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Second-order effects in horizontally loaded reinforced concrete columns

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Abstract: This paper deals with the second-order effects in horizontally loaded reinforced concrete columns. The current standard approach according to Eurocode 2 is the starting point for the considerations. Simplified methods that take into account the secondary effects, that is, the nominal stiffness method and the nominal curvature method, and their limitations are discussed. Most attention is devoted to the general method. As only general guidelines for this method can be found in the literature on the subject, the author presents his own original approach to calculations done using this method. Exemplary analyses for the corbel columns of high bay racked warehouses are made. Columns of different lengths are analyzed. The calculations show the overestimates introduced by the simplified methods and the benefits stemming from the use of the general method, especially in the case of quite slender columns.

Keywords: buckling; column; general method; reinforced concrete; second-order effects.

1 Introduction

A major dynamically growing trend in construction is high bay racked warehouses. Their principal structural components are frames consisting of high reinforced concrete columns on which steel girders are laid. For economic reasons, the main load-bearing frames are spaced as much as 12.0 m apart. Consequently, the wind load transmitted via the enclosure panels to the columns becomes a significant factor. The columns are rigidly fixed in the foundations and their connection with the steel structure is practically of the hinged type, but allowing horizontal displacement. The use of static models with elastic constraints, limiting the displacements of the column's upper joint, introduces the arbitrariness of assuming the stiffness of the constraints, the way of considering the effects of wind action on the building's opposite surface, and so on. With the current state of knowledge, it seems safest to treat such columns as corbel columns loaded with a relatively weak vertical force – a reaction from the roof girder – and a horizontal load (wind pressure).

2 Basic observations and comments

Previously, standard PN-B-03264 (2002) was used in Poland. The standard recommended that the slenderness (l_0/h) of the columns should not exceed 30. For example, for height h=60 cm, the buckling length l_0 should not exceed 0.6 m•30=18 m, which, in the case of corbel columns, comes down to the condition that the column length l_{col}≤9.0 m. In high bay warehouses, the heights of the columns are much greater and their slenderness considerably exceeds the former recommendations. There is no such slenderness limitation in the current European standard EN 1992-1-1 (2004), and very slender members can be designed, but one should properly take into account the resulting second-order effects (see, e.g., Klempka and Knauff, 2005; Knauff, 2022).

Second-order effects in static load calculations are taken into account by rejecting the structural system solidification assumption. In other words, the internal forces are calculated taking into account the values of the displacements they induce. As regards an elastic system with invariable stiffness, this comes down to a relatively simple iterative process. In the first step, internal forces and the displacement they generate are determined. In general, both moments and axial and shearing forces can be considered (Aristizabal-Ochoa, 2007). In the next step, the additional moments generated by the displacements are taken into account. The computations

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can be terminated when the moment increments are infinitesimally small and it is certain that their sequence is convergent.

This procedure becomes more complex when subsequent changes in moment values entail changes in the stiffness of the cross sections. Additional difficulties arise when it is necessary to take into account changes in stiffness over time – rheological phenomena. All these problems must be addressed in the case of reinforced concrete structures.

Standard EN 1992-1-1 (2004) is the main source of information for designers as regards determination of second-order effects. It presents two approximate calculation methods: a nominal stiffness method and a nominal curvature method. How to use the methods and their basic relationships are described in quite detail. Also, an outline of the use of the general method and recommendations concerning its use are given. The description contained in the standard is rather general, requiring much inventiveness from the designer in its use.

The source of the two approximate methods is an analysis of the ultimate strain of a beam with constant stiffness *EI*, axially compressed with force *N*, pinned at both its ends – the Euler model (Fig. 1) (Timoshenko and Gere, 1961).

For the above system, one can write a similar relation as when determining deflections in beam under bending with moment *M* (1).

$$\frac{d^2y}{dx^2} = -\frac{M}{EI} = -\frac{N \cdot y}{EI}. \tag{1}$$

This is a homogenous differential equation of the second order, which can be expressed as follows:

$$\frac{d^2y}{dx^2} + -\frac{N \cdot y}{EI} = 0. \tag{2}$$

The general solution of this equation has the form:

$$y = A \cdot \cos(k \cdot x) + B \cdot \sin(k \cdot x) = 0, \quad \text{where } k^2 = \frac{N}{EI}. \tag{3}$$

In the considered case, the boundary conditions are $y(x{=}0) = y(x{=}l) = 0$, resulting in an equation with unknowns *A* and *B* (note: constants *A*, *B*, *C*, ... have the character of "local variables" and when recalled in the subsequent equations, assume other values):

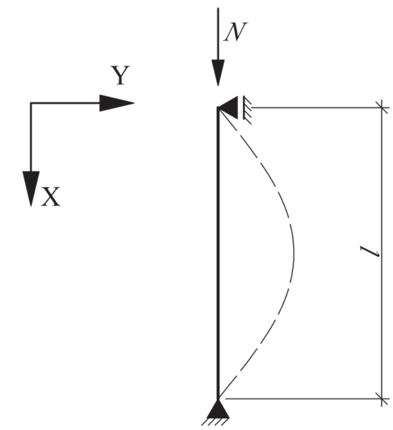

Figure 1: Basic Euler model.

$$\begin{cases} A \cdot 1 + B \cdot 0 = 0 & (4) \\ A \cdot \cos(k \cdot l) + B \cdot \sin(k \cdot l) = 0. & (5) \end{cases}$$

Equations (4) and (5) form the so-called indeterminate system. Either $A = B = 0$, which means that the beam will not deform, or the system's main determinant is equal to 0 and there will be an infinite number of equations. But in a nontrivial case, the following must hold true:

$$1 \cdot \sin(k \cdot l) - 0 \cdot \cos(k \cdot l) = 0 \Rightarrow \sin(k \cdot l) = 0. \tag{6}$$

Function *sinus* equals zero when $\alpha{=}\pi$. Thus, one gets $k \bullet l{=}\pi$. Taking into account (3), one can calculate critical force N_B:

$$N_B = \frac{\pi^2 EI}{l^2}. \tag{7}$$

The solutions of equation (2) have the form:

$$y = B \cdot \sin(k \cdot x) = B \cdot \sin\left(\frac{\pi \cdot x}{l}\right). \tag{8}$$

The modified form of equation (7) constitutes a basis for the nominal stiffness method and that of equation (8) a basis for the nominal curvature method.

