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COMPARISON OF CONGRUENT DYNAMIC ECONOMETRIC MODEL FORECASTS AND SETAR MODEL FORECASTS

Abstract: The paper presents dynamic congruent linear econometric model as a tool for forecasting nonlinear relationships between economic processes. The aim of this paper is comparison of forecast errors based on dynamic congruent model and SETAR class models. The root square mean errors of linear and nonlinear are compared. Analysis is based on simulation. Basing on estimated models, forecast is built and forecast errors are calculated. Simulations assume different combinations of parameters: number of observations, scale of disturbance, relations between processes, type of nonlinearity between generated processes and others. Conclusions and remarks are formulated.

Key words: dynamic congruent model, SETAR models, nonlinear relationships.

1. Introduction

This paper refers to comparison of congruent dynamic econometric model forecasts and forecasts built using SETAR (*self-exciting threshold autoregressive model*) class models. Presented considerations are element of wider research about possibility to use congruent dynamic econometric models to explain nonlinear relationships. This research is based on simulation analysis using Monte Carlo method. Simulations were proceeded in *gretl* package. In all numerical experiments the DGP have the non-linear form and vary types of models are used to explain the DGP. In this paper explanatory models are: congruent dynamic econometric models (which are linear models), autoregressive models and SETAR models. Basing on estimated models forecasts were built and the ex post forecast errors were indicators of models quality. Results of the research are presented in tables and figures. The paper is ended with conclusions based on the results of the research and previous authors' work.

The researcher who examines real time series data must take under consideration the form of relationship between economic phenomena. This relationships can be linear or nonlinear. The range of linear models is wide, but for the nonlinear ones is much more wider. The researcher has precise set of data, economic phenomena and has to choose one type of relations from variety of known models. Some hints are given by the theory of economy, but these considerations are usually theoretical, with many assumptions and refer to specific situations, which sometimes are not satisfying in real.

This paper (and others authors' work) aims to answer the question about possibility to use only linear models (e.g. congruent models) for forecasting. The hypotheses set in this research are:

1. Models with autoregressive structures can be used to explain the nonlinear relationships between economic phenomena.
2. Congruent dynamic econometric model explains well nonlinear relationships.

2. Models used in research

In this research 4 models were used to explain nonlinear relationships:
– congruent dynamic econometric model,
– autoregressive model,
– SETAR model,
– SETAR model with additional exogenous variable.

2.1. Congruent dynamic econometric model

The author of the concept of congruent dynamic modelling is Professor Zygmunt Zieliński. The name of the concept – congruent, refers to harmonic congruency of endogenous processes and joint harmonic structure of exogenous and residual processes. The residual process is independent of the exogenous processes. The simplest and most obvious congruent model is a model built for white noises processes and have the form:

$$\varepsilon_{yt} = \sum_{i=1}^{k} \rho_i \varepsilon_{x_i t} + \varepsilon_t. \tag{1}$$

This model is always congruent, because harmonic structure of ε_{yt} is equal (or spectra of this process are parallel in relation to frequency axis) to joint harmonic structure of $\varepsilon_{x_i t}$ processes and ε_t process.

The congruent dynamic econometric model is using information about internal structure of used processes at the model specification stage. Let Y_t be endogenous process and X_{it} (i = 1, ..., k) are exogenous processes, then the internal structure of processes is described by basic models and concern:
– models describing nonstationary components:

$$Y_t = P_{yt} + S_{yt} + \eta_{yt}, \qquad X_{it} = P_{x_i t} + S_{x_i t} + \eta_{x_i t}, \tag{2}$$

where: $P_{yt}, P_{x_i t}$ – polynomial functions of t variable for appropriate processes, $S_{yt}, S_{x_i t}$ – seasonal components (constant or changing amplitude of fluctuations), $\eta_{yt}, \eta_{x_i t}$ – stationary autoregressive processes refers to appropriate processes,

and:
– autoregressive models:

$$B(u)\eta_{yt} = \varepsilon_{yt}, \qquad A_i(u)\eta_{x_i t} = \varepsilon_{x_i t}, \tag{3}$$

where: $B(u)$, $A_i(u)$ – stationary autoregressive backshift operators for which all roots of equations $|B(u)| = 0$ i $|A_i(u)| = 0$ lie outside the unit circle and $\varepsilon_{yt}, \varepsilon_{x_i t}$ – white noises for appropriate processes.

