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INTERDEPENDENCE BETWEEN THE CAC40 AND THE DAX INDICES IN INTRADAY DATA[1]

Abstract: This paper seeks to find out if shocks observed on the French and German markets are observed at the same moment or are transmitted among the markets within the very short time period. The bivariate GARCH model is estimated with variables standing for shocks in order to examine if there exists interdependence between the markets. The sample consists of 5-minute returns of the CAC40 and the DAX. Using high-frequency data and within the framework of the dynamic conditional correlation models, we show that volatility increases at the same time in the series. Within the nearest 5-minute interval, shocks observed in the DAX returns increase volatility of CAC40. Our results indicate co-movement in this pair of indices which suggests common exogenous sources of volatility shocks.

Key words: high-frequency data, jumps, volatility.

1. Introduction

The recent literature on financial markets shows that there exists strong interdependency between the markets. Shocks or abnormal returns observed in the financial series are irregular. Usually it is assumed that they are caused by unexpected information arriving on the market. These abnormal returns or unanticipated local shocks, observed on one market, may influence other markets' returns or volatility. There is already sizeable literature on the contagion effect and interdependency [Dungey et al. 2004].

Early studies of markets' interdependency are done on daily data. Y. Hamao, R.W. Masulis and V. Ng [1990] use the univariate GARCH models in testing price volatility spillovers for three major international stock markets, New York, Tokyo and London, finding evidence of volatility transmissions in few directions. C. Kearney and A.J. Patton [2000] use the multivariate BEKK model for contagion effect on currency markets. G. Milunovich and S. Thorp [2006] use the dynamic conditional correlation model in examining volatility spillover between the stock exchanges in London, Paris and Frankfurt.

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Another part of literature is concerned with the problem of comparing how arrival of information impacts different financial series. Market reaction to different announcements is considered in more and more popular intraday data. J. Lahaye, S. Laurent, and Ch. Neely [2007] use the Lee-Mykland statistic in detecting jumps and co-jumps in several intraday financial series. K. Harju and S.M. Hussain [2009] use the method of periodicity filtering presented in [Andersen, Bollerslev 1998] for four European indices and concluded that volatility of these indices reacts similarly to American macroeconomic announcements. However, in both studies the problem of co-movement in series is not present explicitly.

This paper contributes to the existing literature in a few aspects. Our motivation is to examine if abnormal returns detected with a jump test on two European markets, the French, and the German, are transmitted between the markets, namely if they influence the other markets' volatilities.

In the paper, jumps are defined following [Lee, Mykland 2008] as shocks in financial series caused by unexpected events. These events are market crashes, corporate defaults, macroeconomic announcements, national bank's decisions, etc. The jump detection test used in our study was originally proposed for a raw series. However, when applied to intraday data, the test may give different results depending on the periodicity treatment. K. Boudt et al. [2008] find that taking into account periodicity improves the correctness of intraday jump detection methods. The series are filtered from periodicity and introduced to the multivariate GARCH (MGARCH) models in order to model their interdependency. With the Dynamic Conditional Correlation models we are able to study if the jumps observed on one index influence conditional variance of another index.

The rest of the paper is as follows: in the first part of Section 2 the sample is described and in the second part the characteristic features of high-frequency data for examined series are presented. In Section 3 we describe the Weighted Standard Deviation, the non-parametric method of periodicity filtering used in the paper. Section 4 is devoted to jump detection methodology: we explain how jumps are detected in the examined series and presents the results of jump detection tests. Section 5 is dedicated to the methodology of the MGARCH models used in the study and Section 6 to the results of MGARCH estimations. Section 7 provides a conclusion.

2. Sample description and periodical patterns in intraday series

2.1. The sample

The sample consists of 5-minute prices of the CAC40 and the DAX indices from 1.09.2008 – 30.04.2010 (415 days), transformed into percentage logarithmic returns. The timing of quotations on these stock markets is the same: there are 102 returns per day in the CAC40 and the DAX series (stock exchanges are open from 9:00 to 17:30). The data is from financial database www.stooq.pl.

In our work we use OxMetrics 6.0, in particular G@RCH 6 software and Ox codes [Laurent, Peters 2002].