It should be noted that in the Euler problem, there are no horizontal loads acting on the beam and affecting the displacements of its axis and, therefore, the curvature. Generalized solutions for a beam with constant stiffness were presented by Yoo and Lee (2011).

2.1 Nominal stiffness method

This method is also called a multiplicative method because the ultimate value of bending moment M_{Ed} is determined by multiplying 1st order moment M_{0Ed} by a coefficient mainly dependent on a ratio of buckling (Euler) force N_B to design column compressing force N_{Ed} according to relation (9).

$$M_{\mathrm{Ed}} = M_{0\mathrm{Ed}}\left(1+\frac{\beta}{\frac{N_{\mathrm{B}}}{N_{\mathrm{Ed}}}-1}\right). \tag{9}$$

The above form means that the sequence of successive moment (eccentricity) increments is geometric progression β=1 or the one very close to it: $\beta \in \langle 0.82;1.23 \rangle$.

The determination of nominal stiffness EI, constant for the whole column and independent of the stress intensity level, is key in this method (hence its name). Several applications of the method are proposed in the literature on the subject, (e.g., Tikka and Mirza, 2008; Bonet et al., 2011). They have been found useful, but only for not too slender columns loaded with a constant moment and an axial force. The current standard provides a relation expressed by equation (10).

$$EI = K_{\mathrm{c}} E_{\mathrm{cd}} I_{\mathrm{c}} + E_{\mathrm{s}} I_{\mathrm{s}}, \tag{10}$$

where K_c is a coefficient dependent on the influences of cracking, creep, etc., E_{cd} is the design E-modulus of the concrete, I_c is the concrete's section moment of inertia, E_s is the E-modulus of steel, and I_s is the moment of inertia of steel.

In the case of columns rectangular in cross section, with the overall reinforcement ratio $\rho=\rho_1+\rho_2$ and a symmetric reinforcement ($\rho_1=\rho_2$), equation (10) assumes the following form:

$$\begin{aligned} EI &= \frac{k_1 k_2}{1+\varphi_{\mathrm{ef}}}\frac{E_{\mathrm{cm}}}{1.2}\frac{bd^3}{12}\left(1+\frac{a}{d}\right)^3 + E_{\mathrm{s}}\frac{1}{2}\rho b d^3\left(1-\frac{a}{d}\right)^2 = \\ &= bd^3\left[\frac{k_1 k_2}{1+\varphi_{\mathrm{ef}}}\frac{E_{\mathrm{c}}}{14.4}\left(1+\frac{a}{d}\right)^3 + \frac{1}{2}E_{\mathrm{s}}\rho\left(1-\frac{a}{d}\right)^2\right], \end{aligned} \tag{11}$$

where E_{cm} is the concrete's mean E-modulus, b is the width of the cross section, d is the effective height of the cross section, and a is the distance between the reinforcement's center of gravity and the nearest edge (the other symbols are explained in the text).

As a result, one gets:

$$\frac{N_{\mathrm{B}}}{N_{\mathrm{Ed}}} = \left(\frac{d}{l_0}\right)^2 \cdot \frac{\pi^2\left[\frac{k_1 k_2}{1+\varphi_{\mathrm{ef}}}\frac{E_{\mathrm{cm}}}{14.4}\left(1+\frac{a}{d}\right)^3 + \frac{1}{2}E_{\mathrm{s}}\rho\left(1-\frac{a}{d}\right)^2\right]}{n \cdot f_{\mathrm{cd}}}. \tag{12}$$

If wind pressure is the horizontal load causing bending, one can assume that the effective creep coefficient $\varphi_{ef}\approx 0$. Coefficient $k_1=\sqrt{f_{ck}}/20$ results from the scaling of the test results of concrete with characteristic compressive strength f_{ck} relative to C20/25 concrete class and it has an empirical character. Similar is the case of parameter $k_2=n\,\lambda/170\leq 0.2$, which takes into account column slenderness λ and the degree of loading with the axial force: $n=N_{Ed}/(A_c f_{cd})$ (where A_c is the concrete cross-sectional area and f_{cd} is the concrete's design compressive strength).

The use of this method for the analysis of horizontally loaded columns raises several questions and doubts enumerated below.

1) According to equation (9), the whole first-order moment M_{0Ed} is subject to multiplication. Actually, its part, which stems from the action of the horizontal force, does not change as the beam's axis displaces (only the moment associated with the eccentricity of vertical force N_{Ed} changes). At significant magnitudes of the horizontal forces, this can lead to significant overdimensioning of the column.
2) The column's nominal stiffness defined by equation (11) takes into account the increase in stiffness due to the presence of a large compressive zone in the cross section, especially at high slenderness ratios, only to a small degree. This stems from parameter k_2 in the equation. For example, for parameter λ=100, k_2=0.2 is obtained for both n=0.35, at which the steel in the tension zone yields, and n=0.8, at which a large cross-sectional zone determines the load capacity. This leads to underestimated results for slender columns highly stressed by the vertical force.
3) Another doubt concerns the design length for corbel columns, defined as $l_0=2l$. It was introduced solely, so that the buckling force, regardless of the column support mode, could be calculated from equation (7). But force N_B itself stems from the form of equation (1). In the case when a horizontal load occurs, the form of this equation is already different.

Because of all the above reservations, the use of the nominal stiffness method becomes highly debatable in some cases.

2.2 Nominal curvature method

This method is also called additive. This is so called because of the structure of the basic equation:

$$M_{\mathrm{Ed}} = M_{0\mathrm{Ed}} + N_{\mathrm{Ed}} \cdot e_2. \tag{13}$$

Owing to the fact that (as opposed to the multiplicative method) the first-order moment is distinguished, one can isolate the moment induced by wind from the one resulting from displacement of the place of action of the vertical force. But problems arise when analyzing the relation for the second-order eccentricity. The latter mainly depends on the curvature and the buckling length. The theoretical basis is the Euler solution in form (8). It is assumed that $B=e_2$ and this eccentricity occurs at half of the column's height. The generalization consists in replacing l by l_0. In this way, the axis curvature $\kappa=1/r$ is interrelated with e_2. The value of the latter can be calculated from the relation:

$$\frac{e_2}{d} = K_\mathrm{r} K_\varphi \frac{1}{r_0}\frac{l_0^2}{c}\frac{1}{d} = K_\mathrm{r} K_\varphi \frac{f_{\mathrm{yd}}}{0.45 \cdot E_\mathrm{s}}\frac{1}{c}\left(\frac{l_0}{d}\right)^2, \tag{14}$$

where f_{yd} is the design yield strength of the reinforcement steel and c is a coefficient dependent on the curvature distribution (usually amounting to 10).

