# Hybrid lens of optimized aberration correction 

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#### Abstract

A hybrid lens is understood as a glass lens with a diffraction microstructure deposited on one of its spherical surfaces. The imaging properties of such a lens depend above all on the curvature radii of the lens surfaces, topography of diffraction microstructure and both position and size of the entrance pupil. The third order aberrations depend directly on these parameters. By choosing properly the above parameters an achromatic hybrid lens of corrected spherical aberration and selected field aberrations can be designed. The imaging quality of the designed hybrid lens has been illustrated by such characteristics as geometrical aberrations, spot-diagrams, diffraction point spread function and wave aberration.


## 1. Introduction

A hybrid lens is understood as a combination of refractive lens (glass lens, for instance) and a diffractive structure deposited on one of its surfaces. The diffraction fringes constituting diffractional part of the hybrid lens can be generated either by interference methods or by numerical calculations followed by drawing. In the hybrid elements working as imaging lenses, being the subject-matter of this paper, the rotational symmetry is required. Therefore, the assumption that the geometry of fringes corresponds to the case of interference of two spherical monochromatic waves is natural. Then, the hybrid lens can be described by several design parameters, i.e., two radii of curvature of the lens $\rho_{1}$ and $\rho_{2}$, the refractive index $n$ of the material of which the lens is produced and two distances $z_{\alpha}$ and $z_{\beta}$ of the light sources, generating spherical waves producing diffractive structure. In most cases the diffraction structure is produced synthetically and then these distances play the part of parameters describing the topography of the structures which may be deposited on the first or second refracting surface (Fig. 1). The analysis of aberrations of such a lens was presented in many works ([1]-[5], for instance). In particular, the possibilities of aplanatic correction were examined [6], [7]. The aberrations of such a lens of the entrance pupil shifted to take a position in front of the lens were calculated [8]. Also, the chromatic properties of hybrid lens were analysed [8]-[11]. Based on the results presented in the above works the construction methodology of a hybrid lens of desired type of aberration correction will be presented.


Fig. 1. Design parameters of the hybrid lens.

### 1.1. Denotations

In this work the following denotations will be used:
$V_{\rho 1}=\frac{1}{\rho_{1}}$ - curvature of the first refracting surface,
$V_{\rho 2}=\frac{1}{\rho_{2}}$ - curvature of the second refracting surface,
$n$ - refractive index of the material of which the lens was made,
$V_{\alpha}=\frac{1}{z_{\alpha}}$ - curvature of the first wavefront creating the diffraction structure,
$V_{\beta}=\frac{1}{z_{\beta}}$ - curvature of the second wavefront creating the diffraction structure, $V=\frac{1}{s}-$ reciprocal of the distance of the lens from the object point,
$V^{\prime}=\frac{1}{s^{\prime}}$ - reciprocal of the distance of the lens from the image point.
The hybrid lens being an optical system composed of two parts: refractive and diffraction ones, is assumed for simplicity to be a thin lens. The power of the refractive part is a sum of powers of both the surfaces $\varphi_{R 1}$ and $\varphi_{R 2}$, where:

$$
\begin{align*}
\varphi_{R 1} & =(n-1) V_{\rho 1}  \tag{1a}\\
\varphi_{R 2} & =(1-n) V_{\rho 2} \tag{1b}
\end{align*}
$$

The diffraction part is of power

$$
\begin{equation*}
\varphi_{D}=\mu\left(V_{\alpha}-V_{\beta}\right) \tag{2}
\end{equation*}
$$

The coefficient $\mu$ is taken from holography, where it denotes the ratio of the wavelengths used to reconstruction and recording, respectively. In our case, where the diffraction structure is, as a rule, created artificially, $\mu$ is usually equal to 1 and therefore will be omitted in further considerations.

