Giuseppe Calzoni<br>Wrocław University of Economics

# THE SYSTEMS OF INCOME TAXATION AND THEIR SIGNIFICANCE FOR ECONOMIC AND SOCIAL DEVELOPMENT OF HUMAN SOCIETIES 


#### Abstract

Summary: Studying the inequality of family income distribution, the possibility of implementing a system of taxation is analyzed, which allows getting a predetermined revenue decreasing the degree of differentiation and so strengthening social cohesion.


Key words: tax, income, distribution, well-being, cohesion.

## 1. The changes of income taxation systems

It is clear that the importance of income taxation in financing public expenditure has been considered particularly significant since ancient times. For example, in the hierarchy of the Roman Empire the censor played a major role, second only to the figure of the Emperor. His role was to assess the census, that is, the individual wealth of the most important citizens and to decide what share of the income they should have to pay to contribute to public expenditure. This role is famous in history partly due to the figure of the censor Marcus Porcius Cato, who lived between the third and second century BC, and imposed severe taxes on luxury, since he was convinced that excessive wealth could help lead to a weakening of morals and, as a final consequence, to a crisis in the social order and the decline of the power of Rome.

After these preliminary remarks, it may be deduced that taxation has been considered important for reasons far beyond that of the role, also significant, of the contribution to the expenditures of the state, and that its function has been to ensure the achievement of fundamental results in the maintenance of social order and cohesion.

Of course, there were no algorithms for the calculation of tax or levy systems in those days, and the situation continued without major changes until the time of the French Revolution, during which, following the revolutionary motto "Liberté Egalité - Fraternite", it was established to charge a levy on the income of the citizens at a constant rate, independent of the income amount. This was a decisive step forward in French history, in which people had previously been taxed indiscriminately by
the king. This policy had created an extremely confused and tense situation, which the Marshal de Vauban had vainly attempted to remedy, proposing that the king replace all existing contributions and tributes with a proportional tax, equal to ten per cent [de Vauban 1707]. It is particularly interesting to observe that De Vauban, to highlight the conditions of extreme hardship and discontent of the population, describes in his work the situation of extreme inequality of income distribution for classes and thus the importance that economic problems may have on the stability and cohesion of social relations [Projet d'une... 1851, p. 36].

The king took no action and, less than a century after, the French Revolution broke out, actually confirming how right was Aristotle's statement (from Politica), that is, the more the distribution of honours and wealth is unfair and unequal within a state, the more social anxiety can give rise to the outbreak of a revolutionary conflict.

Later J. Stuart Mill [1848] proposed a proportional tax with an exemption, which, all in all, deducted from the amount of all income an amount equal to the minimum income (and, therefore, exempted the latter from tax) and applied a proportional rate on the remaining income.

This system, which has been considered very favourably, turns out to be a progressive charging, faulty because a constant rate is applied to curtailed income but, of course, the curtailment weight decreases as the value of income increases.

The principle upon which the tax is based is certainly acceptable, but it does not allow obtaining a significant rebalancing of the distributive structure, because of the absence of official statistics on the distribution of individual incomes, which would allow for an adequate analysis of real situations. When this occurred, towards the end of the nineteenth century, the tax levy issue could be studied in depth and in the variety of its aspects, thus acquiring importance as a foundation of stability as well as economic and social progress.

What Pareto wrote on this subject clearly shows the importance that the author attributed to this subject: "It is the observation that should inform us about the causes that determine the distribution of wealth. If we find that the distribution of wealth varies widely and irregularly, we conclude that 'chance' plays a considerable part in the occurrence of this phenomenon. If variations in the distribution of wealth follow changes in economic organization, we have to attribute a major part of them to this organization. If, finally, the distribution of wealth does not vary greatly across regions, periods and different organizations, we have to conclude that, without wanting to neglect other causes, we must seek the root cause of the problem in human nature" [Pareto 1897, p. 974].

Continuing the analysis, he states: "Despite the uncertainties implied in taxpayers income tax declarations, today these are the most secure basis for achieving knowledge, even if approximate, of the way in which wealth is distributed" [Pareto 1897, p. 974].