If one disregards the influence of creep (K_φ=1), the relative force n will be weaker than n_{bal} (K_r=1), and so, the relative curvature κd will be always the same (15), and the relative eccentricity will depend exclusively on $(l_0/d)^2$ (16).

$$\kappa d = \frac{d}{r_0} = \frac{f_{\mathrm{yd}}}{0.45 \cdot E_\mathrm{s}} = \frac{435}{0.45 \cdot 200} \cdot 10^{-3} = 4.83 \cdot 10^{-3}, \tag{15}$$

$$\frac{e_2}{d} = \kappa d \frac{1}{c}\left(\frac{l_0}{d}\right)^2 = 0.483 \cdot 10^{-3} \cdot \left(\frac{l_0}{d}\right)^2. \tag{16}$$

The above fact and the strongly emphasized influence of l_0/d raise doubts as they do not find confirmation in building practice. The curvature correction stemming from the influence of the axial force on curvature is assumed to have the form:

$$K_\mathrm{r} = \frac{n_\mathrm{u} - n}{n_\mathrm{u} - n_{\mathrm{bal}}} \leq 1. \tag{17}$$

This form of the formula means that the force–curvature relationship is linearized and reaches its maximum value at $n=n_{\mathrm{bal}}$ and zero when $n=n_\mathrm{u}$. The assumption that for $n<n_{\mathrm{bal}}$, the curvature remains constant and for $n>n_{\mathrm{bal}}$, it decreases so quickly raises serious doubts.

In the literature, one can find several references to this method applied in engineering design (e.g., Bond et al., 2006; Mosley et al., 2007) and to other conceptions concerning a way of determining the maximum curvature in a cross section. Connections between the maximum curvature and deformations not in the reinforcement, but in the outer fibers of concrete under compression are described by Barros et al. (2010).

3 Usage of the general method

This method has no connection whatsoever with the assumptions made in the simplified methods. Its idea was most fully described in CEB/FIP Manual (1978). In Poland, the method was presented in outline by Knauff et al. (2006) and in a much more extended form by Pędziwiatr (2019) and Pędziwiatr and Musiał (2023). It consists in solving the equation:

$$\frac{d^2 y}{dx^2} = -\frac{M(x)}{EI(x)}. \tag{18}$$

This entails the following:
1) the determination of the relationship between bending moment and stiffness for a given value of the axial force;
2) the solution of the differential equation. In the adopted approach, it is changed into a difference equation, and solutions (beam axis displacements $y(x_i)$) in selected column cross sections are obtained. On this basis, one can determine the current moment paths and solve the difference equation again. This procedure ends when the differences between the results (moment values) in successive steps are already infinitesimally small or if the sequence of the results becomes divergent – the column fails under the set configuration of load, geometry, and reinforcement.

For determining the moment–stiffness relationship or the moment–curvature relationship in a second-order analysis, the standard recommends the following relation for stress in concrete:

$$\sigma_\mathrm{c} = \sigma_\mathrm{c}(\varepsilon_\mathrm{c}) = f_{\mathrm{cm}} \frac{k\eta - \eta^2}{1 + (k-2)\eta}, \tag{19}$$

where f_{cm} is the mean compressive strength of the concrete. The coefficients occurring in equation (19) are defined by the following relations:

Table 1: Basic parameters of concrete grades.

Grade	C30	C35	C40	C45	C50	C55	C60	C70	C80	C90
f_{cm} [MPa]	38	43	48	53	58	63	68	78	88	98
ε_{c1} [‰]	2.2	2.25	2.3	2.4	2.45	2.5	2.6	2.7	2.8	2.8
E_{cm} [GPa]	32	34	35	36	37	38	39	41	42	44
A	0.884	0.830	0.766	0.713	0.670	0.633	0.602	0.552	0.501	0.471
B	-0.207	-0.198	-0.189	-0.174	-0.167	-0.160	-0.148	-0.137	-0.128	-0.128
C	-0.025	-0.059	-0.104	-0.120	-0.146	-0.167	-0.167	-0.189	-0.213	-0.243
k	1.945	1.868	1.761	1.712	1.641	1.583	1.566	1.490	1.403	1.320
η_{max}	1.591	1.556	1.522	1.458	1.429	1.280	1.154	1.037	1.000	1.000

$$k = 1.05 E_{\mathrm{cm}} \frac{\varepsilon_{\mathrm{c1}}}{f_{\mathrm{cm}}}, \tag{20}$$

$$\eta = \frac{\varepsilon_{\mathrm{c}}}{\varepsilon_{\mathrm{c1}}}, \tag{21}$$

where E_{cm} is the mean E-modulus of the concrete and ε_{c1} is the strain corresponding to the largest deformation. It is convenient to write equation (19) in the form with the highlighted variable ε_c representing strains in the concrete:

$$\sigma_{\mathrm{c}} = f_{\mathrm{cm}} \frac{A\varepsilon_{\mathrm{c}} + B\varepsilon_{\mathrm{c}}^2}{1 + C\varepsilon_{\mathrm{c}}} \tag{22}$$

Table 1 gives values of the parameters which make it possible to use relation (22) for a large group of concrete grades. However, the form of this equation is problematic, leading to quite complicated relations for the value of force in the concrete. It is much convenient to replace this formula with its polynomial approximation. This is illustrated in Fig. 2 for C30/37 concrete.

One can use relation $\sigma_c=\sigma_c(\varepsilon_c)$ to determine the force in the compressed concrete for all the stress intensity levels. Then the relations between the moment value and the beam axis curvature can be determined.