The location of the image will be found taking advantage of the dependence $V^{\prime}=V+\varphi$
where

$$
\begin{equation*}
\varphi=\varphi_{R}+\varphi_{D}=\varphi_{R 1}+\varphi_{R 2}+\varphi_{D} \tag{4}
\end{equation*}
$$

It is convenient to introduce some dimensionless parameters [12], [13]

$$
\begin{equation*}
\eta=\frac{\varphi_{D}}{\varphi_{R}}=\frac{\varphi_{D}}{\varphi_{R 1}+\varphi_{R 2}} \tag{5}
\end{equation*}
$$

which describes the chromatic properties of the lens,

$$
\begin{equation*}
\zeta=\frac{\varphi_{R 2}}{\varphi_{R 1}}=-\frac{V_{\rho 2}}{V_{\rho 1}}=-\frac{\rho_{1}}{\rho_{2}} \tag{6}
\end{equation*}
$$

which describes the geometrical form of the refractive part,

$$
\begin{equation*}
\beta=\frac{V_{\beta}}{V_{\alpha}}=\frac{z_{\alpha}}{z_{\beta}} \tag{7}
\end{equation*}
$$

which describes the topography of the diffraction structure.
Now, the power of the refractive part can be expressed as follows:

$$
\begin{align*}
& \varphi_{R 1}=\varphi \frac{1}{(1+\zeta)(1+\eta)}  \tag{8a}\\
& \varphi_{R 2}=\varphi \frac{\zeta}{(1+\zeta)(1+\eta)}  \tag{8b}\\
& \varphi_{R}=\varphi_{R 1}+\varphi_{R 2}=\frac{1}{(1+\eta)} \tag{8c}
\end{align*}
$$

therefore:

$$
\begin{align*}
& V_{\rho 1}=\frac{\varphi}{n-1} \frac{1}{1+\eta} \frac{1}{1+\zeta}  \tag{9a}\\
& V_{\rho 2}=\frac{\varphi}{1-n} \frac{1}{1+\eta} \frac{\zeta}{1+\zeta} \tag{9b}
\end{align*}
$$

The power of the diffraction parts is

$$
\begin{equation*}
\varphi_{D}=\varphi \frac{\eta}{(1+\eta)} \tag{10}
\end{equation*}
$$

therefore:

$$
\begin{align*}
& V_{\alpha}=\varphi \frac{\eta}{1+\eta} \frac{1}{1-\beta}  \tag{11a}\\
& V_{\beta}=\varphi \frac{\eta}{1+\eta} \frac{\beta}{1-\beta} \tag{11b}
\end{align*}
$$

The real construction parameters of the hybrid lens of power $\varphi$, i.e., $\rho_{1}=1 / V_{\rho 1}$, $\rho_{2}=1 / V_{\rho 2}, z_{\alpha}=1 / V_{\alpha}$ and $z_{\beta}=1 / V_{\beta}$ can be expressed directly by the coefficients $\eta$, $\zeta$ and $\beta$, and the refractive index $n$. The design procedure consists in calculating the values of these parameters.

### 1.2. Monochromatic aberrations

When developing suitably the eiconal into a power series the coefficients describing the particular third order aberrations can be determined for a thin hybrid lens suspended in the air [1], [6]. In particular:

- spherical aberration

$$
\begin{align*}
& S=V\left(V-\frac{\varphi_{R 1}}{n-1}\right)^{2}-\left(\frac{1}{n}\right)^{2}\left(V+\varphi_{R 1}+\varphi_{D 1}\right)\left(V+\varphi_{R 1}+\varphi_{D 1}-\frac{n}{n-1} \varphi_{R 1}\right)^{2} \\
& -\left[(V+\varphi)\left(V+\varphi-\frac{1}{1-n} \varphi_{R 2}\right)^{2}\right]+\left(\frac{1}{n}\right)^{2}\left(V+\varphi_{R 1}+\varphi_{D 1}\right)\left(V+\varphi_{R 1}+\varphi_{D 1}-\frac{n}{1-n} \varphi_{R 2}\right)^{2} \\
& \quad+\frac{\varphi_{D 1}}{1-\beta_{1}}\left[\left(\frac{\varphi_{D 1}}{1-\beta_{1}}-\frac{\varphi_{R 1}}{n-1}\right)^{2}-\beta_{1}\left(\frac{\varphi_{D 1} \beta_{1}}{1-\beta_{1}}-\frac{\varphi_{R 1}}{n-1}\right)^{2}\right] \\
& \quad+\frac{\varphi_{D 2}}{1-\beta_{2}}\left[\left(\frac{\varphi_{D 2}}{1-\beta_{2}}-\frac{\varphi_{R 2}}{1-n}\right)^{2}-\beta_{2}\left(\frac{\varphi_{D 2} \beta_{2}}{1-\beta_{2}}-\frac{\varphi_{R 2}}{1-n}\right)^{2}\right], \tag{12}
\end{align*}
$$

- coma
$C=\tan w\left[V\left(V-\frac{\varphi_{R 1}}{n-1}\right)+\frac{V+\varphi_{D 1}+\varphi_{R 1}}{n(n-1)}\left(\varphi_{R 1}+\varphi_{R 2}\right)-(V+\varphi)\left(V+\varphi+\frac{\varphi_{R 2}}{n-1}\right)\right]$,
- astigmatism
$A=-\varphi \tan ^{2} w$,
- field curvature
$F=-\tan ^{2} w\left(\varphi+\frac{\varphi_{R}}{n}\right)$,
- distortion
$D=0$.
The magnitude $w$ denotes the field angle while the indices 1 and 2 denote the first and the second surfaces, respectively. The pupil overlaps the lens.


## 2. Aberration correction

### 2.1. Chromatic aberration

The design procedure is started from the division of the total hybrid lens power between its refraction and diffraction parts. The ratio of these powers (5) is decisive
so far as chromatic aberration is concerned [8], [10]. The hybrid lens can be understood as a doublet of components characterized by the Abbe v-number and its diffractive analog, respectively:

$$
\begin{align*}
& v_{R}=\frac{n-1}{n_{1}-n_{2}}  \tag{17a}\\
& v_{D}=\frac{\lambda}{\lambda_{1}-\lambda_{2}} \tag{17b}
\end{align*}
$$

as well as by the partial dispersion and its diffraction analog:

$$
\begin{align*}
& P_{R}=\frac{n_{1}-n}{n_{1}-n_{2}}  \tag{18a}\\
& P_{D}=\frac{\lambda_{1}-\lambda}{\lambda_{1}-\lambda_{2}} \tag{18b}
\end{align*}
$$

where: $n_{1}, n_{2}$ correspond to $\lambda_{1}, \lambda_{2}$, respectively.
The chromatic aberration for the wavelengths $\lambda_{1}$ and $\lambda_{2}$ is corrected if

$$
\begin{equation*}
\frac{\varphi_{D}}{\varphi_{R}}=\eta=-\frac{v_{D}}{v_{R}} . \tag{19}
\end{equation*}
$$

Having $\eta$ the magnitudes $\varphi_{R}, \varphi_{D}$ can be determined from formulae (8c) and (10). The value of parameter $\eta$ depends directly on the wavelength range for which the chromatic aberration is corrected. Denoting this range by $\Delta=\lambda_{2}-\lambda_{1}$ and assuming that the refractive index $n$ can be described by Cauchy's formula

$$
\begin{equation*}
n=a+\frac{b}{\lambda^{2}}, \tag{20}
\end{equation*}
$$

we have

$$
\begin{equation*}
\eta=\frac{b\left(2+\Delta / \lambda_{1}\right)}{\left(\lambda_{1}+\Delta\right)^{2}\left(n_{1}-1\right)} \tag{21}
\end{equation*}
$$

where $n_{1}$ is the refractive index for $\lambda_{1}$. For the given sort of glass, basic wavelength and the correction range the parameter $\eta$ can be determined. An example of the result for BK7 glass is presented in Fig. 2.