The graphical representation of the distribution of income that Pareto presents is extremely simple: $x$ points out a certain level of income, $N$ the number of income earners, with incomes higher than $x, \alpha$ the level of inequality distribution. Describing on the abscissa, the logarithm of $x$ and on the ordinate the logarithm of $N$, the author notes that in different periods and different countries "these so fixed points have a very accentuated tendency to arrange themselves in a descending straight line" [Pareto 1897, p. 974] described by the equation [from Pareto 1897, p. 975]:

$$
\begin{equation*}
\log N=\log A-\alpha \log x \tag{1}
\end{equation*}
$$

where $A$ is a constant equal to the total number of earners. Writing the formula (1) in term of numbers the result is [Pareto 1897, p. 976]:

$$
\begin{equation*}
N=A / x^{\alpha} \tag{2}
\end{equation*}
$$

with $\alpha>1$.
The conclusion about the meaning of the variations of inequality is related by Pareto to the movements of the income distribution system, in the sense that, as he says, a minor difference between the extremes classes of income "[...] can be approached in two essentially different ways: as if the rich become poor, as if the poor become rich", and concludes, agreeing with Lassalle, that a reduction of inequality is defined by the fact that the number of poor people is decreasing in relation to the total number of members of society, which means that the extreme classes of income are approaching each other.

This statement leads us to establish that it is possible and reasonable to encode a rational system of progressive taxation that, saving the basic fundament of the distribution structure of incomes as described by (1) and (2), reduces the inequality of the distribution keeping unchanged the minimum income, ensuring at the same time the charging of a predetermined tax yield.

## 2. Quantitative determination of the fundamental elements of a rational system of taxation

If we consider any algebraic formula from a logical point of view, it is not difficult to ascertain that very often between the independent variable and the function there is a relation of cause and effect, which in this analysis is explicit in the relation existing between income earners and incomes.

Since in this case the question is to study the structure of the incomes from which a given tax yield is to be drawn, the Pareto function has to be adapted to this purpose, that is, to construct it to express, in numerical terms, the income that every single person earns, which allows us to establish the place which it takes in a list where income earners are arranged from the richest to the poorest, and therefore in reverse order in comparison with the Lorenz-Gini curve. This is accomplished by reversing the formula (2) and obtaining

$$
x=f(n, \rho)
$$

where $n$ indicates the ranking place of a person who earns the income $x$, and $\rho=$ $1 / \alpha$ the degree of inequality of income distribution, ranking from a minimum of 0 (equal distribution of income among the people ( when $\alpha=\infty$ ) and a maximum of 1 (attribution of the entire income to the richest person (when $\alpha=1$ ).

By making explicit the functional relation, we obtain the formula:

$$
\begin{equation*}
x=(n)^{\rho} \tag{3}
\end{equation*}
$$

in which the income of each person is expressed in terms of the average income in the system.

Assuming for the sake of simplicity that the total income to be distributed is equal to 100 , and equal to 100 is also the number of earners-taxpayers, the value of each individual income is expressed in terms of average income equal to 1 according to the data of the problem.




Figure 1. Income distribution in situations of growing inequality
Source: author's own work.
The trends of the graphs that describe the extreme situations of distribution are represented in Figure 1 respectively by the maximal initial ordinate at the height of
the total income, indicating the situation of maximum inequality, that is, the values of income equal to 0 for all earners of an income following the first, and by the diagonal of the quadrant, which describes a constant increase in income, that is, a situation of an even income distribution.

The intermediate trends are described in Figure 1, which shows three situations in which total income 100 is divided between deciles of populations, from poorest to richest, in situations in which $\rho$ is equal to $0.2,0.5$, and 0.9 , i.e., a situation of increasing inequality.

This process can be defined as the systematic expropriation of income earners, leading to the extreme situation in which the entire available income is transferred to the richest of them and the distribution line coincides with the ordinate axis.

Reading data in cross-sectional direction is without doubt more significant - it can be connected with the process of subsequent withdrawals from the income of a taxpayer $n$, caused by increased inequality index of distribution from 0.1 to 0.9 , being fixed total income.

The tendency of the subsequent decrease in income owing to the taxpayer with an order of precedence of 10 is shown in the set of data in Table 1, which corresponds to Figure 2, which, in essence, coincides with the hyperbolic curves described by Pareto.

Table 1. Decrease in income

| 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.79 | 0.63 | 0.5 | 0.4 | 0.32 | 0.25 | 0.2 | 0.16 | 0.13 |

Source: author's own work.


Figure 2. Curve of the income owing to the taxpayer with an order of precedence of 10
Source: author's own work.

In relation to the situation of an increasing inequality from 0.1 to 0.9 , the case of a reduction in inequality from 0.9 to 0.1 , shows a tendency clearly ascendant and symmetrical in relation to the trajectory indicating an increase from the midpoint of the range.

The same pattern can be applied to people with an increasing amount of wealth, with similar results, except that the decrease due to increasing of inequality is more and more attenuated, because, for values of inequality close to the maximum, wealth is already almost completely transferred to them.