2.2. The intraday periodicity pattern

The intraday periodical pattern is very well described in a number of papers (see e.g. [Dacorogna et al. 2001]. In short, periodical pattern is recognized as a U-shaped strong autocorrelation function in intraday absolute returns and an inverted J-shaped curve of averages of absolute returns. What is characteristic for the shapes of averages of absolute returns for European stock markets is a sharp increase in volatility at the time of American macroeconomic announcements at 14:30 and 16:00 (see [Harju, Hussain 2009; Będowska-Sójka 2010]).

The comparison of the average absolute returns for the CAC40 and the DAX is shown in Figure 1. The averages are almost identical with high volatility at the beginning of the trading day, calm lunch time and a slow rise in volatility once the American stock exchange opens.

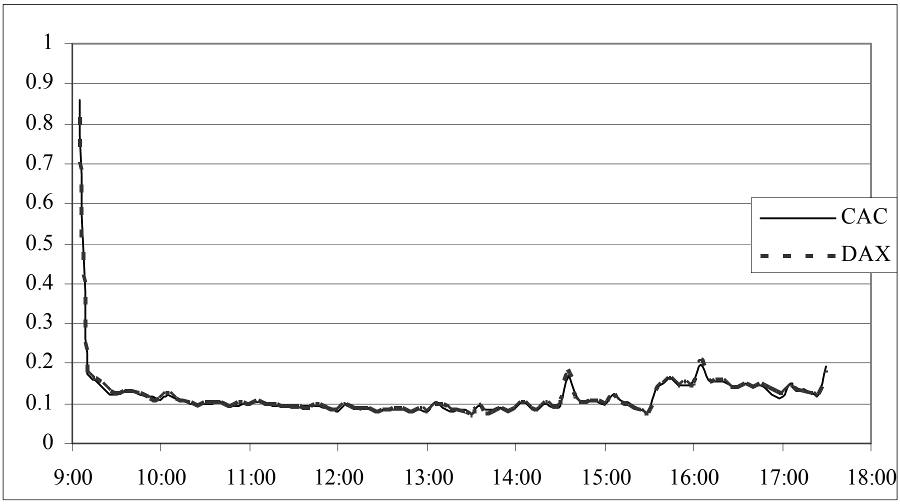

Figure 1. Average of absolute returns of CAC40 and DAX within the day

Source: own calculations.

The autocorrelation functions for the considered series (not shown here) indicate strong periodical pattern repeated every 102 returns (1 day). T. Andersen and T. Bollerslev [1998] suggest the series with such a pattern cannot be modelled with GARCH models directly, but need periodicity filtering first. In the study we use one of the non-parametric methods of filtering returns from periodicity.

3. Removing the periodical pattern with the Weighted Standard Deviation

In high frequency data, the return is a normal random variable with zero mean, whereas the standard deviation is a product of deterministic component, f_i, and average volatility factor, s_i (see [Andersen, Bollerslev 1998]. We rely on a general framework proposed by K. Boudt et al. [2008] for analyzing high-frequency returns:

$$r_i = f_i s_i u_i + b_i, \tag{1}$$

where: r_i – high frequency return,

f_i – deterministic component which is a function of the day of the week and the time of the day,

s_i – average volatility factor constant over "local window"; $\sigma_i = f_i s_i$; the choice of the local window depends on the frequency of the data,

u_i – independent and identically distributed; $u_i \sim N(0, 1)$,

b_i – is non-zero in the presence of jump and may be perceived as an outlier with respect to $f_i s_i u_i$.

A number of both parametric and nonparametric formulations can be used to approximate f_i. Here we use Weighted Standard Deviation (WSD), an approach proposed by K. Boudt et al. [2008]. Within WSD the data is filtered with respect both to the time and to the day-of-the-week effect. Let $\overline{r}_{1,i}$, ..., $\overline{r}_{m,i}$ be a collection of standardized returns of the same periodicity feature as $\overline{r}$. The $\overline{r}_{1,i}$, ..., $\overline{r}_{m,i}$ are the returns which are observed at the same time of the day and the same day of the week as r_i. The WSD periodicity estimator is as follows:

$$\hat{f}_i^{WSD} = \frac{WSD_i}{\sqrt{\frac{1}{M}\sum_{j=1}^{M} WSD_j^2}}, \tag{2}$$

where:

$$WSD_j = \sqrt{1.081 * \frac{\sum_{l=1}^{m_j} w_{lj}\overline{r}_{lj}^2}{\sum_{l=1}^{m_j} w_{lj}}}, \tag{3}$$

M – the number of equally spaced observations within the day (intraday returns),

$i = 1, 2, \ldots, M$ is a consecutive intraday return,

w_{lj} – weights which depend on the value of the standardized returns divided by the Shortest Half periodicity estimate (see: [Rousseeuw, Leroy 1988]). In the paper we use the rejection with threshold equal to 99% quantile of the $\chi^2(1)$:

$$w(z) = \begin{cases} 1, & z \leq 6.6349 \\ 0, & z > 6.6349 \end{cases}. \tag{4}$$

The correction value 1.081 makes the filter a consistent scale estimator [Boudt et al. 2008]. WSD filter is both robust to jumps and efficient.