The secondary spectrum described by the expression

$$
\begin{equation*}
\Delta \varphi=\frac{P_{D}-P_{R}}{v_{D}-v_{R}} \varphi \tag{22}
\end{equation*}
$$

is unfortunately too great. The longitudinal chromatic aberration for several hybrid lenses designed on the basis of selected sorts of glasses is presented in Fig. 3.

Selecting the example illustrating our considerations we decided to choose a relatively narrow range of achromatisation. For a hybrid lens produced of BK7 glass and achromatized for the wavelength $\lambda_{1}=\lambda_{F}=0.4861 \mu \mathrm{~m}$ and $\lambda_{21}=$


Fig. 2. Values of parameter $\eta$ as a function of the wavelength $\lambda_{1}$ and the correction range $\Delta$ for BK7 glass.


Fig. 3. Longitudinal chromatic aberration for four hybrid lenses designed using: BK7 (A), SF2 (B) LaK1 (C), and fluorite (D) glasses, respectively.
$=\lambda_{D}=0.5893 \mu \mathrm{~m}$ (of refractive indices $n_{1}=1.52236$ and $n_{2}=1.51671$ ) the Abbe numbers are as follows: $v_{R}=92.45$ and $v_{D}=-4.710$, and thus the achromatization parameter should be $\eta=0.051$.

### 2.2. Coma

The expression describing coma for the hybrid lens with diffraction structure deposited on the first surface expressed by the dimensionless parameters introduced by formulas (5) - (7) has the form

$$
C_{1}(\varphi, v, \tan w, n, \zeta, \eta)=
$$

$$
\begin{align*}
& =\varphi^{2} \tan w\left\{v\left[v-\frac{1}{(n-1)(1+\zeta)(1+\eta)}\right]+\left[v+\frac{1}{(1+\zeta)(1+\eta)}+\frac{\eta}{(1+\eta)}\right]\left[\frac{1}{n(n-1)(1+\eta)}\right]\right. \\
& \left.-(v+1)\left[v+1-\frac{\zeta}{(1-n)(1+\zeta)(1+\eta)}\right]\right\}, \tag{22a}
\end{align*}
$$

and if the diffraction structure is located on the second lens surface then

$$
\begin{align*}
& C_{2}(\varphi, v, \tan \omega, n, \zeta, \eta)= \\
& \varphi^{2} \tan w\left\{v\left[v-\frac{1}{(n-1)(1+\zeta)(1+\eta)}\right]+\left[v-\frac{1}{(1+\zeta)(1+\eta)}\right]\left[\frac{1}{n(n-1)(1+\eta)}\right]\right. \\
& \left.-(v+1)\left[v+1-\frac{\zeta}{(1-n)(1+\zeta)(1+\eta)}\right]\right\} \tag{23b}
\end{align*}
$$

where

$$
\begin{equation*}
v=\frac{V}{\varphi} . \tag{24}
\end{equation*}
$$

It assures typical normalization applied in optics.
In both cases the aberration coefficients are independent of the parameter $\beta$ but depend on parameters $\eta$ and $\zeta$. The first parameter $\eta$ is defined uniquely by the conditions of chromatism correction. Additionally, in order to obtain the correction of coma it is sufficient and necessary to determine the parameter $\zeta$ by solving the equation

$$
\begin{equation*}
C(\varphi, v, \tan w, n, \xi, \eta)=0 . \tag{25}
\end{equation*}
$$



Fig. 4. Value of parameter $\zeta$ assuring correction of coma as a function of object position $v$ and the division of power $\eta$ between the refractive and diffraction parts of the lens when the diffraction structure is located on the first surface of the refractive lens (a) and on the second one (b).

