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## **THEORETICAL BASIS FOR DETERMINING ROLLING RESISTANCE OF BELT CONVEYORS**

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**Abstract:** Conveyor belts are the most common way of mechanical handling equipment in industry, especially in underground and open-pit mines. Maintenance of conveyor belts generate high costs, therefore energy savings belts have become popular in recent years. Constant development of continuous transport equipment and looking for savings implies necessary of carrying advanced theoretical research and analysis. In this case determine of indentation rolling resistance is the clue. Based on previous research authors suggested new theoretical model of determination rolling resistance. Authors proofed that stress distribution described in time coordinates coexists with lateral deformation of belt. In two dimensional Kelvin–Voigt model it has two components: the particular solution of the differential equation of belt’s model and an general solution (which is typical for harmonic load). In this new model authors included overlooked by others particular solution; that gives a possibility of designating the whole spectrum of changes in the value of the rolling resistance. The obtained results allow to specify new and more accurate damping factor.

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**Keywords:** *rolling resistance, belt, idler, conveyor belt, damping factor*

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### 1. INTRODUCTION

Conveyor belts are the most common way of mechanical handling equipment in industry, especially in underground and open-pit mines. Despite the many advantages, maintenance of conveyor belts generate high costs, therefore energy savings belts have become popular in recent years (Gładysiewicz, 2003). The effect of lowering the rolling resistance as a result of using energy savings belts with special parameters was proved by many experiments, for example in the process of the cyclic compression of belt or in rig with inclined plane (Wrocław University of Science and Technology).

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Constant development of continuous transport equipment and looking for savings implies necessary of carrying advanced theoretical research and analysis (Król, 2013). Such analysis has to be based on full recognition and description of phenomena on belt conveyors (Harrison, 2009). In this case, determine of indentation rolling resistance is the clue because it is the main part of the primary resistances (Gładysiewicz, 2003).

#### SYMBOLS

$q_t$  – unite vertical load  
 $\rho$  – the radius of curvature of the tape at the point of support by idler  
 $\varphi_1, \varphi_2$  – wrap angle and convergence belt from idler  
 $\sigma$  – compressive stresses  
 $\xi$  – measure of belt's damping  
 $a$  – shift the axis of the load  
 $w_e$  – unit rolling resistance of the belt  
 $t_0$  – duration of a single cycle load  
 $\omega_0$  – angular velocity of idler  
 $\delta$  – phase lag angle  
 $\varepsilon$  – transverse strains  
 $\varepsilon_{max}$  – maximal transverse strains in belt  
 $\varepsilon_1$  – particular solution (viscous flow)  
 $\varepsilon_2$  – general solution (typical for harmonic load)  
 $\tau_0$  – time constant model of belt  
 $t_m$  – time to maximum deflection of belt  
 $D_k$  – diameter idler roll  
 $E_c$  – modulus of elasticity  
 $\Phi$  – damping function  
 $z_f$  – linear contact length of idler and belt  
 $z_e$  – effective contact length of idler and belt  
 $s_0$  – depth crumple zones  
 $\lambda$  – coefficient of bending belt on idler  
 $h_0$  – open belt thickness participating in the process of compression  
 $c_e$  – unit stiffness transverse belt  
 $F$  – damping factor  
 $e_1$  – unit elastic energy transferred to belt  
 $e_2$  – energy recovered

## 2. BELT CONVEYOR MAIN RESISTANCE COMPONENTS

Belt's progressive move generates lots of occurrence causing various forms of energy conversion. Every one of them is responsible for different component of movement resistance. There are three main groups of movement resistance movement of conveyor belts (Gładysiewicz, 2003):

- occurring at the headend, drive feedback , tension and loading focused resistance
- accompanying the movement of the belt along the entire route of the conveyor spread resistance (also called primary resistance)
- occurring only on the sloping sections of conveyor lifting resistance.

In over 80 m long belt conveyors dominate primary resistance. Depending on the energy conversion they can be are divided into (Gładysiewicz, 2003):

- idler rotational resistance  $W_k$
- belt-on-idler rolling (indentation) resistance  $W_e$
- belt bending resistance (flexure resistance of a belt)  $W_b$
- flexure resistance of bulk material  $W_f$
- sliding resistance of a belt on idlers  $W_r$ .