The dynamic congruent model is built by substituting to equation (1) processes $\varepsilon_{yt}, \varepsilon_{x_i t}$, which have white noise properties and received from equations (3), then according to (2) the autoregressive processes $\eta_{yt}, \eta_{x_i t}$ are calculated and substituting to previous received equations. Transforming, one gets the congruent dynamic econometric model for real processes Y_t and X_{it}:

$$B(u)Y_t = \sum_{i=1}^{k} A_i(u)X_{it} + P_t + S_t + \varepsilon_t. \tag{4}$$

The residual process from the above model has the same properties as the one in model (1), so has the white noise properties and the congruent condition is satisfied.

Dynamic congruent econometric models comply with information about internal structure (trend, seasonal and autoregressive components) and relations between used economic phenomena. This information is used at the specification stage of model building.

2.2. SETAR models

SETAR models are *self-exciting threshold autoregressive models* and can be classified as piece linear models. These constructions allow to describe relations according to condition, regime of process. This class of models is used to explain unemployment rate or industrial production in economic cycle stages (depending on economic stage, the different regime is used). It can also be applied to financial data, especially for asymmetric changes. Autoregressive threshold model with r regimes and number of p lags, noticed as SETAR(r, p), is as follows:

$$Y_t = \sum_{j=1}^{r} \left(\alpha_j^{'} X_t + \varepsilon_t \right) 1 \ \left\{ c_{j-1} < Y_{t-d} < c_j \right\} =$$

$$= \begin{cases} \alpha_{10} + \alpha_{11} Y_{t-1} + \cdots + \alpha_{1p} Y_{t-p} + \varepsilon_t \ \ for\ Y_{t-d} \le c_1 \\ \alpha_{20} + \alpha_{21} Y_{t-1} + \cdots + \alpha_{2p} Y_{t-p} + \varepsilon_t \ \ for\ c_1 < Y_{t-d} \le c_2 \\ \vdots \\ \alpha_{r0} + \alpha_{r1} Y_{t-1} + \cdots + \alpha_{rp} Y_{t-p} + \varepsilon_t \ \ for\ c_{r-1} < Y_{t-d} \end{cases},$$

where: $X_t = (1, Y_{t-1}, \ldots, Y_{t-p})$'' is vector of lagged endogenous variable, α_j' is coefficient vector for $j = 1, \ldots, r$, $1\{\cdot\}$ is Heavside function, c_j are threshold parameters and $\infty = c_0 < c_1 < \ldots < c_{r-1} < c_r = \infty$, $Y_{t\text{-}d}$ is the threshold variable.

In this research the SETAR(2,1) model was used with Y_{t-1} as threshold variable and the threshold parameter was mean of the process. Second used model of this class was model SETAR(2,1) with additional exogenous variable X_t.

3. Numerical experiments scenario

In this research 2 numerical experiments were made. In each experiment non-linear process was generated and next explained by 4 models described in point 2 of this paper. Basing on estimated models, forecasts were built and the forecast errors were compared. Additionally values of first order partial autocorrelation function of residual processes were analyzed. Scenarios of the experiments were as follows:

1. Generating 2 nonlinear processes:

– experiment 1 – $y_t = 5 / (0.8x_{1t} + 1.2x_{2t} + u_t)$,

– experiment 2 – $y_t = 5 + 1.05x_{1t} + 0.95x_{2t} + 0.5x_{1t}x_{2t} + u_t$,

with the assumption that processes x_{1t} and x_{2t} are autoregressive processes of order one:

$$x_{1t} = 5 + \beta_1 x_{1t-1} + \varepsilon, \text{ where } \beta_1 = \{0.8, 0.95\}, \varepsilon \sim N(0,1),$$
$$x_{2t} = 7 + \beta_2 x_{12-1} + \varepsilon, \text{ where } \beta_2 = \{0.7, 0.9\}, \varepsilon \sim N(0,1).$$

