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## An Efficient Elastooptic Light Modulator


#### Abstract

An elastooptic modulator is used in a polarimeter as a replacement of a Pockels cell. The performed calculations and experimental results show that the efficiency of the used modulator is higher if the aperture is smaller than the whole modulating transparent bar in which a $\lambda / 2$ ultrasonic standing wave is generated. The predicted modulation efficiency reaches $99 \%$.


## 1. Introduction

Measurements of the emission anisotropy of molecular fluorescence provide information about the structure of molecules and their environment. A photoelectric method for accurate measurements of the polarization of low light levels from fluorescent solutions based on modulation of linearly polarized part of the light was reported by BaUER and Rozwadowski [1]. A Pockels cell has been utilized as a 1 kHz light modulator [2]. This cell produces a $\lambda / 4$ phase shift (for $\lambda=500 \mathrm{~nm}$ ) between the extraordinary beams when 4500 V is applied. Due to small angular aperture a parallel light beam is required. To further improve the signal-to-noise ratio a new 50 kHz elastooptic standing wave modulator was developed which in comparison with Pockels cells has extremely low power requirements. This method of light intensity modulation was described first by Hiedeman and Hoesch [3] as well as Bonch-Bruievich [4] and later utilized in a phase fluorometer [5], in optical communication [6], for measurements of circular dichroism and detection of a weak birefringence [7].

## 2. Theory

An elastooptic modulator (Fig. 1) consists of a polarizer $P$, a quartz or glass bar (a perpendicular parallelepiped) with an ultrasonic

[^0]

Fig. 1. Elastooptic light modulator
$L S$ - light source, $L$ - lenses, $P$ - polarizer, $K$ ultrasonic transducer, $\lambda / 4$ - quarter-wave plate, $A$ analyzer, $P h M$ - photomultiplier
transducer $K$ generating a fundamental $\Lambda / 2$, mode in it, an analyzer $A$ and eventually a $\lambda / 4$ plate. The polarization planes are mutually perpendicular and positioned under the angle of $45^{\circ}$ to the direction of the ultrasonic wave propagation ( $z$-axis).

### 2.1. Modulation without a $\lambda / 4$ plate

The exit flux is described by the following expression:

$$
\begin{align*}
& I=\frac{I_{0}}{D} \int \sin ^{2}[\alpha(z) / 2] d z \\
& \quad=\frac{I_{0}}{2 D} \int[1-\cos (A \cos K z)] d z \tag{1}
\end{align*}
$$

where
$I_{0}$ - exit flux for $A$ and $P$ parallel, withont stress in the bar,

$$
a(z)=\frac{2 \pi}{\lambda} c p_{0} \cos \Omega t \cos K z=a \cos \Omega t \cos K z
$$

$$
=A \cos K z
$$

$D$ - length of aperture along the $z$-axis,
$\lambda$ - light wavelength,
$c$ - elastooptic coefficient,
$p_{0}$ - strain amplitude introduced in the bar by the ultrasonic wave,
$\Omega, K=2 \pi / \Lambda$ - the frequency and the wave vector of the ultrasonic wave, respectively.

For the aperture covering the front face of the bar i.e. for $D=\Lambda / 2$ calculations of integral (1) were performed by Bonch-Bruievich [4], who obtained the following expression for the exit flux:

$$
\begin{align*}
& I(t) \\
& \begin{aligned}
&=I_{0}\left[W_{0}(a)+W_{2}(a) \cos 2 \Omega t+W_{4}(a) \cos 4 \Omega t+\right. \\
&+\ldots] .
\end{aligned}
\end{align*}
$$

The coefficient $W_{0}(a), \quad W_{2}(a), \quad W_{4}(a)$ and the maximum of the modulated light intensity as a function of $a$ are presented in Fig. 2.


Fig. 2. Expansion coefficients (eq. (2)) vs. parameter a

$$
\begin{array}{ll}
---- & W_{0}(a) \\
--\cdots- & W_{2}(a) \\
-\cdots-\cdots- & W_{4}(a) \\
& W_{0}(a)+W_{2}(a)+W_{4}(a)
\end{array}
$$

As shown below the symmetrical reduction of the aperture results in a higher efficiency and a lower content of harmonics. Calculation of the integral (1) for a rectangular aperture of the height $D=\Lambda / 4$, which in our case happens to be a $30 \times 30 \mathrm{~mm}$ square, gives the following expression:

$$
\begin{align*}
& I(t) \\
& \begin{aligned}
= & I_{0}\left[W_{0}(a)+W_{2}(a) \cos 2 \Omega t+W_{4}(a) \cos 4 \Omega t+\right. \\
& +\ldots]
\end{aligned}
\end{align*}
$$

The parameters $W_{0}(a), W_{2}(a), W_{4}(a)$ and the maximum light intensity $I(t=0, a) / I_{0}$
$=W_{0}(a)+W_{2}(a)+W_{4}(a)$ were calculated by means of a computer as a function of $a$ and are presented in Fig. 3.