It is convenient to use nondimensional quantities, whereby much higher universality of the results can be achieved. The following denotations and relations were assumed for the parameters (the axial force, the bending moment, and the relative curvature, respectively):

$$n = \frac{N_{\mathrm{Ed}}}{f_{\mathrm{cd}} b d}, \tag{23}$$

$$m = \frac{M_{\mathrm{Ed}}}{f_{\mathrm{cd}} b d^2}, \tag{24}$$

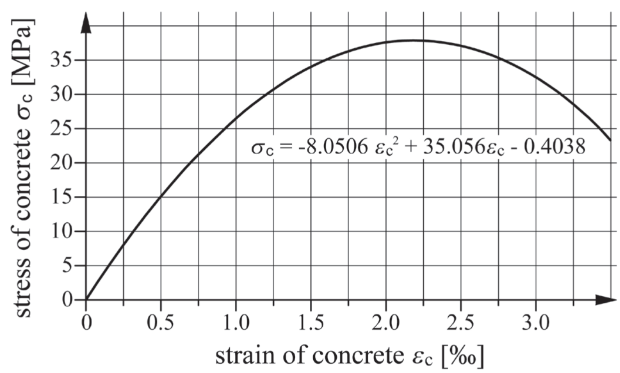

Figure 2: $\sigma_c=\sigma_c(\varepsilon_c)$ diagram for C30/37 concrete with equation approximating this relation.

$$\kappa d = \frac{1}{r} d. \tag{25}$$

To plot the $\kappa d=f(m)$ curve, one must determine this relation for a given value of n at changing values of strain ε_c in the concrete in the cross section's outer fibers. The initial value is $\varepsilon_c=\varepsilon_{c,min}$, which perfectly matches the axially compressed cross section. The subsequent values increase gradually up to ε_{cu1}. When calculating the successive values of κd and m, one uses the equation for the equilibrium of forces and moments and the law of flat sections. Steel yield is taken into account at appropriate stress intensity levels when strain ε_s in the steel reaches plastic strain $\varepsilon_{pl}=f_{yd}/E_s$. Exemplary results of such calculations are presented in Table 2.

The independent variable in the calculations is ε_c. The strain (ε_d) of the least compressed fibers (if the cross section is compressed) or the relative height (ξ) of the compression zone is determined on the basis of the equilibrium-of-forces condition, whose particular form depends on the stage of stressing. Thanks to the

Table 2: Exemplary results of $\kappa d=f(m)$ relation determination for n=0.8, concrete C30/37, $\rho_1=\rho_2$=1%.

ε_c	ε_d	ε_{s1}	ε_{s2}	n_c	n_{s1}	n_{s2}	m	$m/\kappa d$	κd	Notes
[‰]	[‰]	[‰]	[‰]	[-]	[-]	[-]	[-]	[-]	[-]	
0.456	0.456	0.456	0.456	0.715	0.043	0.043	0.000	181.5	0.00000	whole cross section compressed (part 1 in figs 3 and 4)
0.490	0.422	0.428	0.484	0.715	0.040	0.045	0.011	181.5	0.00006	
0.600	0.315	0.341	0.574	0.714	0.032	0.054	0.047	181.4	0.00026	
0.700	0.222	0.265	0.657	0.714	0.025	0.061	0.079	181.1	0.00043	
0.800	0.131	0.192	0.739	0.713	0.018	0.069	0.110	180.7	0.00061	
0.900	0.043	0.121	0.822	0.712	0.011	0.077	0.140	180.2	0.00078	
ε_c	ε_d	ε_{s1}	ε_{s2}	n_c	n_{s1}	n_{s2}	m	$m/\kappa d$	κd	Notes
[‰]	[-]	[‰]	[‰]	[-]	[-]	[-]	[-]	[-]	[-]	
0.951	1.100	0.086	0.864	0.711	0.008	0.081	0.155	179.9	0.00086	$\varepsilon_{s1}<\varepsilon_{pl}$, $\varepsilon_{s2}<\varepsilon_{pl}$ cross section with tension zone (part II in figs 3 and 4)
0.960	1.091	0.080	0.872	0.711	0.007	0.081	0.158	179.7	0.00088	
1.000	1.053	0.051	0.905	0.711	0.005	0.085	0.169	178.2	0.00095	
1.100	0.973	-0.031	0.987	0.711	-0.003	0.092	0.194	171.7	0.00113	
1.300	0.855	-0.220	1.148	0.713	-0.021	0.107	0.236	155.2	0.00152	
1.500	0.775	-0.436	1.306	0.719	-0.041	0.122	0.271	139.8	0.00194	
1.700	0.718	-0.669	1.463	0.726	-0.062	0.137	0.301	127.2	0.00237	
2.000	0.660	-1.030	1.697	0.738	-0.096	0.159	0.341	112.6	0.00303	
2.200	0.634	-1.271	1.853	0.746	-0.119	0.173	0.365	105.1	0.00347	
2.400	0.615	-1.504	2.010	0.753	-0.141	0.188	0.386	98.9	0.00390	
2.600	0.601	-1.727	2.167	0.759	-0.161	0.203	0.405	93.5	0.00433	
2.605	0.601	-1.733	2.171	0.759	-0.162	0.203	0.405	93.4	0.00434	
2.650	0.600	-1.770	2.208	0.762	-0.165	0.203	0.407	92.2	0.00442	$\varepsilon_{s2}\geq\varepsilon_{pl}$
2.700	0.599	-1.808	2.249	0.766	-0.169	0.203	0.410	90.9	0.00451	
2.900	0.598	-1.948	2.415	0.779	-0.182	0.203	0.416	85.8	0.00485	
3.100	0.600	-2.063	2.584	0.790	-0.193	0.203	0.419	81.2	0.00516	
3.355	0.607	-2.172	2.802	0.800	-0.203	0.203	0.419	75.8	0.00553	
3.400	0.608	-2.194	2.841	0.800	-0.203	0.203	0.417	74.6	0.00559	R*)
3.500	0.610	-2.239	2.926	0.800	-0.203	0.203	0.414	72.1	0.00574	

*) – failure (part II_R in figs 3 and 4)

law of flat sections, one can calculate the strains in the reinforcement – ε_{s1}, ε_{s2} – and the corresponding forces – n_{s1}, n_{s2}. The force in the compressed concrete is calculated by integrating $\sigma_c=\sigma_c(\varepsilon_c)$ over ζ within the limits ζ=0 to $\zeta=\xi$, having previously written relation $\varepsilon_c=\varepsilon_c(\zeta)$ stemming from the law of flat sections. In the next step, using the equilibrium conditions, one can calculate the value of relative moment m corresponding to determined strain ε_c. Relative curvature κd is calculated from appropriate relation (26) when the whole cross section is compressed or (27) when a tension zone appears in the cross section.