This equation can be solved for the case of the diffraction structure deposited on the first or second refracting surface using computer symbolic calculations. Due to significant degree of complexity of the solution it is most convenient to present it in a graphic form (Fig. 4). Thus, it can be stated that after having fulfilled the condition of achromatization the requirement of coma correction, for the given object distance and converging power, leads to unique determination of the curvature radii of the two refracting surfaces. The remaining degree of freedom is determined by parameter $\beta$ and can be useful for correction of spherical aberration. In Table 1 the corresponding parameters for three exemplified lenses (collimating, imaging and focusing ones, all of focal length $f^{\prime}=100 \mathrm{~mm}$ ) are given.

Table 1. Design parameters for selected hybrid aplanatic lenses.

| Kind of lens | Focusing | Imaging | Collimating |
| :---: | :---: | :---: | :---: |
| $v$ | 0 | -0.5 | -1 |
| $s$ | $-\infty$ | -200 | $-100$ |
| Diffraction structure on the first surface | Example A | Example B | Example C |
| $\zeta$ | 0.07112 | 1.03998 | 20.3657 |
| $\rho_{1}$ | 58.80 mm | 111.99 mm | 824.10 mm |
| $\rho_{2}$ | $-826.77 \mathrm{~mm}$ | -107.68 mm | -58.82 mm |
| $\beta$ | 0.98984 | 0.988805 | 0.987005 |
| 2 | 20.958 mm | 23.093 mm | 26.806 mm |
| $z_{\beta}$ | 21.173 mm | 23.354 mm | 27.651 mm |
| Diffraction structure on the second surface | Example D | Example E | Example F |
| $\zeta$ | 0.04912 | 0.9616 | 14.06027 |
| $\rho_{1}$ | 57.59 mm | 107.68 mm | 826.77 mm |
| $\rho_{2}$ | $-1172.92 \mathrm{~mm}$ | $-111.99 \mathrm{~mm}$ | - 58.80 mm |
| $\beta$ | 0.986595 | 0.984560 | 0.980635 |
| $z_{\text {a }}$ | 27.652 mm | 31.850 mm | 39.946 mm |
| $z_{\beta}$ | 28.028 mm | 32.349 mm | 40.735 mm |

### 2.3. Spherical aberration

Spherical aberration of a hybrid lens with diffraction structure can be expressed as follows:

$$
\begin{aligned}
& S(\varphi, n, v, \eta, \zeta, \beta)= \\
& \varphi^{3}\left\{v\left[v+\frac{1}{(n-1)(\zeta+1)(\eta+1)}\right]^{2}-(v+1)\left[v+1+\frac{\xi}{(n-1)(\zeta+1)(\eta+1)}\right]^{2}\right.
\end{aligned}
$$

$$
\begin{align*}
& -\frac{1}{n^{2}}\left[v+\frac{1}{(\zeta+1)(\eta+1)}+(2-k)-\frac{\eta}{\eta+1}\right] \\
& {\left[v+\frac{1}{(\zeta+1)(\eta+1)}+(2-k) \frac{\eta}{\eta+1}+\frac{n}{(n-1)(\zeta+1)(\eta+1)}\right]^{2}} \\
& +\frac{1}{n^{2}}\left[v+\frac{1}{(\zeta+1)(\eta+1)}+(2-k) \frac{\eta}{\eta+1}\right] \\
& {\left[v+\frac{1}{(\zeta+1)(\eta+1)}+(2-k) \frac{\eta}{\eta+1}+\frac{\xi n}{(n-1)(\zeta+1)(\eta+1)}\right]^{2}} \\
& +\frac{\eta}{(1-\beta)(\eta+1)}\left[\frac{\eta}{(1-\beta)(\eta+1)}-\frac{1}{(n-1)(\zeta+1)(\eta+1)}\right]^{2} \\
& -\frac{\beta \eta}{(1-\beta)(\eta+1)}\left[\frac{\beta \eta}{(1-\beta)(\eta+1)}-\frac{1}{(n-1)(\zeta+1)(\eta+1)}\right]^{2} \tag{26}
\end{align*}
$$

where $k$ takes the values 1 or 2 , respectively, depending on the location of the diffraction structure on the first or second surface of the lens.