Idler rotational resistance is due to the phenomena of energy conversion in knots and seals bearing idler rollers. Belt bending resistance is connected with cyclical bends belt between the sets of idlers. Flexure resistance of bulk material is caused by cyclical deformations of ore stream during belt bending. Belt-on-idler rolling resistance is connected with indentation of belt's bottom cover by idler. Sliding resistance of a belt on idlers appeared in contact zone between belt and idler. The percentage of individual components of primary resistance in top tendon of conveyor is shown below in fig. 1 (Gładysiewicz, 2003).

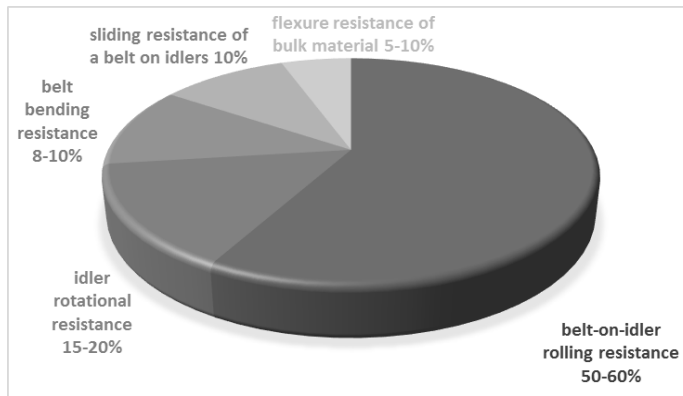


Fig. 1. The percentage of individual components of primary resistance in top tendon of conveyor

As it is seen in the graph (fig. 1), rolling (indentation) resistance represents almost 60% of the whole primary resistance. Therefore seeking belts that generate lower rolling resistance is fully justified.

### 3. METHODS OF EXAMINATION THE ROLLING RESISTANCE OF CONVEYOR BELTS

#### 3.1. VISCOELASTIC PROPERTIES OF A BELT

The indentation of the rubber bottom cover depends mainly on its viscoelastic properties and in lesser extent from the core. Theoretical analysis are based on experiments, therefore examination of belt and defining its parameters are so important. The most significant part of it is defining damping factor of rubber  $tg\delta$ . This factor can be expressed as the ratio of the loss modulus to the storage modulus (Gładysiewicz, 2003).

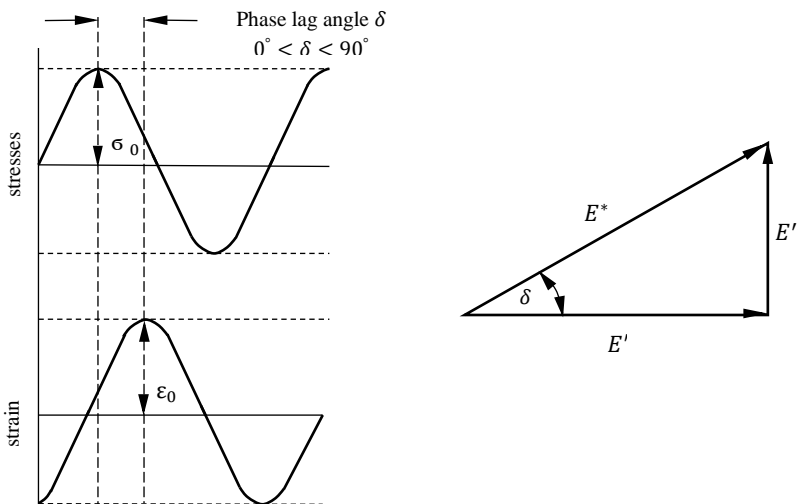


Fig. 2. Correlation between stresses and strain for visco-elastic rubber (O'Shea et al., 2014)

Damping factor can be examined by squeezing belt periodically. The lower damping factor is the lower would be rolling resistance for the rubber compound. It is important to remember that this factor depends on many parameters such as the load exerted on the belt or environment temperature (Drenkelford, 2015).

