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# Optical bistability in double quantum dot molecule with inter-dot tunneling

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We have theoretically studied the optical bistability behaviors under an external electric field and a coupling laser field in double quantum dot molecule system with π-type four energy levels. It can be adjusted by the system parameters such as the electronic cooperation parameter, the tunneling strength, the coupling laser field, the probe and coupling laser detuning. These results may be useful in the experiment and provide new types of all-optical switching.

Keywords: optical bistability, quantum dot molecule, tunneling.

## 1. Introduction

It is well known that, in the all-optical and quantum networks, controlling light by light is very significant. So, during the last decades, optical bistability (OB) has been studied extensively for its potential application in all-optical switching, optical logic, and optical transistors. At the very beginning, OB is investigated in atoms confined in an optical ring cavity [1–16] which can be controlled by phase fluctuation, electromagnetically induced transparency, spontaneously generated coherence, and so on.

On the other hand, OB in the quantum wells and dots has attracted attention recently because of its inherent advantages such as large electric dipole moments, great flexibilities in devices, high nonlinear optical coefficients, and so on. One recent study is OB and multistability in one-dimensional photonic crystals doped with carbon-nanotube quantum dots [17] in which the threshold of optical bistability can be adjusted due to spin-orbit coupling in QDs. Karabulut [18] investigated theoretically optical bistability in a symmetric quantum well by choosing appropriate values of the electron sheet density and the intensity of the optical radiation. Tian *et al.* [19] reported that OB can be controlled via tunneling induced quantum interference in triangular quantum dot molecules (QDMs). Taherzadeh *et al.* [20] proposed a scheme using a dielectric slab via inter-dot tunneling to achieve low threshold OB by at least one order of magnitude in respect to free QDMs. Of course, the other OB schemes in quantum wells and dots are also reported [21–31].

In this work, we investigated the OB in a QDM system with π-type four energy levels driving by an external electrical field and a coupling laser field inside an optical ring cavity. The OB behavior can be easily controlled by an extra coupling laser field and a tunneling electric field via adjusting properly the corresponding system parameter.

## 2. The model and theory

A QDM system consisting of two QDs with π-type four energy levels as shown in Fig. 1 is considered in this paper. The two QDs are different in size and have different band structure. Such a QDM can be produced by self-assembled dot growth technology [32]. $|1\rangle$ and $|4\rangle$ are the ground states without excitations for the two QDs. $|2\rangle$ and $|3\rangle$ are the excited states which are coupled by the inter-dot tunneling with tunneling strength $T_e$. A probe laser field (with frequency $\omega_p$ and Rabi frequency $\Omega_p$) couples the states $|1\rangle$ and $|2\rangle$. A coupling laser field (with frequency $\omega_c$ and Rabi frequency $\Omega_c$) couples the states $|3\rangle$ and $|4\rangle$.

Under the rotating-wave and the electric-dipole approximation, the system Hamiltonian with the state $\{|1\rangle, |2\rangle, |3\rangle, |4\rangle\}$ can be written as (assumption of $\hbar = 1$)

$$H_{\text{int}}^{\text{I}} = \begin{bmatrix} 0 & -\Omega_p & 0 & 0 \\ -\Omega_p & \Delta_2 & -T_e & 0 \\ 0 & -T_e & \Delta_2 & -\Omega_c \\ 0 & 0 & -\Omega_c & \Delta_2 + \Delta_3 \end{bmatrix} \tag{1}$$

where $\Omega_p = \mu_{12} E_p$ ($\Omega_c = \mu_{34} E_c$) is the Rabi frequency of the probe (coupling) laser field, $\mu_{ij}$ being the electric dipole moment of transition $|i\rangle$ and $|j\rangle$. $\Delta_2 = \omega_{21} - \omega_p$ and