$$\kappa d=\frac{\varepsilon_c-\varepsilon_d}{1+\frac{a}{d}}, \tag{26}$$

$$\kappa d=\frac{\varepsilon_c}{\xi}. \tag{27}$$

On the basis of the calculation results, one can draw graphs as the ones shown in Figs 3 and 4. The relation of the type shown in Fig. 4 is used for further calculations aimed at determining the displacements of the column axis. Equation (18) can be written in the form:

$$\kappa(x) \cdot d = \frac{m(x) f_{cd} b d^3}{EI(x)} = m(x) \left[\frac{f_{cd} b d^3}{EI(x)}\right] = F(m). \qquad (28)$$

Function $F(m)$ in (28) is represented by the relations shown in Fig. 4. By replacing the differential equation with a difference equation, one gets:

$$\kappa_i d = \frac{w_{i-1} - 2w_i + w_{i+1}}{l_i^2} d = F(m). \qquad (29)$$

One should follow the procedure:

1) divide the column using points (the more points there are, the higher the solution precision),
2) determine the value of relative moment m in each of the points,
3) calculate the relative curvature on the basis of appropriate relations (see Fig. 4),
4) determine displacements w_i – according to (29) with the boundary conditions taken into account,
5) in successive points, calculate moment increments $N_{Ed} w_i$ and the corresponding nondimensional quantities, and
6) calculate new moment values and repeat the procedure.

The procedure ends when moment increments become minimal or when it is found that increments form an increasing sequence.

Figure 5a shows a corbel beam. In point **0,** the displacement and the angle of rotation are equal to zero. This leads to the following two relations: $w_0=0$, $w_1=w_{-1}$. The forms of equation (29) in the particular points are as follows:

– **0**: $\kappa_0 d = \frac{2w_1}{l_i^2} d \Rightarrow w_1 = \kappa_0 d \frac{l_i^2}{2d} = \frac{\kappa_0 l_i^2}{2}$,

– **1**: $\kappa_1 d = \frac{w_0 - 2w_1 + w_2}{l_i^2} d = \frac{2w_1 + w_2}{l_i^2} d \Rightarrow w_2 = 2w_1 + \kappa_1 l_i^2$,

– **2**: $\kappa_2 d = \frac{w_1 - 2w_2 + w_3}{l_i^2} d \Rightarrow w_3 = 2w_2 - w_1 + \kappa_2 l_i^2$,

– **(n-1)**: $\kappa_{n-1} d = \frac{w_{n-2} - 2w_{n-1} + w_n}{l_i^2} d \Rightarrow w_n =$

$= 2w_{n-1} - w_{n-2} + \kappa_{n-1} l_i^2$.

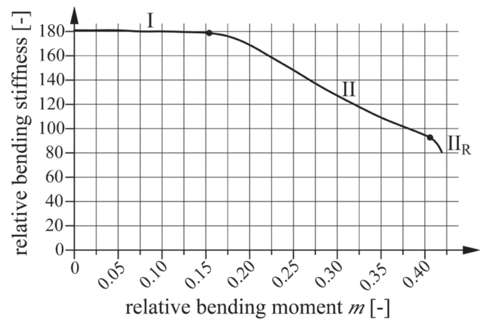

Figure 3: Relative stiffness-relative moment diagram.

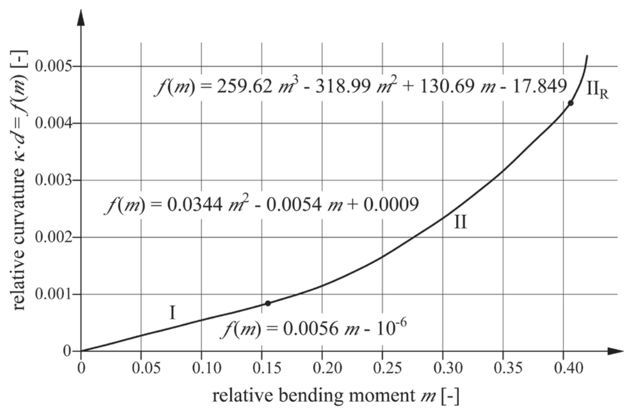

Figure 4: Relative curvature $\kappa d = f(m)$ versus relative moment for $n=0.8$, concrete C30/37, $\rho_1 = \rho_2 = 1\%$.

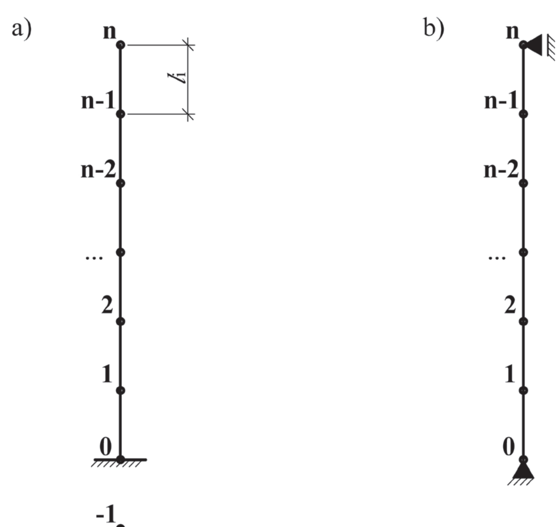

Figure 5: Beam division taking fixing conditions into account: a) corbel column, b) pin-supported column.

It is evident that in the case of a corbel beam, one can directly determine the displacements of successive points along the beam axis.

Table 3: Exemplary results of iterative determination of 2nd order moment for 6.0 m high column.