### 3.2. DESIGNATION OF DAMPING FACTOR IN THE PROCESS OF THE CYCLIC COMPRESSION

Designation of damping factor in laboratory is carried out on sample between two parallel steel plates. In experiment harmonically variable compressive load is forced and simultaneously deformation and stress are recorded. Based on these measurements can be created hysteresis loop. In first cycles of loading is observed variables behavior of the sample. Only after a few cycles stabilization is reached and after that hysteresis loop can be designed. Hysteresis loop is used to determine damping factor and transverse elasticity module of belt. According to the scheme in fig. 3 damping factor responds the ratio of the contained inside the hysteresis loop (energy changed) to the field below the load curve (energy delivered). Elasticity modulus of the belt can be defined as the angle created by the slope of a straight line which connects two vertices of the loop (Gładysiewicz, 2003).

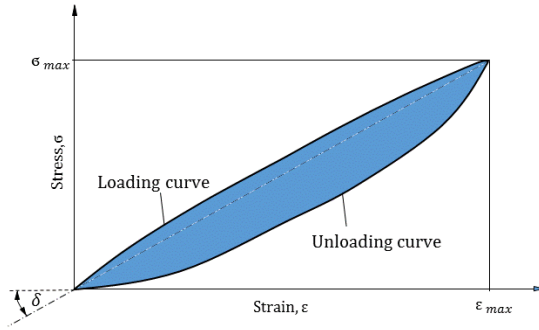


Fig. 3. Example of hysteresis loop in cyclical compression

## 4. BASIC ASSUMPTIONS OF THE MODEL

Model of belt laying on idler with vertical unit load  $q_T$  (generated by weights of belt and transported ore) is shown on fig. 4. The longitudinal axis of the strip in contact zone with idler has a curvature with radius  $\rho$ . Contact zone between belt and idler in cross-section to idler axis can be described by two angles:  $\varphi_1$  i  $\varphi_2$ . For small angles it can be simply transformed into linear system, where the section due to damping inside the belt and way of support is not symmetrical and  $\varphi_1 > \varphi_2$ . Belt's damping ( $\xi$ ) can be defined as ratio of the wrap angle to the convergence angle of belt from idler.

$$\xi = \frac{\varphi_2}{\varphi_1} \quad (1)$$

It is easy to see that when damping do not appear  $\xi = 1$  and  $\varphi_1 = \varphi_2$  while vertical unit load  $q_T$  operates along a vertical axis of idler. For small angles shift lines of vertical unit  $a$  with respect to idler axis equals:

$$a = \frac{D_K}{2} \cdot \frac{\varphi_1 - \varphi_2}{2} \tag{2}$$

$$a \cdot q_T = w_e \cdot \frac{D_K}{2} \tag{3}$$

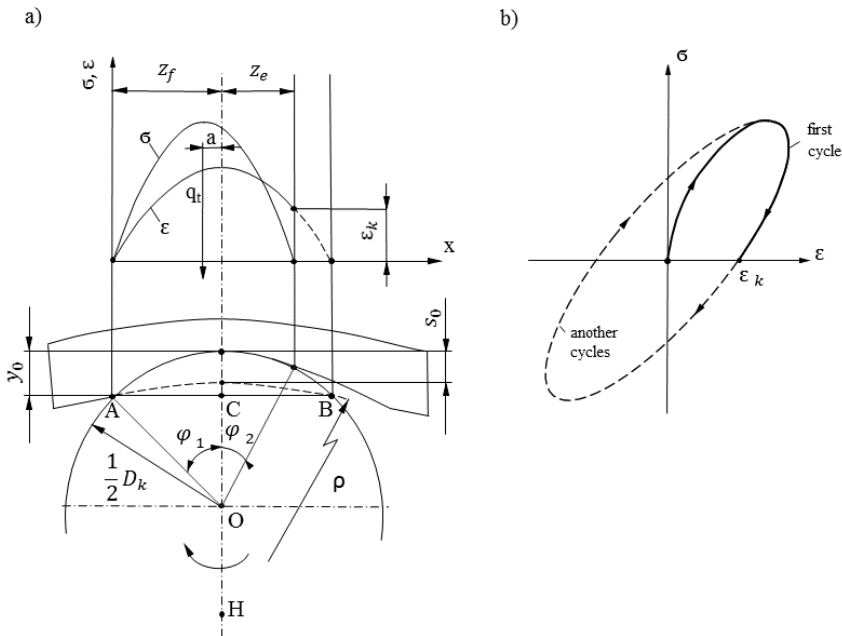


Fig. 4. a) stress and strain distribution in contact zone between belt and idler set  
 b) hysteresis loop for first and another cycles of loading (based on Gładysiewicz, 2003)

