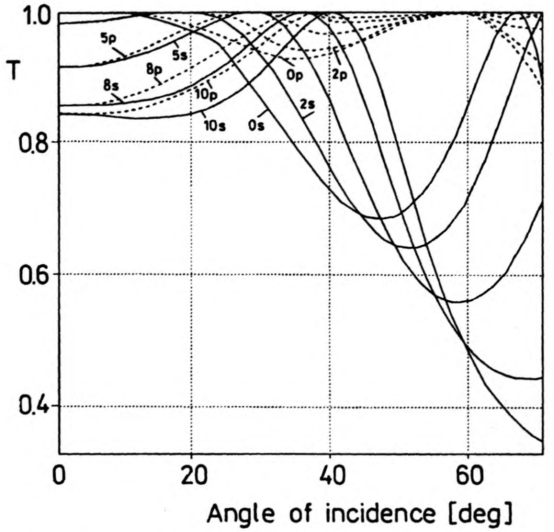

Fig. 3. Dependence of the transmissivity of a plane-parallel plate of BK7 glass for coherent radiation wavelength 632.8 nm on the angle of incidence. The thicknesses are expressed in quarter-wave optical thickness for 632.8 nm: 0*s* and 0*p* — 10.0, 2*s* and 2*p* — 10.2, 5*s* and 5*p* — 10.5, 8*s* and 8*p* — 10.8, 10*s* and 10p— 11.0; corresponding do *s*- and *p*-type polarization

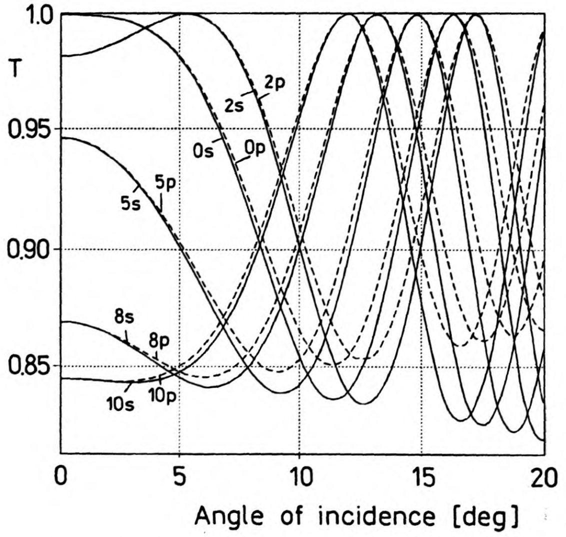

Fig. 4. Dependence of the transmissivity of a plane-parallel plate of BK7 glass for coherent radiation wavelength 632.8 nm on the angle of incidence. The thicknesses are expressed in quarter-wave optical thickness for 632.8 nm: 0*s* and 0*p* — 100.0, 2*s* and 2*p* — 100.2, 5*s* and 5*p* — 100.5, 8*s* and 8*p* — 100.8, 10*s* and 10*p* — 101.0; corresponding to *s*- and *p*-type polarization

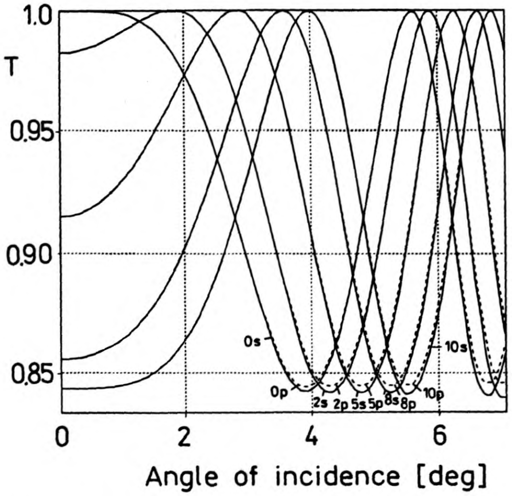

Fig. 5. Dependence of the transmissivity of a plane-parallel plate of BK7 glass for coherent radiation wavelength 632.8 nm on the angle of incidence. The thicknesses are expressed in quarter-wave optical thickness for 632.8 nm: 0*s* and 0*p* — 1000.0, 2*s* and 2*p* — 1000.2, 5*s* and 5*p* — 1000.5, 8*s* and 8*p* — 1000.8, 10*s* and 10*p* — 1001.0, corresponding to *s*- and *p*-type polarization