Fig. 3. Expansion coefficients (eq. (3)) vs. parameter a

$$
\begin{array}{ll}
\cdots \cdots \cdots-\cdots & W_{0}(a) \\
-\cdots-\cdots- & W_{2}(a) \\
\cdots \cdots-\cdots- & W_{4}(a) \\
& W_{0}(a)+W_{2}(a)+W_{4}(a)
\end{array}
$$

From Fig. 2 and Fig. 3 the optimum values of the parameter $a=\frac{2 \pi}{\lambda} c p_{0}$ and the corresponding values of efficiency and signal distortion can be determined. The results are presented in table.

Calculated optimum values of efficiency and signal distortion

| Method | $a=\frac{2 \pi c p_{0}}{\lambda}$ | Efficiency <br> $I_{\text {max }}$ | $W_{2}$ | $W_{4}$ | Signal <br> distor- <br> tion <br> $W_{4}!W_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Bonch- <br> Bruie- <br> vich [4] | 3.83 | $70 \%$ | 0.34 | 0.11 | $33 \%$ |
| Present <br> authors | 3.36 | $96 \%$ | 0.43 | 0.14 | $33 \%$ |

### 2.2. Modulation with a $\lambda / 4$ plate

In this case

$$
\begin{align*}
I= & \frac{I_{0}}{D} \int_{-D / 2}^{+D / 2} \sin ^{2}[\alpha(z) / 2+\pi / 4] d z \\
& =\frac{I_{0}}{2 D} \int_{-D / 2}^{+D / 2}[1+\sin (A \cos K z)] d z \tag{4}
\end{align*}
$$

The same procedure as above gives the following formula for the exit flux:

$$
\begin{align*}
& I(t) \\
& \begin{aligned}
=I_{0}\left[0.5+W_{1}(a) \cos \Omega t+W_{3}(a)\right. & \cos 3 \Omega t+ \\
& +\ldots]
\end{aligned}
\end{align*}
$$

The values $W_{1}(a), W_{3}(a)$ and the maximum light intensity $I(t=0, a) / I_{0}=0.5+W_{1}(a)+$ $+W_{3}(a)$ calculated as a function of $a$ are presented in Fig. 4.


Fig. 4. Expansion coefficients (eq. (5)) vs. parameter a

$$
\begin{array}{ll}
---\cdots- & W_{1}(a) \\
-\cdots-\cdots- & W_{3}(a) \\
- & 0.5+W_{1}(a)+W_{3}(a)
\end{array}
$$

In this case for $a=1.7$ it is possible to obtain $99 \%$ efficiency and $13 \%$ signal distortion and the modulation frequency is equal to that of ultrasonic wave.

Assuming that $a$ is a monotonically increasing function of the applied voltage it should be expected that power requirements are lower if the aperture is smaller than the whole transparent bar section and is located in the middle of the bar, i.e. where the strain reaches its maximum value. If frequency doubling is not essential this power requirements are even lower due to the use of the $\lambda / 4$ plate.

## 3. Experimental

### 3.1. Elastooptic element

The measurements were carried out by means of the arrangement presented in Fig. 1. The elastooptic element consisted of a quartz
or glass bar and a piezoelectric ceramic transducer (Fig. 5) bonded with "epoxy" resin. The cuboidal ceramic transducer was made of modified lead zirconate titanate (PZT). Two opposite surfaces of the transducer were silvered and served as electrodes. The chemical