2. Processes y_t were explained by the models:

– congruent model: $y_t = \alpha_0 + \alpha_1 x_{1t} + \alpha_2 x_{1t-1} + \alpha_3 y_{t-1} + \alpha_4 y_{t-2} + \varepsilon_t$,

– autoregressive model: $y_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \varepsilon_t$,

– SETAR model: $y_t = (\alpha_0 + \alpha_1 y_{t-1} + \alpha_2 y_{t-2}) 1\{ y_{t-1} > \overline{y}_t \} + \varepsilon_t$,

– SETARX model: $y_t = (\alpha_0 + \alpha_1 y_{t-1} + \alpha_2 x_{1t}) 1\{ y_{t-1} > \overline{y}_t \} + \varepsilon_t$.

Notice that in above explanatory models the x_{2t} process was not used, but was used to generate y_t processes. This is typical situation in modelling real data, when researcher do not have whole information about processes which have the influence on described process.

3. For each estimated models, forecasts for 5 observations were made and root square mean errors were calculated.

Additionally, in each scenario some parameters were changed:

– observation number – n = {20, 60, 120, 300},
– disturbance of y_t process – $u_t \sim N(0,1)$, $N(0,2)$, $N(0,3)$,
– autoregressive coefficients of generated processes $x_{1t} - \beta_1$={0.8, 0.95}, $x_{2t} - \beta_2$ = {0.7, 0.9}.

For each experiment each combination was repeated 1000 times, what gives whole number of replications equal to 96,000.

4. Results of experiments

Results of experiments are presented in two aspects. The first one concerns autocorrelation of residual processes of explanatory models, the second – forecast errors of 4 used models.

Table 1 presents percentage of models, where first order of autocorrelation of residual process is insufficient on 5% of significance level according to the value of partial autocorrelation function. Table contains kind of experiment (form of nonlinearity), number of observations and kind of explanatory model.

Table 1. Percentage of models, where first order of autocorrelation of residual process is insignificant towards nonlinear form, number of observations and explanatory model

Number of observations	Model	Experiment 1	Experiment 2
n = 20	congruent	99.75	99.81
	autoregressive	99.83	99.87
	SETAR	99.69	99.78
	SETARX	99.65	99.55
n = 60	congruent	99.97	99.96
	autoregressive	99.97	99.97
	SETAR	99.97	89.97
	SETARX	99.96	99.96
n = 120	congruent	99.99	100.00
	autoregressive	99.98	100.00
	SETAR	99.99	100.00
	SETARX	99.99	99.99
n = 300	congruent	100.00	100.00
	autoregressive	100.00	100.00
	SETAR	100.00	99.96
	SETARX	100.00	99.95

Source: own calculations.

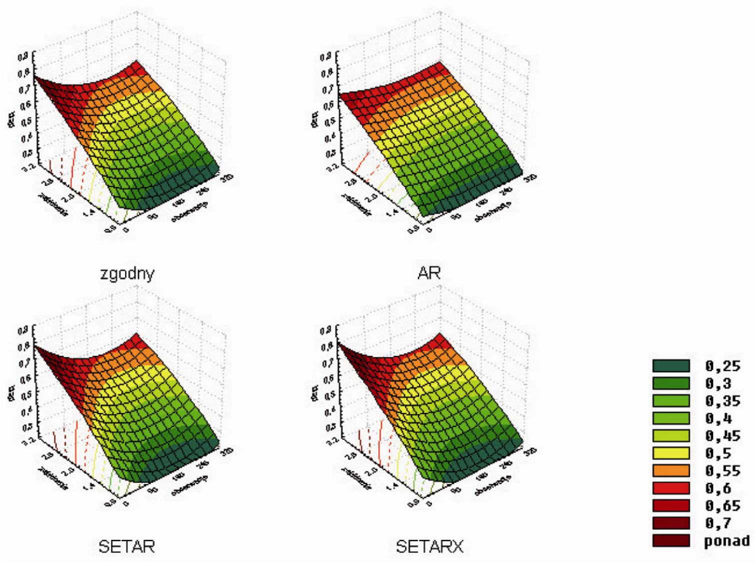

Figure 1. Root square mean errors versus number of observations, disturbance of generated process and explanatory models for experiment 1

Source: own calculations.