In Figure 2 we show the autocorrelation function for raw and filtered absolute returns.

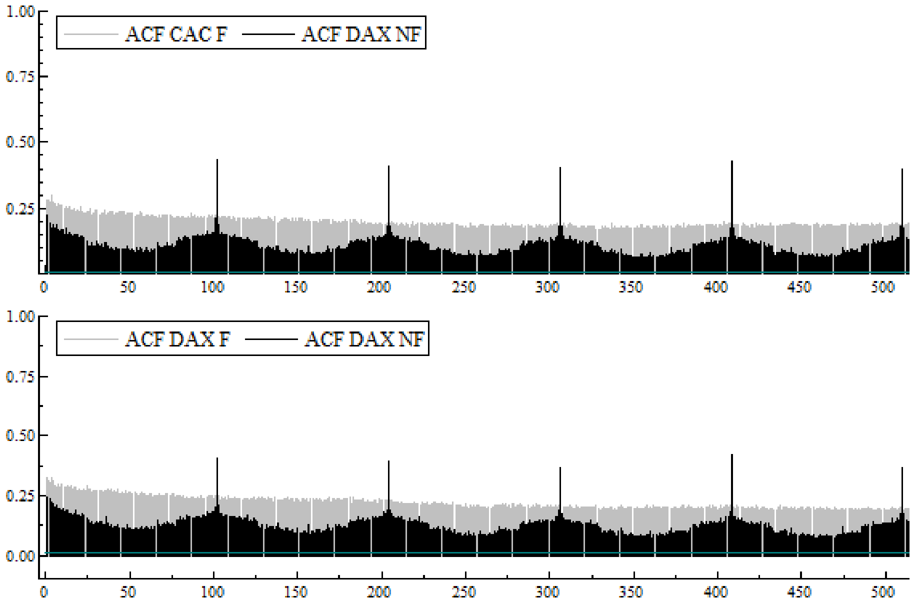

Figure 2. Autocorrelation function for absolute values of raw (NF) and filtered (F) returns

Source: own caluclations done in OxMetrics 6.0.

The series of absolute raw returns display very strong autocorrelation with peaks observable every 102 returns (one day). After periodicity removal we obtain the series without periodical pattern (no peaks), but with strong and slowly decaying autocorrelation function.

4. Jump detection test

The approaches to jumps in the literature may be divided into two groups: parametric and non-parametric. From the broad group of non-parametric tests (see, e.g., [Barndorff-Nielsen, Shephard 2004; Aït-Sahalia 2004]) we choose one presented in [Lee, Mykland 2008]. It is used in order to find which returns observed in the series are perceived as abnormal. If one considers the relationship between jumps and abnormal returns, the approach in this test is that the outlying (abnormal) returns match to jumps in the price series [Boudt et al. 2008]. We additionally assume that the arrival of jumps is usually determined by information and therefore jumps occur irregularly.

The Lee-Mykland test examines if a single return may be treated as containing a jump. This would be true, if the return's absolute value is abnormally high [Lahaye et al. 2007]. Because volatility clustering is the strong characteristic feature for the financial series, estimates of mean and volatility in the test are done in a short local window. As a result in times of high volatility a return to be perceived as containing a jump must be in absolute values really high, while in low volatility time an abnormal return may be much lower in absolute values. As a by-product of the test we obtain the time and the size of the jump.

Test statistic for the null hypothesis that r_i is not affected by a jump is defined as an absolute value of return, divided by the local standard deviation of volatility:

$$J_i = \frac{|r_i|}{\hat{\sigma}_i}, \tag{5}$$

where $\hat{\sigma}_i$ stands for square root of realized bipower variation estimated as a sum of products of consecutive absolute returns in a local window [Barndorff-Nielsen, Shephard 2004].