Iteration	Quantity [-]	Cross section no.							Moment increment [%]
		0	**1**	**2**	**3**	**4**	**5**	**6**	
0	m_0	**0.2500**	**0.2083**	**0.1667**	**0.1250**	**0.0833**	**0.0417**	**0.0000**	
	κd	0.0022	0.0016	0.0011	0.0007	0.0004	0.0002	0.0000	
	w	0.0000	0.0022	0.0076	0.0151	0.0240	0.0337	0.0438	
	Δm	0.0219	0.0208	0.0181	0.0144	0.0099	0.0051	0.0000	
1	m	0.2719	0.2291	0.1848	0.1394	0.0933	0.0467	0.0000	8.760
	κd	0.0025	0.0019	0.0013	0.0008	0.0005	0.0002	0.0000	
	w	0.0000	0.0025	0.0088	0.0177	0.0282	0.0396	0.0515	
	Δm	0.0258	0.0245	0.0213	0.0169	0.0117	0.0060	0.0000	
2	m	0.2758	0.2328	0.1880	0.1419	0.0950	0.0476	0.0000	1.417
	κd	0.0026	0.0019	0.0013	0.0008	0.0005	0.0002	0.0000	
	w	0.0000	0.0026	0.0091	0.0182	0.0290	0.0407	0.0529	
	Δm	0.0265	0.0252	0.0219	0.0174	0.0120	0.0061	0.0000	
4	m	0.2766	0.2336	0.1887	0.1425	0.0954	0.0478	0.0000	0.048
	κd	0.0026	0.0019	0.0013	0.0008	0.0005	0.0002	0.0000	
	w	0.0000	0.0026	0.0091	0.0183	0.0291	0.0409	0.0532	
	Δm	0.0266	0.0253	0.0221	0.0175	0.0121	0.0061	0.0000	
5	m	0.2766	0.2336	0.1887	0.1425	0.0954	0.0478	0.0000	0.009
	κd	0.0026	0.0019	0.0013	0.0008	0.0005	0.0002	0.0000	
	w	0.0000	0.0026	0.0091	0.0183	0.0291	0.0409	0.0532	
	Δm	0.0266	0.0253	0.0221	0.0175	0.0121	0.0061	0.0000	
6	m	0.2766	0.2336	0.1887	0.1425	0.0954	0.0478	0.0000	0.002
	κd	0.0026	0.0019	0.0013	0.0008	0.0005	0.0002	0.0000	
	w	0.0000	0.0026	0.0091	0.0183	0.0291	0.0409	0.0532	
	Δm	0.0266	0.0253	0.0221	0.0175	0.0121	0.0061	0.0000	
7	m	**0.2766**	**0.2337**	**0.1887**	**0.1425**	**0.0954**	**0.0478**	**0.0000**	**0**

Table 3 shows an exemplary iteration sequence illustrating the presented procedure. The column is rigidly fixed in the bottom node. It is 6.0 m high and its effective cross-sectional height amounts to d=h-a=0.5 m. The relative axial force value amounts to n=0.5, and the relative value of the first-order moment varies linearly from 0.25 in the fixing to zero at the free end. Seven consecutive cross sections spaced l_i=1 m apart are distinguished in the column. The number of the iteration step is given in the first column of the table. For the initial state, the values of relative curvatures κd, horizontal displacements w_i, and moment increments $\Delta m_i = n(w_6 - w_i)$ induced by the displacements were determined one by one in all the selected cross sections. Six iterations were performed. The last column shows the relative moment increments in the successive iteration steps. In the considered case, the sequence is decreasing very quickly. Taking into account second-order effects, the relative moment is m=0.2766. The increment relative to initial value m_0=0.25 amounts to about 11%.

It is important to pay careful attention during the successive iterations. Too early termination of the iteration process may turn out to be a wrong decision. Table 4 shows fragments of the iteration for a 12.0-m-high column with maximum first-order moment m_0=0.18. Up to iteration 14, the moment increments slightly decreased. But after iteration 14, the tendency changed. The eccentricity increments began to gradually increase. This meant that the column failed. This was due to the fact that a load level was reached at which the increment in column curvature significantly accelerated.

4 Examples and comparisons

To examine the differences between the results yielded by the approximate methods and the ones obtained using the general method, calculations were carried out assuming the following: concrete C30/37 (f_{ck}=30 MPa), steel with f_{yk}=500 MPa, and reinforcement ratio $\rho_1=\rho_2$=1%. Four corbel columns with heights l=6.0,8.0,10.0,12.0 m were considered. To increase the generality of the results, the nondimensional parameters relative axial force n, relative first-order moment m_0, relative curvature κd, and moment increment md induced by second-order effects were used. Results for three stress intensity levels induced by axial force n=0.1,0.5,0.8 are given in the examples. At these levels of relative force n, the maximum values of moment m_{Rd} can be determined from the equilibrium equations whose specific form depends on the assumed concrete model:

a) The nonlinear model from the general method
- For n=0.1, the steel in the compression zone is not utilized and the equilibrium-of-forces condition has the form of a quadratic equation with regard to ξ:

$$n = \frac{\xi}{f_{cd}}\left(\frac{1}{3}A\varepsilon_c^2 + \frac{1}{2}B\varepsilon_c + C\right) + \varepsilon_c \frac{E_s}{f_{cd}} \frac{\xi - \frac{a}{d}}{\xi}\rho_2 - \frac{f_{yd}}{f_{cd}}\rho_1. \tag{30}$$

  The solution is ξ=0.149. After determining the appropriate strains and forces, one gets the maximum moment m_{Rd}=0.233.
- For n=0.5, both the steels can be utilized, but m_{Rd}=0.381 is reached just before the yielding of the steel in the compression zone at ε_c=3.10‰.
- For n=0.8, both the steels can be utilized, but m_{Rd}=0.419 is reached just before the yielding of the steel in the compression zone at ε_c=3.10‰.

b) The simplified model
- For n=0.1, the steel in the compression zone is not utilized, the equilibrium-of-forces condition has the form (31), and the relative resistance to bending is calculated from equation (32).

$$n = \xi_{eff} - \rho_1 \frac{f_{yd}}{f_{cd}} \Rightarrow \xi_{eff} = 0.1 + 0.01\frac{435}{21.4} = 0.3033, \tag{31}$$

$$m_{Rd} = \xi_{eff}(1 - 0.5\xi_{eff}) - 0.45n = 0.1223. \tag{32}$$

  If the bilinear model were used, m_{Rd}=0.232 would be obtained.
- For n=0.5, both the steels are utilized, the equilibrium-of-forces condition has the form (33), and the relative resistance to bending m_{Rd} calculated from (32) amounts to 0.3329.

$$n = \xi_{eff} \Rightarrow \xi_{eff} = 0.5. \tag{33}$$

- For n=0.8, only the steel in the compression zone is utilized, relative effective height ξ_{eff} of the compression zone amounts to 0.5967, and relative bending strength m_{Rd}=0.2637.