Unit indentation resistance of belt  $w_e$  determined from the condition of equilibrium of moments the axis of the idler:

$$w_e = q_T \frac{\varphi_1 - \varphi_2}{2} = \frac{1}{2} q_T \varphi_1 (1 - \xi) \tag{4}$$

Distribution of compressive stress of belt in the contact zone with the idler shows the equation:

$$\sigma = \sigma_0 \cdot \sin\left(\frac{\pi \cdot \varphi}{\varphi_1 + \varphi_2}\right) \quad (5)$$

In analysis it is also possible to turn into a description in coordinates of time. Then we use the following compounds:

$$\varphi = \omega_0 \cdot t \quad (6)$$

With the transition to time coordinates commonly used in the consideration of belts models it is important to keep in mind the following boundary conditions:

$$\text{for } \varphi = 0 \quad t = 0 \quad \omega_0 \cdot t = 0 \quad (7)$$

$$\text{for } \varphi = \varphi_1 + \varphi_2 \quad t = t_0 \omega_0 \cdot t_0 = \pi \quad (8)$$

If rotation angle  $\varphi = \varphi_2$  reached after the time  $t = t_m$  then:

$$\frac{\varphi_1}{\varphi_1 + \varphi_2} = \frac{t_m}{t_0} \quad \text{or} \quad t_m = t_0 \cdot \frac{\varphi_1}{\varphi_1 + \varphi_2} = \frac{t_0}{1 + \xi} \quad (9)$$

Including boundary condition (8) which is  $\omega_0 \cdot t_0 = \pi$  we get:

$$\omega_0 \cdot t_m = \frac{\pi}{1 + \xi} \quad (10)$$

During passing through supporting idlers, belt a part of the rubber is indented as a result of external forces. This phenomena repeated cyclically and because of visco-elastic properties of belt every time part of energy is dissipated. Deformation of belt extend beyond contact zone between belt and idler. Such deformation are called viscous flow (Gładysiewicz, 2003).

Distribution of stress in the function of time can be described by equation:

$$\sigma = \sigma_0 \cdot \sin(\omega_0 t) \quad (11)$$

Accompanied by lateral deformation of belt, which for two dimensional model of Kelvin-Voigt has two components:  $\varepsilon_1$  (particular solution of the differential equation of belt's model) and  $\varepsilon_2$  (general solution). Particular solution  $\varepsilon_1$  described viscous flow caused by subsequent reactions on the idler support with load breaks, when belt goes to another idler set. General solution is typical for harmonic load and characterized by a phase shift  $\delta$ (delay distortion relative to forcing stress).

$$\varepsilon_1 = \frac{\sigma_0}{2E_c} \cdot \sin(2\delta) \cdot e^{\frac{-t}{\tau_0}} \quad (12)$$

$$\varepsilon_2 = \frac{\sigma_0}{E_c} \cdot \cos \delta \cdot \sin(\omega_0 t - \delta) \quad (13)$$

Connection between coordinate time in belt's model  $\tau_0$  and phase shift  $\delta$  is described below:

$$\omega_0 \cdot \tau_0 = tg\delta \quad (14)$$

Phase shift angle  $\delta$  can be described as:

$$\delta = \frac{\pi}{2} \cdot \frac{\varphi_1 - \varphi_2}{\varphi_1 + \varphi_2} = \frac{\pi}{2} \cdot \frac{1 - \xi}{1 + \xi} \quad (15)$$

The maximum lateral deformation of belt appeared for the angular coordinate  $\varphi_1$ , which corresponds to the time coordinate  $t_m$ .