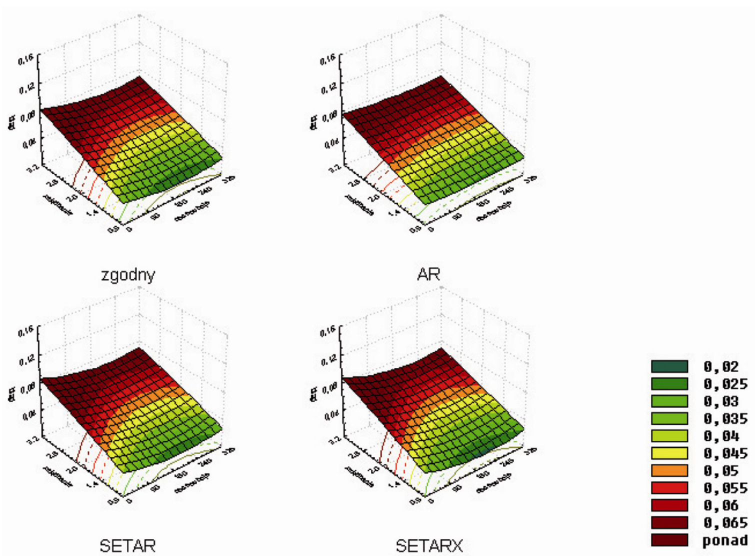

Figure 2. Root square mean errors versus number of observations, disturbance of generated process and explanatory models for experiment 2

Source: own calculations.

Figures 1 and 2 present root square mean errors versus number of observations and disturbance of generated process for 2 experiments.

All values presented on figures were square adjusted to obtain directions and relations of values of forecast errors. Green colour refers to low forecast errors and red colour refers to high forecast errors.

5. Conclusions

According to the results of the research some conclusions and remarks can be formulated. The analysis of autocorrelation of residual processes indicates that residual processes have white noise properties. Models with autoregressive structures can be used to describe and forecast nonlinear relationships.

Analysis of forecast errors shows the following conclusions. Values of errors are independent of the number of observations and change with the change of disturbance of generated process. Disturbance growth impacts the growth of value of forecast errors, which is obvious conclusion. Further analysis of forecast errors shows that values of errors are similar to explanatory models. This suggests that it is not necessary to use complicated models which are hard to estimate and calculate. Similar remarks can be found in the work of Kufel [2009]. This research shows that the forecast for autoregressive models and congruent models is similar to the forecast based on SETAR class models which are more difficult in modelling real economic phenomena.

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PORÓWNANIE JAKOŚCI PROGNOZ ZGODNEGO DYNAMICZNEGO MODELU EKONOMETRYCZNEGO Z PROGNOZAMI OPARTYMI NA MODELACH SETAR

Streszczenie: referat dotyczy prezentacji liniowego, dynamicznego, zgodnego modelu ekonometrycznego jako narzędzia do predykcji nieliniowej zależności między procesami ekonomicznymi. Referat jest kontynuacją prowadzonych badań dotyczących zgodnego dynamicznego predykatora.

Zależności rzeczywistych procesów ekonomicznych można podzielić na dwie grupy – zależności liniowe oraz nieliniowe. Dynamiczny model zgodny dobrze opisuje zależności liniowe.

Celem referatu jest analiza predyktorów liniowych otrzymanych na podstawie ekonometrycznego modelowania zgodnego do opisu zależności nieliniowych oraz porównanie ich z predyktorami opartymi na modelach SETAR. Ocenie podlega średni błąd prognozy dla predyktorów liniowych i nieliniowych.

Analizę przeprowadzono na podstawie danych symulacyjnych. Na podstawie oszacowanych modeli dokonano prognozy, a średni błąd prognozy jest miarą porównawczą. Analizę przeprowadzano przy różnych założeniach dotyczących badanych procesów: typu nieliniowej zależności, liczby obserwacji, stopnia zakłócenia, korelacji między zmiennymi niezależnymi, różnych parametrów symulowanych procesów.