When comparing the results, one can either use the m_{Rd} values calculated from the above formulas or assume the values obtained using the simplified concrete model commonly used for dimensioning as the limit values. Comparisons were made for both these cases.

4.1 Nominal stiffness method

The results of the calculations based on the nominal stiffness method are presented in Tables 5 and 6. Ratio N_B/N_{Ed} was calculated from equation (12) and m_{Ed}/m_{0Ed} from relation (9).

At light vertical loads (n=0.1), the moment increment due to second-order effects obviously depends on the column height, ranging from 1.15 to 1.82. When the load is increased to n=0.5, it becomes necessary to exclude 10- and 12-m-high columns as they would fail – N_B/N_{Ed}<1.0. The maximum column height at n=0.8 is 6.0 m.

Table 5 shows the values of maximum first-order moment $m_{0ED,max}$ which can act on the column at set l and n combinations. The moment was calculated as an appropriate m_{Rd} divided by m_{Ed}/m_{0Ed}. If this moment is interpreted as, for example, the effect of wind action via the curtain walls on the column, it turns out that even at n=0.1, the use of columns with l=12.0 m is dubious. At n=0.5, only a less than 8.0-m-high column can carry considerable horizontal loads.

Table 4: Exemplary results of iterative determination of 2nd order moment for 12.0 m high column.

Iteration	Quantity [-]	Cross section no. 0	1	2	3	.	9	10	11	12	Moment increment [%]
0	m	0.18	0.165	0.15	0.135	.	0.045	0.03	0.015	0	
	κd	0.001	0.001	0.001	0.001	.	0.000	0.000	0.000	0.000	
	w	0.000	0.001	0.005	0.010	.	0.065	0.077	0.089	0.101	
	Δm	0.051	0.050	0.048	0.046	.	0.018	0.012	0.006	0.000	
1	m	0.231	0.215	0.198	0.181	.	0.063	0.042	0.021	0.000	28.10
	κd	0.002	0.002	0.001	0.001	.	0.000	0.000	0.000	0.000	
	w	0.000	0.002	0.007	0.015	.	0.103	0.122	0.140	0.159	
	Δm	0.080	0.079	0.076	0.072	.	0.028	0.019	0.009	0.000	
2	m	0.260	0.244	0.226	0.207	.	0.073	0.049	0.024	0.000	12.59
	κd	0.002	0.002	0.002	0.002	.	0.000	0.000	0.000	0.000	
	w	0.000	0.002	0.009	0.019	.	0.129	0.152	0.175	0.199	
	Δm	0.099	0.098	0.095	0.090	.	0.035	0.023	0.012	0.000	
3	m	0.279	0.263	0.245	0.225	.	0.080	0.053	0.027	0.000	7.60
	κd	0.003	0.002	0.002	0.002	.	0.000	0.000	0.000	0.000	
	w	0.000	0.003	0.010	0.022	.	0.148	0.175	0.201	0.228	
	Δm	0.114	0.113	0.109	0.103	.	0.040	0.027	0.013	0.000	
.	.	.	.	.	.	.	.	.	.	.	
7	m	0.323	0.306	0.287	0.265	.	0.095	0.063	0.032	0.000	2.56
	κd	0.003	0.003	0.003	0.002	.	0.000	0.000	0.000	0.000	
	w	0.000	0.003	0.013	0.028	.	0.196	0.230	0.265	0.300	
	Δm	0.150	0.148	0.144	0.136	.	0.052	0.035	0.018	0.000	
.	.	.	.	.		.	.	.	.	.	
11	m	0.347	0.330	0.310	0.287	.	0.103	0.069	0.035	0.000	1.52
	κd	0.004	0.004	0.003	0.003	.	0.001	0.000	0.000	0.000	
	w	0.000	0.004	0.015	0.032	.	0.224	0.264	0.304	0.344	
	Δm	0.172	0.170	0.165	0.156	.	0.060	0.040	0.020	0.000	
.	.	.	.	.	.	.	.	..	.	.	
15	m	0.365	0.348	0.327	0.303	.	0.110	0.073	0.037	0.000	1.17
	κd	0.004	0.004	0.004	0.003	.	0.001	0.000	0.000	0.000	
	w	0.000	0.004	0.017	0.036	.	0.248	0.291	0.336	0.380	
	Δm	0.190	0.188	0.182	0.172	.	0.066	0.044	0.022	0.000	
16	m	0.370	0.353	0.332	0.307	.	0.111	0.074	0.037	0.000	1.32
	κd	0.005	0.004	0.004	0.003	.	0.001	0.000	0.000	0.000	
	w	0.000	0.005	0.018	0.038	.	0.258	0.303	0.349	0.395	
	Δm	0.197	0.195	0.188	0.178	.	0.068	0.046	0.023	0.000	
17	m	**0.377**	**0.360**	**0.338**	**0.313**	.	**0.113**	**0.076**	**0.038**	**0.000**	**1.96**

Table 5: Selected calculation results for different column heights l and relative forces n, obtained using nominal stiffness method.

Quantity	n=0.1				n=0.5				n=0.8			
l [m]	6.0	8.0	10.0	12.0	6.0	8.0	10.0	12.0	6.0	8.0	10.0	12.0
l_0 [m]	12	16	20	24	12	16	20	24	12	16	20	24
λ	75	100	125	150	75	100	125	150	75	100	125	150
k_2	0.044	0.059	0.073	0.088	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
N_B/N_{Ed}	7.76	4.60	3.10	2.26	2.45	1.38	0.88	0.61	1.53	0.86	0.55	0.38
m_{Ed}/m_{0Ed}	1.15	1.29	1.49	1.82	1.71	3.70	-	-	2.93	-	-	-

Table 6: Results of calculations of maximum 1st order moment $m_{0ED,max}$ for different column heights l and relative forces n done acc. to standard stiffness method.