If the considered return is not affected by a jump, the statistic J_i follows approximately the same distribution as the absolute value of a standard normal random variable (see [Lee, Mykland 2008].

We obtain altogether 67 significant jumps in the CAC40 filtered series and 61 in the DAX series.

5. Methodology

Our approach is to focus on interdependence between two European markets: the German and the French. After detecting jumps in the index return series, the filtered series are introduced into multidimensional models with additional variables. We examine if shocks observed on one of the markets are transformed to other markets using dynamic conditional correlation model.

After periodicity filtering, the series display long memory. Therefore the conditional mean equation for every series

$$\mathbf{r}_t = \mathbf{c} + \mathbf{a}_t \tag{6}$$

is modelled by an ARFIMA process given by:

$$\Psi(L)(1-L)^{\zeta}(r_t - \mu_t) = \Theta(L)a_t. \tag{7}$$

The residuals, $\mathbf{a}_t$, are introduced into the Dynamic Conditional Correlation model in the following way [Engle 2002]:

$$\mathbf{a}_t = \mathbf{H}_t^{1/2}\varepsilon_t, \tag{8}$$

$$\mathbf{a}_t \left| \Omega_{t-1} \right. \sim EC_k(\mathbf{O}, \mathbf{H}_t, g), \tag{9}$$

where EC_k stands for elliptical (here Student-*t*) distribution and g is a scalar function, referred to as density generator [Pelagatti, Rondena 2004]. The multivariate Student-*t* distribution is used due to high excess kurtosis in the series. Then:

$$\mathbf{H}_t = \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t, \tag{10}$$

$$\mathbf{D}_t = diag(\sqrt{h_{11,t}}, \dots, \sqrt{h_{kk,t}}), \tag{11}$$

where conditional volatilities $h_{ii,t}$ are modelled with FIGARCH (1, *d*, 1) process with specification given by Chung [1999]:

$$(1-\alpha_1 L)(1-L)^d (a_t^2 - h_{ii}) = (1-\beta_1 L)(a_t^2 - h_{ii,t}), \tag{12}$$

where: h_{ii} – unconditional variance of a_t and $0 \le d < 1$.

Additionally variables standing for jumps that occur at time t and at time $t-1$ are included in conditional variance equation. These are dummy variables: if a jump occurs, the variable is equal to 1, if not, its value is zero. The lagged values of dummies are used in order to examine if the shocks are transmitted within a 5-minute period.

The choice of the model from GARCH family is based on information criteria and the value of logarithm of likelihood function.

In the DCC model the R matrix is given by:

$$\mathbf{R}_t = (diag(\mathbf{Q}_t))^{-1/2}\mathbf{Q}_t(diag(\mathbf{Q}_t))^{-1/2}. \tag{13}$$

The dynamics of symmetric and positively defined matrix $\mathbf{Q}$ is given by the following equation:

$$\mathbf{Q}_t = (1-\sum_{m=1}^{M}\alpha_m - \sum_{n=1}^{N}\beta_n)\bar{\mathbf{Q}} + \sum_{m=1}^{M}\alpha_m \mathbf{u}_{t-m}\mathbf{u}'_{t-m} + \sum_{n=1}^{N}\beta_n \mathbf{Q}_{t-n}, \tag{14}$$

where vectors $\mathbf{u}$ are standardized residuals from univariate FIGARCH models:

$$\mathbf{u}_t = \mathbf{D}_t^{-1}\mathbf{a}_t. \tag{15}$$

We assume Student-*t* distributed errors in the estimation, and therefore we use the following quasi-maximum likelihood function:

$$L(\psi \left| \phi \right.) = \sum_{t=1}^{T}(\ln c - \frac{1}{2}\ln\left|\mathbf{R}_t\right| + \ln g(\mathbf{u}'_t \mathbf{R}_t^{-1}\mathbf{u}_t). \tag{16}$$

The MaxSQPF algorithm which implements a sequential quadratic programming technique to maximize the log-likelihood function is used [Laurent, Peters 2002]. The covariance matrix of the estimates is computed with the Quasi-Maximum Likelihood method.

6. Results

The series were introduced into the MGARCH models. The model is estimated with dummy variables standing for jumps detected in series at time t and $t-1$. Parameter estimates with standard errors in italics are presented in the upper part of Table 1, while estimates from the DCC models are in the lower part.