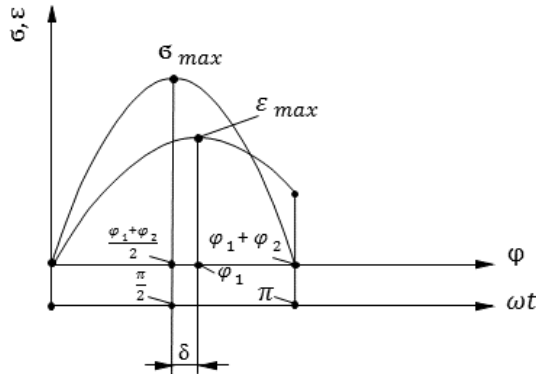


Fig. 5. Deformation in belt generated by harmonic load

Components of the maximum deformation of belt equals:

$$\varepsilon_{1m} = \frac{\sigma_0}{2E_c} \cdot \sin(2\delta) \cdot e^{\frac{-t_m}{\tau_0}} \quad (16)$$



$$\varepsilon_{2m} = \frac{\sigma_0}{E_c} \cdot \cos \delta \cdot \sin(\omega_0 \cdot t_m - \delta) = \frac{\sigma_0}{E_c} \cdot \cos \delta \quad (17)$$

because:

$$\omega_0 \cdot t_m - \delta = \frac{\pi}{1 + \xi} - \frac{\pi}{2} \cdot \frac{1 - \xi}{1 + \xi} = \frac{\pi}{2} \quad (18)$$

eventually:

$$\sin(\omega_0 t_m - \delta) = 1 \quad (19)$$

Exponent in the equation 16 equals:

$$-\frac{t_m}{\tau_0} = -\frac{\pi}{(1 + \xi) \cdot \operatorname{tg}\left(\frac{\pi}{2} \cdot \frac{1 - \xi}{1 + \xi}\right)} \quad (20)$$

Finally the maximum deformation of belt is described by the following equation:

$$\varepsilon_{\max} = \varepsilon_{1m} + \varepsilon_{2m} = \frac{\sigma_0}{E_c} \cdot \left[ \frac{1}{2} \sin\left(\pi \frac{1 - \xi}{1 + \xi}\right) \cdot e^{-\frac{\pi}{(1 + \xi) \operatorname{tg}\left(\frac{\pi}{2} \cdot \frac{1 - \xi}{1 + \xi}\right)}} + \cos\left(\frac{\pi}{2} \cdot \frac{1 - \xi}{1 + \xi}\right) \right] = \frac{\sigma_0}{E_c} \cdot \Phi(\xi) \quad (21)$$

Where damping function  $\Phi(\xi)$  equals:

$$\Phi(\xi) = \frac{1}{2} \sin\left(\pi \cdot \frac{1 - \xi}{1 + \xi}\right) \cdot e^{-\frac{\pi}{(1 + \xi) \operatorname{tg}\left(\frac{\pi}{2} \cdot \frac{1 - \xi}{1 + \xi}\right)}} + \cos\left(\frac{\pi}{2} \cdot \frac{1 - \xi}{1 + \xi}\right) \quad (22)$$

Integrating relation which described stress distribution (5) by entirety contact zone we get:

$$q_T = \frac{D_K}{2} \int_0^{(\varphi_1 + \varphi_2)} \sigma(\varphi) d\varphi = \frac{\sigma_0 D_K}{2} \cdot \int_0^{(\varphi_1 + \varphi_2)} \sin\left(\frac{\pi\varphi}{\varphi_1 + \varphi_2}\right) d\varphi = \frac{\sigma_0 \cdot D_K}{\pi} \cdot (\varphi_1 + \varphi_2) \quad (23)$$

In further considerations we will use:

$$q_T = \frac{\sigma_0 D_K}{\pi} \cdot \varphi_1 (\xi + 1) \quad (24)$$

Another step would be considering maximal deformation of belt over idler. Geometrical connections result from the arrangement shown in fig. 4. The length of the line segment of contact zone between belt and idler  $z_f$  (fig. 2) for lower angles  $\varphi$  is:

$$z_f = \frac{D_K}{2} \cdot \varphi_1 \quad (25)$$

Considering the geometric relationship for a right triangle OAC (fig. 4) we get:

$$\left( \frac{D_K}{2} - y_0 \right)^2 + z_f^2 = \left( \frac{D_K}{2} \right)^2 \quad (26)$$

so:

$$\left( \frac{D_K}{2} \right)^2 - D_K y_0 + y_0^2 + z_f^2 = \left( \frac{D_K}{2} \right)^2 \quad (27)$$