Considering now the value of income as an independent variable, the derived function describes the frequency of earners of a certain level of income when inequality changes.

Given a certain value of income level, higher than the average income, the resulting profile of the level curve shows the variation in the number of people who receive that income when inequality changes, and this frequency first increases for low values of the index, when the area of middle/low income grows rich, and then decreases, tending to 0 at the highest value of the inequality, when wealth is owned by a single person.

The analysis of the relation between the amount of income, the ranking of income earners, and values of inequality is based, as stated earlier, in formula (2):

$$
\begin{equation*}
x=(n)^{\rho} \tag{4}
\end{equation*}
$$

in which a trajectory of a income, generated by the variable power ( $\rho$ ) of a constant basis ( $n$ ) develops, necessarily increasing, because it is included between two extremes: the absolute equality of income level (equal to the average income) when inequality is 0 , income diversification for intermediate values of inequality, and zero income (except for the richest beneficiary) when inequality is at the maximum.

Table 2 shows the development of the income received by earners with orders of precedence between 50 and 95, and then in the middle/high income area, depending on the inequality index values from 0.1 to 0.99 ; the situation of initial egalitarianism should be noted, as well as the situation of the richest earner of the group who, when inequality increases from 0.50 to 0.99 , almost doubles the received income (an increase of $88.8 \%$ ), while the least rich remains almost with the same income as that of the starting point (an increase of $6.76 \%$ ). The transition to the highest limit of inequality, as already noted, clears all the incomes, assigning the whole amount to the richest of the group.

Table 2. Development of income

| 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 0.95 | 0.99 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1.57876 | 2.58625 | 3.92023 | 6.18144 | 9.74679 | 15.36859 | 24.23296 | 38.21016 | 60.24919 | 75.65496 | 90.77084 |

Source: author's own work.


Figure 3. Development of the incomes recived by the earners with the order of precedence between 50 and 95 , depending on the inequality index values from 0.1 to 0.99

Source: author's own work.
Representing together the course of the three functions described on three-axis Cartesian, and describing on the basis the values of inequality index of distribution and of earnings precedence order from the poorest to the richest, the income received by each rentier in terms of its order of precedence could be represented on a third axis perpendicular to the base and a representation related to several levels of income will produce a stereogram in which are described the various values of the three variables of the formula.

If we now use the Bernoulli hypothesis of the measurability of welfare as the logarithm of real income [Bernoulli 1738, p. 178], it is possible to pass from the representation of individual income to that of individual welfare in a specific distribution structure for given values of the index of inequality.

This level of analysis reveals as the best solution the building of a system of taxation that allows for a programmed yield to be obtained, decreasing the inequality of income distribution. In this way, without altering the structure of the social order, it is possible to achieve two fundamental goals of political action: to ensure adequate resources for financing public expenditure and achieve the aims of greater welfare and social cohesion, guaranteed by the rational adjustment of the distributive structure of wealth.

In fact, the index of inequality in the Pareto distribution is based on the relation between two variables: minimum income and average income. The first is exempted from the payment of tax and the second is decreased in proportion to the tax levy.

This makes it possible to determine in advance the value of the index of inequality, when the levy has been collected, and allows the effect of the levy to be defined with reference not only to the financing of public expenditure, but also to the reduction of the degree of distributive inequality.

It should finally be noted that a progressive taxation of the type shown here, described by an exponential function with regards to changes in income, is equivalent in substance to a proportional charging, carried out on the logarithm of income, as shown in Table 3, which uses the logarithms of the data included in the previous table, related to the earners of an income with ranking numbers equal to $55,65,75$, 85 , and 95 as a function of the values of the inequality index from 0.1 to 0.9 .

Table 3. Logarithms of incomes related to earners with ranking from 55 to 95 in situation of growing inequality of distribution

|  | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | $0, .7$ | 0.8 | 0.9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 55 | 0.401 | 0.801 | 1.202 | 1.602 | 2.004 | 2.404 | 2.805 | 3.206 | 3.607 |
| 65 | 0.417 | 0.835 | 1.252 | 1.67 | 2.087 | 2.505 | 2.922 | 3.34 | 3.757 |
| 75 | 0.432 | 0.863 | 1.295 | 1.727 | 2.159 | 2.59 | 3.022 | 3.454 | 3.886 |
| 85 | 0.444 | 0.889 | 1.333 | 1.777 | 2.221 | 2.666 | 3.11 | 3.554 | 3.998 |
| 95 | 0.455 | 0.911 | 1.366 | 1.822 | 2.277 | 2.712 | 3.188 | 3.643 | 4.098 |

Source: author's own work.