Table 1. Estimates of DCC model

	CAC		DAX	
mean equation	coefficient	standard error	coefficient	standard error
μ	–0.0011	*0.0009*	–0.0005	*0.0008*
ζ	**–0.0085**	*0.0038*	**–0.0198**	*0.0037*
volatility equation				
ω	**0.0076**	*0.0028*	**0.0079**	*0.0030*
d	**0.3615**	*0.0145*	**0.3621**	*0.0153*
α_1	**0.3196**	*0.0223*	**0.3697**	*0.0324*
β_1	**0.6190**	*0.0242*	**0.6383**	*0.0329*
$J_{CAC}(t)$	**0.1096**	*0.0096*	**0.0569**	*0.0076*
$J_{CAC}(t–1)$	**–0.0029**	*0.0014*	0.0099	*0.0055*
$J_{DAX}(t)$	**0.0839**	*0.0091*	**0.1438**	*0.0104*
$J_{DAX}(t–1)$	**0.0101**	*0.0033*	0.0012	*0.0035*
Model DCC				
α(DCC)	**0.0241**	*0.0038*		
β(DCC)	**0.9540**	*0.0101*		
LL	62,147.26			
df Student-t	7.3118			
no obs.	42,330			

Coefficient μ is the constant in the mean equation, whereas ζ is a fractional integration of the ARFIMA process. ω is the constant in the variance equation, d is a parameter of fractional integration, α_1 is a coefficient of the lagged squares of errors, β_1 is a coefficient of the lagged variance of the error term. The distribution is t Student and *df* Student-t stands for degrees of freedom in that distribution. Estimation is done by the quasi maximum likelihood method. J stands for parameter of dummy variable which is equal to 1 if a jump is detected and 0 otherwise. (t) and (t–1) stand for a jump detected in time t and t–1, respectively. LL is a value of logarithm of likelihood function. The statistically significant coefficients are in bold.

Source: own calculations.

Estimates from the models show long memory both in the conditional mean and conditional variance equation. There is strong persistence in the GARCH models for both series. The jumps observed on one of the markets increase volatility on both markets, which suggests co-existence of abnormal returns. Volatility of the index from the market, where the jump is detected, increases more than volatility of another series in examined pair, e.g. when the pair of CAC40-DAX is considered, volatility of the CAC40 responds twice as large to the jumps detected on the French market than volatility of the DAX responds to these jumps. When the lagged values of dummies standing for jumps are introduced into the conditional variance equation, the estimated parameters are of both signs, but the values are quite small and often insignificant. The jumps observed on the German market affect volatility on the French market within 5 minutes and jumps observed on the CAC40 decrease volatility of this index within 5 minutes.

7. Conclusion

Shocks observed on one market influence other markets. There is a question if there exist an interdependence or volatility spillover on the financial markets. We examine if there is a common reaction to shocks on two huge European indices, the CAC40 and the DAX. This study presents an approach where abnormal returns are identified within a jump detection test and then used as additional variables in conditional variance equations in the GARCH framework. We examine the interdependence between two indices, the French and the German. The approach allows us to estimate if the reaction to shocks is simultaneous and if there are any jumps transmissions between two markets, the French and the German.

Generally at the time when jumps are detected, volatility on other markets increases, which suggests that there is interdependence between these markets and common exogenous shocks. When the impact of the jumps on the next return is examined, there is only one-way dependence: jumps detected in the DAX returns increase volatility of the next return in the CAC40.

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ZALEŻNOŚCI POMIĘDZY INDEKSAMI CAC40 I DAX W DANYCH ŚRÓDDZIENNYCH

Streszczenie: W artykule zbadano, czy szoki zaobserwowane w indeksach francuskim i niemieckim występują w tym samym czasie, czy też są przenoszone pomiędzy rynkami w bardzo krótkim przedziale czasu. Oszacowano dwuwymiarowy model GARCH z dodatkowymi zmiennymi objaśniającymi w celu zbadania współzależności rynków. Próba składa się ze zwrotów 5-minutowych w okresie 18 miesięcy. Wykazano, że zmienność wzrasta w obu badanych szeregach w tym samym czasie oraz że szoki wykryte w indeksie DAX mają wpływ na zwiększenie zmienności indeksu CAC40. Takie rezultaty wskazują na występowanie wspólnych źródeł szoków dla zmienności.

Słowa kluczowe: dane wysokiej częstotliwości, skoki, zmienność.