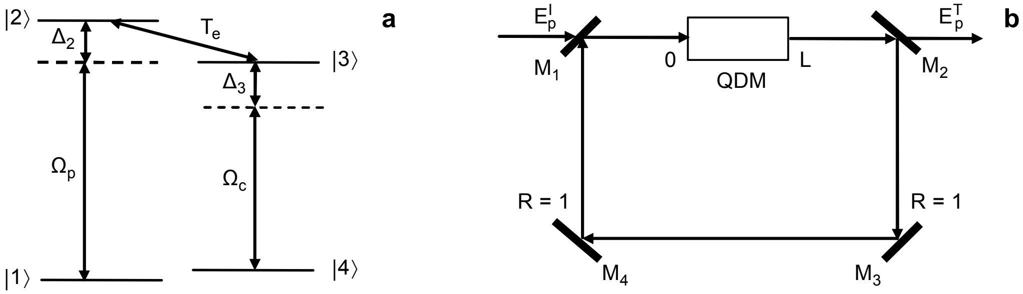

Fig. 1. QDM diagram interacting with a strong coupling field and a weak probe optical field. Tunneling effect born from the external electric field occurs between the state $|2\rangle$ and $|3\rangle$ (**a**). $L$-length QDM sample is inserted into a unidirectional ring cavity with four mirrors. $E_p^{\text{I}}$ ($E_p^{\text{T}}$) is the incident (transmitted) field. For mirrors $M_1$ and $M_2$, $R + T = 1$ and $R$ ($T$) is the reflection (transmission) coefficient. Reflection coefficient $R$ of mirrors $M_3$ and $M_4$ is 1 (**b**).

$\Delta_3 = \omega_{34} - \omega_c$ are the detuning of the probe and the coupling laser field, respectively, and $\omega_{ij}$ is the transition frequency of the state $|i\rangle$ and $|j\rangle$.

At any time, the state vector $\Psi(t) = a_1|1\rangle + a_2|2\rangle + a_3|3\rangle + a_4|4\rangle$ obeys the Schrödinger equation as follows:

$$\frac{\mathrm{d}}{\mathrm{d}t}|\Psi(t)\rangle = -iH_{\mathrm{int}}^{\mathrm{I}}(t)|\Psi(t)\rangle \tag{2}$$

From Eqs. (1) and (2), we can get the following dynamical equations:

$$i\frac{\mathrm{d}}{\mathrm{d}t}a_1 = -\Omega_\mathrm{p}a_2 \tag{3a}$$

$$i\frac{\mathrm{d}}{\mathrm{d}t}a_2 = (\Delta_2 - i\gamma_2)a_2 - \Omega_\mathrm{p}a_1 - T_\mathrm{e}a_3 \tag{3b}$$

$$i\frac{\mathrm{d}}{\mathrm{d}t}a_3 = (\Delta_2 - i\gamma_3)a_3 - T_\mathrm{e}a_2 - \Omega_\mathrm{c}a_4 \tag{3c}$$

$$i\frac{\mathrm{d}}{\mathrm{d}t}a_4 = (\Delta_2 + \Delta_3 - i\gamma_4)a_4 - \Omega_\mathrm{c}a_3 \tag{3d}$$

$$|a_1|^2 + |a_2|^2 + |a_3|^2 + |a_4|^2 = 1 \tag{3e}$$

where the total decay rates $\gamma_i$ ($i = 2, 3, 4$), embodying the radiative decay rate and the pure dephasing rate, are added phenomenologically in the above equations. Solving Eq. (3) by the method given in [28], we can get

$$\rho_{21} = a_2 a_1^* = \frac{A}{1 + |A|^2 + |AB|^2 + |ABD|^2} \tag{4}$$

where

$$A = \frac{\Omega_\mathrm{p}(\Delta_2 - i\gamma_3 - \Omega_\mathrm{c}D)}{(\Delta_2 - i\gamma_2)(\Delta_2 - i\gamma_3 - \Omega_\mathrm{c}D) - T_\mathrm{e}^2} \tag{5a}$$

$$B = \frac{T_\mathrm{e}}{\Delta_2 - i\gamma_3 - \Omega_\mathrm{c}D} \tag{5b}$$

$$D = \frac{\Omega_\mathrm{c}}{\Delta_2 + \Delta_3 - i\gamma_4} \tag{5c}$$

We study the OB behavior of the above-described four-level QDM system in a unidirectional cavity (see Fig. 1**b**). Under the slow envelope approximation, the dynamics of the probe field in the optical cavity is governed by the Maxwell's equation,