The condition for spherical aberration correction

$$
\begin{equation*}
S(\varphi, v, n, \eta, \zeta, \beta)=0 \tag{27}
\end{equation*}
$$

can be found using the computer symbolic calculations. This can be shown to be possible if

$$
\begin{equation*}
\xi=-\frac{\rho_{1}}{\rho_{1}} \neq-1 \tag{28a}
\end{equation*}
$$

and

$$
\begin{equation*}
\beta=\frac{z_{\alpha}}{z_{\beta}} \neq-1 . \tag{28b}
\end{equation*}
$$

are fulfilled.
For a given value of parameter $\eta$ (assuring achromatisation, for instance) there exists a connection between the parameters $\zeta$ and $\beta$ for which Eq. (27) is satisfied. Graphic illustration of this relation is shown in Fig. 5, for different positions of the point objects. The result of this stage of designing is the determination of connection between the parameters $\beta$ and $\zeta$ assuring the correction of the spherical aberration. The distances $z_{\alpha}$ and $z_{\beta}$ are calculated from formula (11). Assuming the values of the parameter $\xi$ determined during the process of coma correction (Fig. 4) and choosing suitable value of the parameter $\beta$ (Fig. 5) a hybrid lens of both coma and spherical aberration being corrected is possible to obtain. In the last three lines of Tab. 1, the parameters describing the diffraction part of a hybrid lens free from spherical aberration have been given. In this way we have obtained all the design parameters $\eta, \zeta, \beta$ of a hybrid lens assuring achromatisation and aplanatic correction for a given position of the object, given power of the lens and defined achromatisation range.


Fig. 5. Relation between the parameters $\beta$ and $\zeta$ assuring spherical aberration correction for different positions of the object point described by the parameter $v$ for the case when the diffraction structure is located on the first surface of the refractive lens (a) and on the second one (b).

### 2.4. Aplanatic lens - illustration

In order to illustrate the correction obtained, the imaging quality of two exemplified lenses denoted in Tab. 1 as E and F has been analysed. The quality of imaging can be examined using the methods of geometrical and wave optics. The methods based on geometrical optics are insufficient for many optical systems though on their basis it is easy to determine the correction of the aberrations of interest. The examinations of aberration correction of our lenses were carried out by determining the selected characteristics from the point of view of both geometrical optics and wave optics. The complete characteristic of imaging would be redundant and its presentation is not necessary. Both the lenses were of $f$-number 10 and focal length 100 mm . In Figure 6, the spot-diagrams for the lens $E$ are shown for selected field angles $\tan w=0,0.03,0.05$. In Figure 7, geometrical aberrations are shown for the same


Fig. 6. Hybrid lens $E$ - spot-diagrams for selected field angles: $\tan w=0$ (a), $\tan w=0.03$ (b), $\tan w=0.05$ (c).


Fig. 7. Hybrid lens E: a - transversal spherical aberration, b - deviation from the sine condition, c - field curvature.


Fig. 8. Hybrid lens $F-$ aberration spot for selected field angles: $a-\tan w=0, b-\tan w=0.03$, c $-\tan w=0.05$.


Fig. 9. Hybrid lens $\mathbf{F}-$ wave aberrations for selected field angles: $\mathbf{a}-\tan w=0, b-\tan w=0.03$, $c-\tan \boldsymbol{w}=0.05$.
lens. This is a typical description of lens aberration used in geometrical optics. Figure 8 deals with the hybrid lens $\mathbf{F}$ and shows the point spread function for three field angles, while in Fig. 9 the wave aberrations for the same field angles are presented. In this case, the description typical of wave optics was exploited.