For real values  $y_0^2 \cong 0$ , therefore:

$$y_0 = \frac{z_f^2}{D_K} = \frac{D_K}{4} \cdot \varphi_1^2 \quad (28)$$

It is similar for HBC triangle (fig.4) :

$$[\rho - (y_0 - s_0)]^2 + z_f^2 = \rho^2 \quad (27)$$

therefore:

$$\rho^2 - 2(y_0 - s_0) \cdot \rho + (y_0 - s_0)^2 + z_f^2 = \rho^2 \quad (29)$$

In equation (28) for lower values  $y_0$  and  $s_0$  we can assume  $(y_0 - s_0)^2 = 0$  and we get:

$$y_0 - s_0 = \frac{z_f^2}{2\rho} = \frac{D_K^2 \cdot \phi_1^2}{8\rho} \quad (30)$$

From dependence (26) and (29) results:

$$s_0 = \frac{D_K \phi_1^2}{4} \cdot \left(1 - \frac{D_K}{2\rho}\right) = \frac{D_K \phi_1^2}{4} \cdot \lambda \quad (30)$$

Where factor of flexion belt on idler is:

$$\lambda = 1 - \frac{D_K}{2\rho} \quad (31)$$

Parameter  $s_0$  is maximal deformation of belt. Assuming that active belt thickness compress on a single idler set equals  $h_0$  that means:

$$s_0 = h_0 \cdot \varepsilon_{max} \quad (32)$$

Including (21), (30) and (31) we get:

$$\frac{h_0}{E_c} \sigma_0 \Phi(\xi) = \frac{D_K \phi_1^2 \lambda}{4} \quad (33)$$

Combining dependence (33) and (24) after eliminating the variable  $\sigma_0$  we obtained an equation which shows:

$$\phi_1 = \sqrt[3]{\frac{4\pi \Phi(\xi)}{\xi + 1} \cdot \frac{q_T}{D_K^2 \lambda c_e}} \quad (34)$$

Where unit transverse stiffness of belt is:

$$c_e = \frac{E_c}{h_0} \quad (35)$$

Substituting equation (34) into (4) after transformation we obtained formula on the unit rolling resistance:

$$w_e = \sqrt[3]{\frac{\pi \cdot \Phi(\xi) \cdot (1-\xi)^3}{2(\xi+1)} \cdot \frac{q_T^4}{D_K^2 \lambda c_e}} = F(\xi) \cdot \sqrt[3]{\frac{q_T^4}{D_K^2 \lambda c_e}} \quad (36)$$

Where factor which depends from belt's damping  $F(\xi)$  is:

$$F(\xi) = \sqrt[3]{\frac{\pi \cdot (1-\xi)^3}{2 \cdot (\xi+1)} \cdot \left\{ \frac{1}{2} \sin\left(\pi \cdot \frac{1-\xi}{1+\xi}\right) \cdot e^{-\frac{\pi}{(1+\xi) \operatorname{tg}\left(\frac{\pi \cdot 1-\xi}{2 \cdot 1+\xi}\right)}} + \cos\left(\frac{\pi \cdot 1-\xi}{2 \cdot 1+\xi}\right) \right\}} \quad (37)$$

The same factor described as a function of the phase angle  $F(\delta)$  would be:

$$F(\delta) = \sqrt[3]{\delta \cdot \left(1 - \frac{\pi - 2\delta}{\pi + 2\delta}\right)^2 \cdot \left\{ \frac{1}{2} \sin(\delta) \cdot e^{-\frac{\pi}{\left(1 - \frac{\pi - 2\delta}{\pi + 2\delta}\right) \operatorname{tg} \delta}} + \cos(\delta) \right\}} \quad (38)$$