Figure 4. Logarithms of incomes related to earners with ranking from 55 to 95 in the situation of the growing inequality of distribution

Source: author's own work.

If we refer to the hypothesis formulated by Daniel Bernoulli, describing the rising trajectories of the curves on a logarithmic scale, they become straight lines, drawn at different heights in relation to different levels of income. In my opinion, this allows us to reach the conclusion that a rational system of taxation can be considered, from this point of view, as the application of a proportional tax on welfare.

The principle that derives from this is that a rational taxation system should achieve a withdrawal that, leaving unchanged the minimum income distribution, strives to diminish the inequality of income distribution, guarding the assets necessary to support public expenditure. Obviously, the intervention should be realized in such a way as to ensure the functionality of the productive system and therefore not overly burden profits.

A policy which, in fact, tends to reach these targets with appropriate and effective interventions to changing economic conditions would achieve strategic action of enormous importance to achieve objectives of great significance for the development of the population in the fields of political, social, and economic life.

I think that we may say that current studies of economic issues are increasingly considering, as well, social and political matters closely related to them; so we can affirm that, when we analyse situations of particular complexity and importance, we can see that they contain some aspects that can be defined as pertaining to a quantitative economy, and others, surely, to a qualitative economy.

This finding is not the product of contemporary thought, if we consider that Malthus wrote that Dr. Smith, in his work, not only analyses the nature and causes of the wealth of nations, but also the more interesting issue of the nature and causes of the happiness of nations, or rather, the happiness and welfare of the poorest classes, which are the major part of every population.

Further, humanity has been defined, since ancient times, as being "naturally social and political", and this, in my opinion, is enough to highlight the fact that, also for this reason, economics, as a human science, must be closely associated with political and social sciences if we want to use all the points of reference needed for the correct course of scientific analysis.

The discussion developed in this short work can be considered significant in this respect, not only for the importance of taxation as an instrument of economic policy oriented to the financing of public expenditure and to economic development, but also as a means to achieve a reorganization of income distribution which could assure a greater homogeneity, cohesion, and functionality of the economic order.

The study of the inequality of income distribution in my opinion is closely related to the field of taxation, and it has to be analysed thoroughly and considered very carefully, because, while on the one hand it can be considered essential to achieve a rational reduction of the inequality degree of income distribution, on the other hand it is necessary to make clear that a too low inequality level would make very difficult the correct functioning of the economic system.

In my opinion, it is obvious that any human structure needs a hierarchy to work properly: this can be a family, a business, an army, or a nation. An economic system therefore needs a hierarchy as well, and this is precisely described by the distribution of income that identifies in it rich people, poor people, and the middle class. But it would be a mistake to believe that a complete levelling of the distribution, which eliminates any hierarchy, could assure permanent welfare to the population and optimal functionality to the economic system.

Finally, I believe that a very difficult problem remains still unresolved: how to achieve this goal in practice. It is clear that for this purpose political action must be engaged in a continuous intervention of fine tuning which, through a proper use of taxation, on the one hand, provides the necessary means for the life and activity of citizens and, on the other hand, ensures rationality and efficiency to the strategies of mutual welfare.

Concluding, I think that, from an economic point of view, this analysis could be considered an useful example of research conducted in the field of the study and comparison of fundamental issues concerning the quantitative and qualitative aspects of economic science as well as the purposes and means of political action. This political action, however, would be insufficient whithout the invaluable support of the community, because the endeavour to reach a "peaceful and good life of the citizens", described by Cacciaguida (from Dante's The Divine Comedy), as the source and the end of the common good, strictly depends on the social cohesion.

## References

Bernoulli D., Specimen theoriae novae de mensura sortis, Academia Scientarum/ Petropolitana, 1738. de Vauban S., Projet d'une dime royale, 1707.
Mill J.S., Principles of Political Economy, 1848.
Pareto V., Corso di economia politica, UTET, Torino 1897.
Projet d'une dime royale, Guillomin, Paris 1851.

## SYSTEMY OPODATKOWANIA I ICH ZNACZENIE DLA EKONOMICZNEGO I SPOEECZNEGO ROZWOJU SPOEECZEŃSTW

Streszczenie: Analizując różne systemy opodatkowania, autor artykułu bierze pod uwage możliwość realizowania systemu podatkowego, który zapewnia ustalony przychód, zmniejszając nierówności w dystrybucji i w ten sposób wzmacniając spójność społeczną.

Słowa kluczowe: podatki, przychód, dystrybucja, spójność społeczna.