$$\frac{\partial E_\mathrm{p}}{\partial t} + c\frac{\partial E_\mathrm{p}}{\partial z} = i\frac{\omega_\mathrm{p}}{2\varepsilon_0}P(\omega_\mathrm{p}) \tag{6}$$

where $c$ is the light speed in the vacuum, $P(\omega_p) = N\mu_{21}\rho_{21}$ is the slowly oscillating term of the induced polarization in the transition $|2\rangle \to |1\rangle$, $N = \Gamma/V$ is the electronic number density, where $\Gamma$ and $V$ are the optical confinement factor and the volume of the QDM, respectively.

Considering the steady state, the time derivative in Eq. (6) is equal to zero. Then, Eq. (6) and the boundary conditions can be written as,

$$\frac{\partial E_p}{\partial z} = iN\frac{\mu_{03}\omega_p}{2c\varepsilon_0}\rho_{21} \tag{7a}$$

$$E_p(0) = \sqrt{T}\,E_p^I + RE_p(L) \tag{7b}$$

$$E_p(L) = E_p^T/\sqrt{T} \tag{7c}$$

where $RE_p(L)$ indicates the feedback mechanism of the mirrors.

In the mean-field limit, we can get the input-output relationship from Eqs. (7),

$$y = x - iC\rho_{21} \tag{8}$$

where the electronic cooperation parameter

$$C = \frac{N\omega_p L|\mu_{21}|^2}{2\hbar c\varepsilon_0 T} \tag{9a}$$

and

$$x = \frac{\mu_{21}E_p^T}{\hbar\sqrt{T}} \tag{9b}$$

$$y = \frac{\mu_{21}E_p^I}{\hbar\sqrt{T}} \tag{9c}$$

## 3. Numerical results and discussions

In this article, QDM is assumed to be produced in advance and the system works in low temperature. For simplicity, all the parameters are scaled by the decay rate $\gamma_2 = 1$ meV, $\gamma_3 = 0.1$, and $\gamma_4 = 0.001$, which is same as those in [33].

Firstly, in Fig. 2, it is shown how the OB is affected by the electronic cooperation parameter $C$. From Fig. 2, we can find that the threshold and the hysteresis cycle change dramatically due to the increasing of the electronic cooperation parameter $C$. It can be explained from Eq. (9a) where $C$ is directly proportional to the electronic number density $N$. So, increasing the density of electrons will lead to the enhancement of the sample absorption and to the rise in the OB threshold.

Secondly, in Figs. 3 and 4, we study the dependence of the OB behavior on the laser detuning $\Delta_2$ and $\Delta_3$ (shown in Fig. 1). As can be seen, in Figs. 3**a**–3**d**, with the

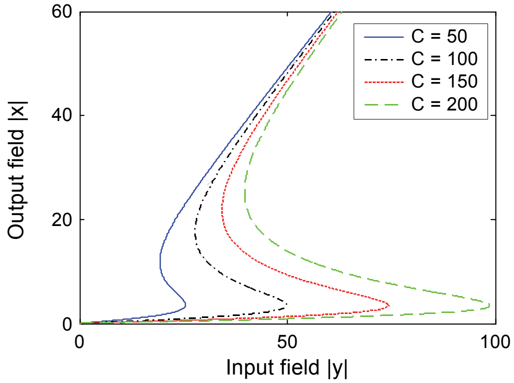

Fig. 2. Output field $|x|$ *vs*. input field $|y|$ for different values of the electronic cooperation parameter $C$; $\varDelta_2 = \varDelta_3 = 5$ and $\varOmega_c = T_e = 1$.