Quantity	n=0.1				n=0.5				n=0.8			
l [m]	6.0	8.0	10.0	12.0	6.0	8.0	10.0	12.0	6.0	8.0	10.0	12.0
m_{Ed}/m_{0Ed}	1.15	1.29	1.49	1.82	1.71	3.70	-	-	2.93	-	-	-
$m_{0ED,max}$	0.202	0.180	0.156	0.127	0.195	0.09	-	-	0.086			

4.2 Nominal curvature method

Similar calculations as the one above were performed using the nominal curvature method. The main results of the calculations are presented in Table 7. Equations (14)–(17) were used in the calculations. Moment increments were calculated from (34) and the value of $m_{0ED,max}$ from (35).

$$\Delta m = \frac{e_2}{d} n, \tag{34}$$

$$m_{0Ed,max} = m_{Rd} - \Delta m. \tag{35}$$

The results (Table 7) are very similar to the ones obtained using the nominal stiffness method, practically regardless of the column height l and the cross section stress intensity level n. In both methods, the deciding factor was the ratio l_0/d.

4.3 General method

The results of the iterative calculations described in Section 3 are presented in Table 8. The table contains appropriate values of maximum moment $m_{0ED,max}$ induced by the horizontal force, assuming that the load-bearing capacity is determined using the concrete strain model in the form of equation (19). The value denoted as $m^{*}_{0ED,max}$ corresponds to the typically adopted simplified dimensioning assumptions. Differences between $m^{*}_{0ED,max}$ and $m_{0ED,max}$ are significant only at n=0.8. At this stress intensity level, the compression zone is extensive and its load-bearing capacity begins to play a decisive role. The mean concrete strength f_{cm} used in equation (19) is much higher than the design concrete compressive strength f_{cd}.

Table 8. Selected calculation results for different column heights l and relative forces n, obtained using general method.

4.4 Discussion of results

Table 9 contains the calculated maximum moments induced by the horizontal load, at which, as the calculations show, the column fails. The results calculated using the nominal curvature method, the nominal stiffness method, and the general method are presented in row 1, row 2, and row 3, respectively. As regards the two simplified methods, the results are similar in all the considered cases. Also, for l≤8.0 m and n=0.1, they are not much different from the results yielded by the general method. The differences become significant at the column heights of 10.0 and 12.0 m. At the height of 10.0 m, they exceed 30% and at 12.0 m, they exceed 50%. When the column's stress intensity level increases to n=0.5, it is found that the considered columns can be designed only up to a height of 8.0 m on the basis of calculations done using the approximate methods. At this column height, the difference in favor of the general method is already very substantial, amounting to almost

Table 7: Selected calculation results for different column heights *l* and relative forces *n*, obtained using nominal curvature method.

Quantity	*n*=0.1				*n*=0.5				*n*=0.8			
l [m]	6.0	8.0	10.0	12.0	6.0	8.0	10.0	12.0	6.0	8.0	10.0	12.0
l_0 [m]	12	16	20	24	12	16	20	24	12	16	20	24
K_r	1.0				1.0				0.669			
$d/r_0=\kappa_0 d$	$4.83\cdot10^{-3}$											
$d/r=\kappa d$	$4.83\cdot10^{-3}$								$3.23\cdot10^{-3}$			
Δm	0.028	0.049	0.077	0.111	0.139	0.247	0.386	0.556	0.149	0.265	0.413	0.595
$m_{0ED,max}$	0.204	0.183	0.155	0.121	0.194	0.086	-	-	0.104	-	-	-

Table 8: Selected calculation results for different column heights *l* and relative forces *n*, obtained using general method.

Quantity	*n*=0.1				*n*=0.5				*n*=0.8			
l [m]	6.0	8.0	10.0	12.0	6.0	8.0	10.0	12.0	6.0	8.0	10.0	12.0
$m_{0ED,max}$	0.22	0.215	0.205	0.195	0.32	0.28	0.23	0.17	0.33	0.27	0.19	0.13
$m^*_{0ED,max}$	0.22	0.21	0.205	0.195	0.29	0.26	0.22	0.17	0.24	0.18	0.16	0.12

Table 9: $m^*_{0ED,max}$ values for three methods used.

	n=0.1				*n*=0.5				*n*=0.8			
l [m]	6.0	8.0	10.0	12.0	6.0	8.0	10.0	12.0	6.0	8.0	10.0	12.0
(1)	0.204	0.183	0.155	0.121	0.194	0.086	-	-	0.104	-	-	-
(2)	0.202	0.180	0.156	0.127	0.195	0.090	-	-	0.086	-	-	-
(3)	0.220	0.210	0.205	0.195	0.290	0.260	0.220	0.170	0.240	0.180	0.160	0.120

190%. Moreover, calculations done using the general method yield considerable values of $m^*_{0ED,max}$ also for 10.0- and 12.0-m-high columns. When the vertical load reaches n=0.8, the approximate methods can be used for up to 6.0-m-high columns. Whereas when the general method is used, positive values of maximum moment $m^*_{0ED,max}$ are obtained also for columns of heights 8.0, 10.0, and 12.0 m.

5 Conclusions

For some time now, a dynamic development of the branch of construction connected with the design and building of high bay racked warehouses has been observed. The basic load-bearing member of such structures is a frame consisting of high, slender, reinforced concrete columns and a lightweight steel girder forming the girt of the frame. Because of the height of the columns and the increasing spacing of the frames, the load acting on the column as a result of wind pressure on the curtain walls of a high bay warehouse becomes a critical factor.

For the safe and at the same economical design of such columns (and their foundations), the second-order effects occurring in this case must be precisely determined. In practice, simplified methods implemented in computer programs are used for this purpose. Most often, the nominal stiffness method is employed. But as demonstrated in this paper, this is an incorrect approach when horizontal forces occur. A somewhat better solution is to use the nominal curvature method, where one can distinguish a part of the first-order moment which is not subject to unwarranted increase.

It is characteristic of the two methods that the calculation results very significantly depend on $(l_0/d)^2$. The use of a fictional value of design length l_0 in a situation when horizontal loads occur is a source of a significant overestimate of the second-order effects. The error increases with column height and the stress intensity level induced by the vertical force. Calculations done in this paper show that the benefits stemming from the use of the general method amount to as much as 200%. Only for l≤8.0 m and n=0.1, the differences between the results yielded by the approximate methods and the general method, respectively, are slight.

The general method is much more labor intensive, requiring a relative moment-relative curvature relation for the given axial force value, reinforcement ratio, and concrete grade to be derived. Moreover, the iterative process, consisting in tracking the sequence of strain increments, is time-consuming. However, in the case of very slender columns, the benefits stemming from the use of general methods are very substantial. As part of further work, one can try to create detailed algorithms to be implemented in relevant computer programs.

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