## 25. Astigmatism, field curvature, distortion

As is well known astigmatism is independent of parameters $\eta, \zeta, \beta$ (14) and, consequently, it cannot be compensated by any choice of their values. The field
curvature depends formally on parameter $\eta$ (see formula (15))

$$
\begin{equation*}
F=-y^{2} v^{2} \varphi^{3}\left(\frac{\eta}{1+\eta}+\frac{1+n}{n(1+\eta)}\right), \tag{29}
\end{equation*}
$$

and thus can be corrected if the condition

$$
\begin{equation*}
\eta=-\frac{1+n}{n} \tag{30}
\end{equation*}
$$

is fulfilled.
In practice, this would mean that the chromatism is unacceptable. However, additional possibilities of correction appear if it is assumed that the entrance pupil of the hybrid lens is shifted with respect to the lens (Fig. 10).


Fig. 10. Hybrid lens with shifted entrance pupil.
The shift of the entrance pupil by the distance $z_{t}$ corresponds to the shift of the active aperture centre by the distance $y_{t}$. For the object located at infinity

$$
\begin{equation*}
y_{t}=-z_{t} \tan w, \tag{31a}
\end{equation*}
$$

while when the object is at a finite distance

$$
\begin{equation*}
y_{\mathrm{t}}=\frac{y v z_{\mathrm{t}} \varphi}{1-v z_{\mathrm{t}} \varphi} . \tag{31b}
\end{equation*}
$$

Formally, this can be accounted for by changing the variables in the eiconal

$$
\begin{equation*}
\bar{y}=y-y_{t} . \tag{32}
\end{equation*}
$$

Grouping suitably the expressions the "new" coefficients describing the third order aberrations can be expressed by the "old" ones as follows:

$$
\begin{align*}
& S_{t}=\bar{S},  \tag{33a}\\
& C_{t}=\bar{C}-y_{t} \bar{S},  \tag{33b}\\
& A_{t}=\bar{A}-2 y_{t} \bar{C}+y_{t}^{2} \bar{S},  \tag{33c}\\
& F_{t}=\bar{F}-2 y_{t} \bar{C}+\bar{y}_{t}^{2} \bar{S},  \tag{33d}\\
& D_{\mathrm{t}}=\bar{D}-2 y_{t} \bar{A}-y_{t} \bar{F}+3 y_{t}^{2} \bar{C}-y_{t}^{3} \bar{S}, \tag{33e}
\end{align*}
$$

where coefficients $\bar{S}, \bar{C}, \bar{A}, \bar{F}, \bar{D}$ have formally the same form as the corresponding aberration coefficients for a lens with the entrance pupil adjacent to the thin lens. However, the coordinate $y$ must be replaced by the coordinate $\bar{y}$ consistently with formula (32). The complete explicit form of formulae (33) is too complex and unreadable. Having the possibility of exploiting the computer symbolic calculation makes writing down these formulae unnecessary.

Since the spherical aberration does not depend on the $y$ coordinate, the following equation holds:

$$
\begin{equation*}
S_{\mathrm{t}}=S \tag{34}
\end{equation*}
$$

If the object is localized at infinity the coefficient describing the other aberrations for the pupil at contact with the lens are independent of $y$ coordinate, as well. Thus:

$$
\begin{align*}
& \bar{C}=C,  \tag{35a}\\
& \bar{A}=A,  \tag{35b}\\
& \bar{F}=F,  \tag{35c}\\
& \bar{D}=D . \tag{35d}
\end{align*}
$$

If the chromatic and spherical aberrations are fully corrected while some uncompensated coma remains one of the field aberrations can be corrected. The corresponding conditions are as follows:

- for astigmatism

$$
\begin{equation*}
y_{t A}=\frac{A}{2 C}, \tag{36a}
\end{equation*}
$$

- for field curvature

$$
\begin{equation*}
y_{t F}=\frac{F}{2 C}, \tag{36b}
\end{equation*}
$$

- for distortion

$$
\begin{equation*}
y_{t D}=\frac{A}{2 C}\left(1 \pm \sqrt{1-\frac{4 C D}{3 A}}\right) . \tag{36c}
\end{equation*}
$$