## 5. OTHER MODELS

There are few models which described rolling resistance known from literature. Every one of them deal with problem of non-linear nature of the deformation of belt in contact zone with idler (Günthner et al., 2010). Uniform solution for all cases is extremely difficult, so to avoid excessive generalizations analyzed model need to be considered in two aspects. This aspects are determined by geometry of system and properties of belt. The most common model are these developed by Jonkers, Spaans and Lodewijks (Lodewijks, 1996). Both, Jonkers and Spaans based on visco-elastic Winklers model and refer it to describe phenomena in belt. In turn, Lodewijks extended theory created by Jonkers by taking into account asymmetric contact zone between belt and idler. Furthermore all models assume constant phase shift (Rudolphi 2008). A common feature of all models is separating the two parts of the product, one of them included design parameters and idler load and the second in different way described damping parameters of belt. Equation scheme for calculation rolling resistance for all models is similar:

$$\text{Rolling resistance} = \text{damping factor} \cdot \text{design parameters} \quad (39)$$

## 5.1. THEORETICAL MODEL BY JONKERS

Jonkers in his model based mainly in energy losses, which can be calculated from hysteresis loop. He included uneven distribution of stress in belt and its visco-elastic properties. Moreover Jonkers assumed that  $tg\delta$  is not bigger than 0,4, so angle  $\delta$  (phase shift angle) can be reached only 0,38 (Jonkers, 1980). Below is equation for rolling resistance created by Jonkers:

$$f_{ij}' = \frac{1}{2} \cdot \pi \cdot tg(\delta) \cdot \left[ \frac{(\pi+2\cdot\delta)\cdot\cos(\delta)}{4\cdot\sqrt{1+\sin(\delta)}} \right]^{\frac{4}{3}} \cdot \left[ \frac{F_z \cdot h}{E' \cdot D^2} \right]^{\frac{1}{3}} \quad (40)$$

Jonkers model is commonly used and it's suitable for quick comparison of belts made of various materials (Drenkelford, 2015).

## 5.2. THEORETICAL MODEL BY SPAANS

Spaans model similar to Jonkers one is based on hysteresis loop (Spaans, 1991). Equation for rolling resistance in this case is as follows:

$$f_{is} = \frac{1}{2} \cdot \eta_i \cdot \frac{F_z^{\frac{1}{3}}}{\left(\frac{2}{3}\right)^{\frac{4}{3}} \cdot E^{\frac{1}{3}} \cdot D_0^{\frac{1}{3}} \left[1 + (1-\eta_i)^{\frac{3}{4}}\right]^{\frac{4}{3}}} \quad (41)$$

Where damping factor by Spaans is:

$$\eta_i(\delta) = \frac{2 \cdot \pi \cdot \tan(\delta)}{2 + (\pi + 2\delta) \cdot \tan(\delta)} \quad (42)$$

It is important to remember, that damping factor  $\eta_i$  is not clearly the same as those suggested in chapter 2 (equation 38). Factor  $\eta_i$  depends only from phase lag angle  $\delta$ , but in equation 40 appeared twice. To make comparison all factors we have to distinguished part which depends entirely on the angle  $\delta$ . After substituting the previous models we receive:

$$f_{is} = \frac{\frac{1}{2} \cdot \frac{2 \cdot \pi \cdot \tan(\delta)}{2 + (\pi + 2\delta) \cdot \tan(\delta)}}{\left(\frac{2}{3}\right)^{\frac{4}{3}} \left[1 + \left(1 - \left(\frac{2 \cdot \pi \cdot \tan(\delta)}{2 + (\pi + 2\delta) \cdot \tan(\delta)}\right)\right)^{\frac{3}{4}}\right]^{\frac{4}{3}}} \cdot \left[ \frac{F_z}{E^* \cdot D_0} \right]^{\frac{1}{3}} \quad (43)$$

Method created by Spaans is less useful than the previous one, because it requires knowledge of more numerous parameters. Spaans relates to the transverse rigidity of

the belt. Nevertheless it still gives a possibility to compare with different models, also Jonkers one (Drenkelford, 2015).