Fig. 5. Elastooptic element
1 - quartz plate, 2 - ceramic element, 3 - silver electrodes
formula of the ceramic material being used is the following:

$$
\begin{equation*}
\mathrm{Pb}_{0.94} \mathrm{Sr}_{0.06}\left(\mathrm{Zr}_{0.52} \mathrm{Ti}_{0.48}\right) \mathrm{O}_{3} \tag{6}
\end{equation*}
$$

and its physical properties are the following: $\varrho($ density $)=7.55 \mathrm{~g} / \mathrm{cm}^{3}$,
$\dot{Y}_{1}^{E}$ (Young's modulus) $=6.2-6.8 \quad 10^{10} \mathrm{~N} / \mathrm{m}^{2}$,
$k_{31}$ (electromechanical coupling coefficient)
$=28 \%$,
$k_{33}$ (electromechanical coupling coefficient) $=65 \%$,
$\varepsilon_{33}^{T} / \varepsilon_{0}$ (dielectric constant) $=900$.
The elastooptic element was assumed to have a resonant frequency - 47.7 kHz . The sizes of $L_{k}$ and $L_{c}$ were chosen so as to form two $\lambda / 2$ sections. The $W_{c}$ and $T_{c}$ dimensions of the ceramic segment were selected to ensure high mechanical strength of the whole element as well as good matching of the acoustic impedances:

$$
\begin{equation*}
Z=W_{k} T_{k} \sqrt{\varrho_{k} Y_{k}}=W_{c} T_{c} \sqrt{\varrho_{c} Y_{1}^{E}} \tag{7}
\end{equation*}
$$

To meet these requirements the cross-section surface of the quartz plate was about 80 percent larger than that of the ceramic element. The final correction of the $L_{c}$ dimension for the two-point supporting system ( $a, b$ lines of Fig. 5) was achieved by grinding the unbonded surface to obtain the lowest possible


Fig. 6. Oscilloscope traces
a) $U_{q}=9 \mathrm{~V}, \quad U_{9}=50 \mathrm{~V}$,
b) $U_{q}=15 \mathrm{~V}, U_{g}=85 \mathrm{~V}$,
c) $U_{q}=40 \mathrm{~V}, U_{g}=170 \mathrm{~V}$,
d) $U_{\boldsymbol{q}}=40 \mathrm{~V}, U_{t}=170 \mathrm{~V}$
( $\mathrm{a}, \mathrm{b}$ and $\mathrm{c}-$ with $\lambda / 4$ plate, $\mathrm{d}-$ without $\lambda / 4$ plate)
level of the input impedance at the resonant frequency. The piezoelectric element was driven directly by an RC generator with an output impedance of $1000 \Omega$.

### 3.2. Results

The modulated light flux, attenuated appropriately to avoid saturation was detected by a photomultiplier. The resulting photocurrent was monitored with a broad band oscilloscope. The oscilloscope traces are presented in Fig. 6. They were obtained using both quartz and glass bars in the modulating arrangement shown in Fig. 1 (except for Fig. 6d where a $\lambda / 4$ was not used). The values given in the captions of Fig. 6 are the effective voltages applied to the piezoelectric transducer ( $U_{q}$ - for quartz, $U_{g}$ - for glass).

One can notice the predicted reduction of the applied voltage for case 6 a (with a $\lambda / 4$ plate) in comparison with case $6 d$ (without a $\lambda / 4$ plate). The degree of modulation, defined as

$$
\begin{equation*}
m=\frac{I_{\max }-I_{\min }}{I_{\max }+I_{\min }} \tag{8}
\end{equation*}
$$

reached $98 \%$, but the efficiency $I_{\text {max }} / I_{0}$ was $88 \%$ (here $I_{0}$ is the unmodulated light intensity observed with parallel polarizers). An almost sine-wave modulation was achieved with a relatively low driving voltage (Fig. 6). Higher driving voltages (higher a values in Eqs. (3) and (5)) produced a modulation with an increasing influence of higher harmonics on the shape of the modulated light intensities ( $W_{3}$ and $W_{4}$ in Eqs. (3) and (5)).

Modulateur efficace photo-élastique de lumière

Au lieu de la cellue de Pockelks dans le polarimètre on a utilisé un modulateur élastoplastique. Sur la base des calculs et des résultats de l'expérience on a constaté, que l'efficacité d'un tel modulateur est plus grande si l'orifice est plus petit que tout le barreau transparent modulant, qui produit une onde stationnaire $\lambda / 2$. L'éfficacité de modulation prévue atteint $99 \%$.

## References

Вместо ячейки Поккельса применили в поляриметре эластооптический модулятор. На основе расчета и результатов эксперимента обнаружено, что эффективность применного модулятора становится выше, если отверстие меньше, чем весь модулирующий прозрачный стержень, в котором возбуждается ультразвуковая стоячая волна $\lambda / 2$. Предусматриваемыћ к. п. д. модуляции достигает 99\%.
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