The desired shift of the pupil is determined from formulae (31). It can be shown that astigmatism is compensated for a given object distance and a definite shift of the entrance pupil, if its coma is

$$
\begin{equation*}
C=\varphi \tan w \frac{z_{t} v \varphi-1}{2 z_{t}} . \tag{37}
\end{equation*}
$$

The lens will be characterized by such a coma if its shape is defined by parameter $\zeta$ following from equation

$$
\begin{equation*}
C(\varphi, n, v, \eta, \xi)=\varphi \tan w \frac{z_{\imath} v \varphi-1}{2 z_{t}} \tag{38}
\end{equation*}
$$

analogically as it was the case in Sec. 2.2.

### 2.6. Lens of corrected field aberrations - an illustration

In order to illustrate the aforementioned possibilities of correction of astigmatism and field curvature by shifting the pupil a hybrid lens of uncompensated coma was

Table 2. Design parameters for selected hybrid lenses of corrected astigmatism and field curvature.
$v_{1}=0$ (focusing lens)
$\eta=0.05095$
Diffraction structure on the first surface of refractive lens
$\zeta_{1}=0$ (plano-convex lens)
$\rho_{1}=\infty$
$\rho_{2}=-54.897 \mathrm{~mm}$
$\beta_{1}=0.986981$
$z_{\mathrm{a}}=26.8544 \mathrm{~mm}$
$\mathrm{z}_{\mathrm{B}}=27.2086 \mathrm{~mm}$
$z_{t}=-21.5 \mathrm{~mm}$ (astigmatism correction - example G)
$z_{t}=-54.5 \mathrm{~mm}$ (field curvature correction - example H )
$\Delta y^{\prime}[\mathrm{mm}]$

a

$\Delta y^{\prime}[\mathrm{mm}]$


C

Fig. 11. Hybrid lens $\mathbf{G}-$ spot-diagrams for selected field angles: $\mathbf{a}-\tan w=0, \mathbf{b}-\tan w=0.03$, c $-\tan w=0.05$.


Fig. 12. Hybrid lens G: a - transversal spherical aberration, b - deviation from the sine condition, c - field curvature.


Fig. 13. Hybrid lens $G-$ aberration spot for selected field angles: $\mathbf{a}-\tan w=0, b-\tan w=0.03$, c $-\tan w=0.05$.


Fig. 14. Hybrid lens $G-$ wave aberration for selected field angles: $a-\tan w=0, b-\tan w=0.03$, $c-\tan w=0.05$.
chosen. Its design parameters are collected in Tab. 2. The value of the pupil shift assuring respective correction of both astigmatism and field curvature was calculated based on formulae (36). They are cited in two last lines of Table 2. The chosen imaging characteristics of the above lenses are shown in Figs. 11-15. Figures 11-14 concern a lens free of astigmatism denoted in Table 2 by symbol $G$. The spot-diagrams for different field angles are shown in Fig. 11, geometrical aberrations are presented in Fig. 12. In Figure 13 the point spread functions are shown while the wave aberrations for selected field angles of the above lens are presented in Fig. 14. The last Figure 15 refers to the hybrid lens $H$ with corrected field curvature. In this figure it is visible that the plane of best imaging is practically flat.


Fig. 15. Hybrid lens $\mathbf{H}$ - longitudinal cross-section of the aberration spot for selected field angles: a $-\tan w=0, b-\tan w=0.03, c-\tan w=0.05$.

## 3. Final remarks

A single hybrid element can have different applications. It can constitute a part of a complex optical system, a photo objective, for instance [1], or be an independent element. The application of hybrid lens to correction of eyepiece seems to be especially interesting. An accurate knowledge of the imaging characteristics of a hybrid lens renders the design of a hybrid eyeglass lens possible. It seems also that the hybrid element can be of significance when looking for new solutions in designing progressive glasses. This problem, however, requires some more examinations and designing works.

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