### 6. COMPARISON OF MODELS

In order to compare the mathematical model of calculation rolling resistance of belt described in previous chapter, part which specified belt’s damping factor need to be extracted. That part depends only from phase lag angle. With these assumption Jonkers equation would be:

$$F_J(\delta) = \frac{1}{2} \cdot \pi \cdot tg(\delta) \cdot \left[ \frac{(\pi+2\delta) \cdot \cos(\delta)}{4 \cdot \sqrt{1+\sin(\delta)}} \right]^{\frac{4}{3}} \tag{44}$$

In turn, Spaans model looks like this:

$$F_S(\delta) = \frac{\frac{1}{2} \cdot \frac{2 \cdot \pi \cdot \tan(\delta)}{2 + (\pi + 2\delta) \cdot \tan(\delta)}}{\left(\frac{2}{3}\right)^{\frac{4}{3}} \cdot \left[ 1 + \left( 1 - \left( \frac{2 \cdot \pi \cdot \tan(\delta)}{2 + (\pi + 2\delta) \cdot \tan(\delta)} \right)^{\frac{3}{4}} \right)^{\frac{4}{3}} \right]} \tag{45}$$

The graph below in fig. 6 shows the function for models of Jonkers, Spaans, and damping -factor suggested by authors in equation 38.

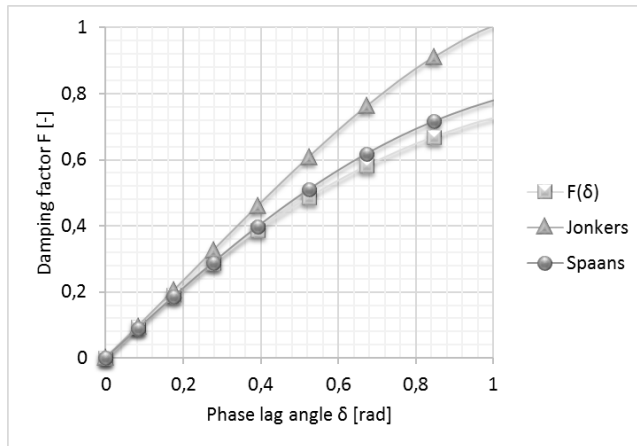


Fig. 6. Correlation between factor which depends from belt’s damping and angle  $\delta$  for different models

The graph above was created for phase lag angle in section from 0 to 1 radians to show the whole spectrum of values for damping factor. It is important to remember

that Jonkers focused only on area from 0 to 0,4 radians and for his method the graph is only hypothetically. For basic assumption, models created by Jonkers or Spaans might be useful. It is true, that phase lag angle between 0,6–0,7 radians is really big and appeared very rare, in turn values above 1 radian are unreachable. It has logical explanation because if damping factor would equal 1 radian damping would be maximal and even the smallest belt's move on idler would be impossible. Because of that values bigger than 1 are completely abstract. Nevertheless, phase lag angle bigger than 0,5 radians is possible to reach and to compare different types of belts we have to know its properties are in different conditions. Due to the graph (fig. 6) damping factor in Jonkers and Spaans models is underestimated even for phase lag angle between 0,2–0,3 radians. The bigger phase lag angle is, the underestimation of damping factor is more significant. This is a consequence of using only general solution of equation of belt's model. Authors of this paper include in their model also particular solution of this equation, which make the prediction of damping factor for every kind of phase lag angle possible.

## 7. CONCLUSION

Based on a comparison of models we can see, that new model suggested by authors is quite similar to models created by Spaans or Jonkers in past. For phase lag angle in range 0,1-0,3 radians values of damping factor are very close and sometimes even equal for different models. Described in article mathematical model includes two cases: particular and general solution of equation of belt's model (equations 12 and 13). It is especially important for larger values of the angle  $\delta$ . Other described models limited in their assumption to the general solution. That is the reason why maximum values of damping factor in each model are so different. Considered solutions make new method created by authors more accurate. Authors also improved their previous work by considering each load cycle as the first one. This two matters (including particular solution and individual approaches to the each load cycle) make the correct calculation and prediction of damping factor possible.

Damping factor is the main part in rolling resistance calculations (39). Rolling resistance are the biggest component of the primary resistance of conveyor, that is why they generate the biggest energy loses. It is possible by recognizing the full phenomena on conveyor belts. Creating new energy saving belts will reduced rolling resistance, thereby lowered costs of conveyor transport.

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