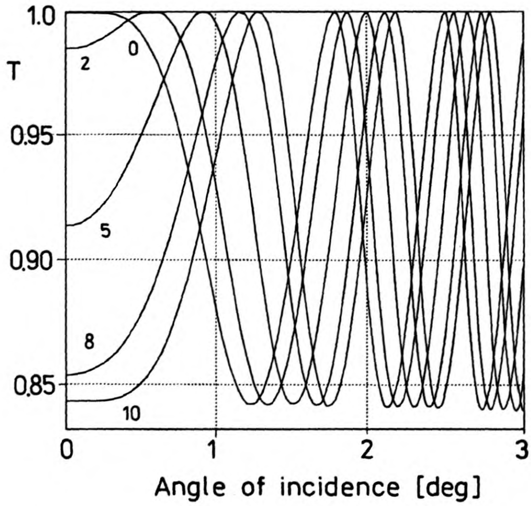

Fig. 6. Dependence of the transmissivity of a plane-parallel plate of BK7 glass for coherent radiation wavelength 632.8 nm on the angle of incidence. The thicknesses are expressed in quarter-wave optical thickness for 632.8 nm: 0 — 10000.0, 2 — 10000.2, 5 — 10000.5, 8 — 10000.8, 10 — 10001.0, corresponding to *s*- and *p*-type polarization

plate thickness is expressed in quarter-wave optical thickness for $\lambda = 632.8$ nm) for coherent radiation of 632.8 nm wavelength as a function of the incidence angles of radiation. Every figure shows the comparison of transmission characteristics of plates whose optical thickness changes up to one phase thickness for wavelength

632.8 nm. The following rules can be noticed:

1. In a wide range of plate thickness the value of transmission coefficient for normal incidence changes from 84.3% to 100% and it depends only on a fractional number of phase thickness for selected wavelength.

2. With increasing plate thickness, the interference fringes fall more within their envelopes.

3. For any thickness the changes in transmission value are slowest for small incidence angles (in the interval of angles from 0° to 20°).

4. With increasing incidence angle, the envelopes of transmissivity for *p*-type polarization converge and osculate at Brewster's angle. Then they diverge continuously for angles greater than Brewster's angle (see Fig. 1).

5. With increasing incidence angle, the envelopes of changes of transmissivity for *s*-type polarization diverge to maximum.

6. In the case of laser beam radiation (quasi-monochromatic, divergent beam and nonuniform wavefront) the value of the transmission can be described as averaging over spectral and angular energy distribution of the incidence beam. The values achieved for non-coherent radiation are also obtained for averaged values of a coherent radiation passing through thick plates (the average results from spectral and angular distribution of laser beam).

7. It is important to remember that the measured values of transmissivity are averaged (they have statistic importance), but a real instantaneous value can be considerably different.

The transmissivity (in the narrow range of spectrum between 626 and 636 nm) of BK7 glass plate is presented in Fig. 7. The thickness of the plate equals to 960.8 in quarter-wave optical thickness for 632.8 nm. The coherent radiation falls normally

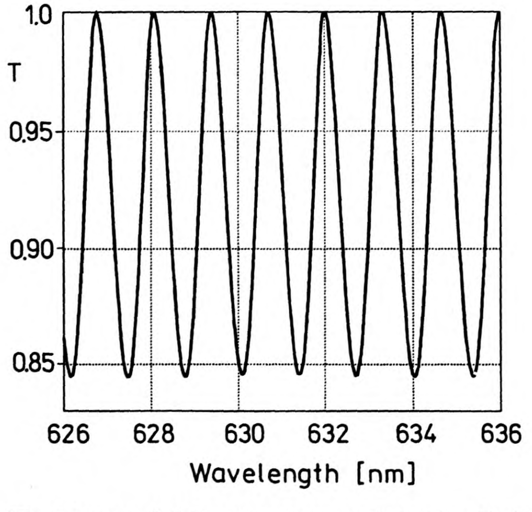

Fig. 7. Transmissivity of a plane-parallel plate of BK7 glass. The plate thickness amounts to 960.8 and is expressed in quarter-wave optical thickness with respect to 632.8 nm. The direction of coherent beam is normal to the plate

upon the plate. The best way to describe whether the plane-parallel plate is suitable, or not is to measure changes of transmissivity for incidence angles close to 0° (items 1–3). In the case of using a standard He-Ne laser (*e.g.*, LG600 PZO) and applying a computer programme of algorithm based on formula (1), it is possible to determine the thickness of plate with accuracy equal to a decimal part of phase thickness. The accuracy of measurement of the transmitted energy (items 6 and 7) limits the improvement of the precision of thickness determination. Pecision improvement is possible by reducing divergence of laser beam, *e.g.*, by using additional elements focusing beam or by using a monomode laser. This method can also be used for less absorbing materials and for computation of the physical thickness of samples and their complex refraction indexes. In these cases the analytical formulas are more complicated.

3. Conclusions

The measurement of transmissivity dependence on the angle of incidence in the range from 0° to 10° provides a really satisfactory estimation of the influence of a plane-parallel plate on the spectral distribution of the coherent beam. The measurement enables determination of fractional part of phase thickness of plate for wavelength applied. The necessity of initial investigation of the plate with the use of laser beam becomes obvious. In order to achieve a higher quality for optical elements, it is necessary to use more precise measurement stands.

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