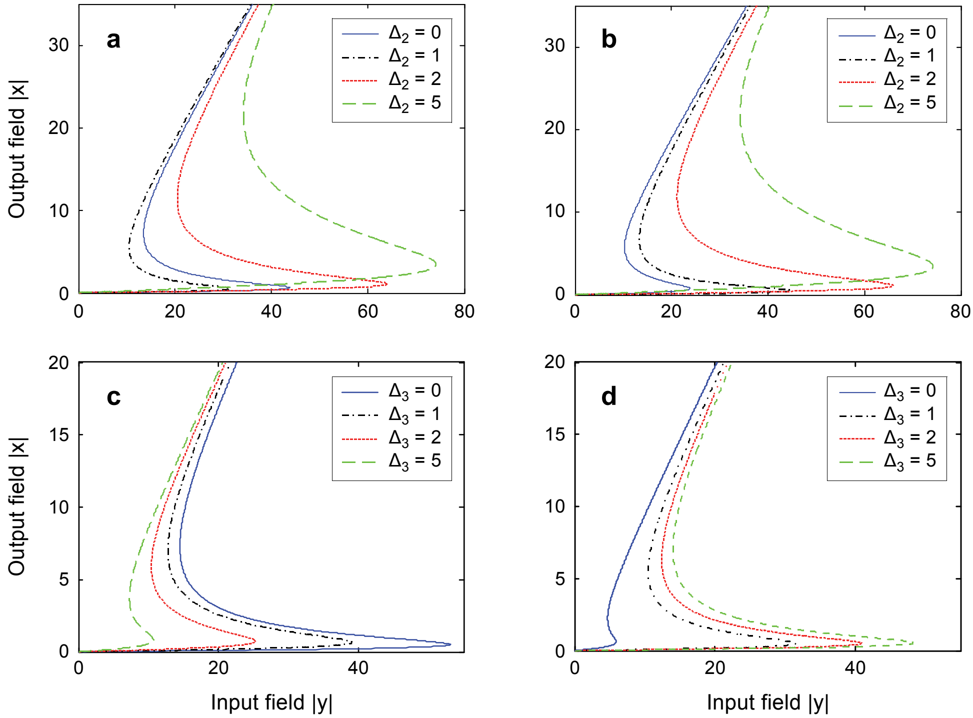

Fig. 3. Output field $|x|$ *vs*. input field $|y|$ for different values of $\varDelta_2$ and $\varDelta_3 = 1$ (**a**), for different values of $\varDelta_2$ and $\varDelta_3 = 3$ (**b**), for different values of $\varDelta_3$ and $\varDelta_2 = 0.1$ (**c**), for different values of $\varDelta_3$ and $\varDelta_2 = 1$ (**d**). The other parameters are: $\varOmega_c = T_e = 1$, $C = 150$.

increasing of the detuning $\varDelta_2$ and $\varDelta_3$, the hysteresis cycles change dramatically. And it is interesting that, when $\varDelta_3 = 1$, the OB threshold decreases firstly, and then increases in Fig. 3**a**. To explain this phenomenon, we plot the imaginary parts of $\rho_{21}$ as a function

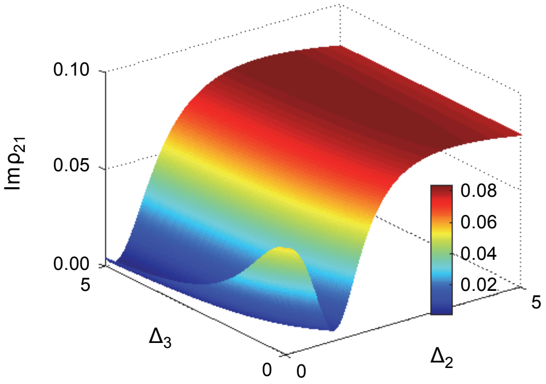

Fig. 4. Im $\rho_{21}$ as a function of $\Delta_2$ and $\Delta_3$. The other parameters are: $\Omega_p = 10$, $\Omega_c = T_e = 1$, $C = 150$.

of $\Delta_2$ and $\Delta_3$ in Fig. 4. It can easily be seen that when $\Delta_3 = 1$, the Im $\rho_{21}$ also decreases firstly, and then increases, which leads to the absorption and the OB threshold changes similarly. But, in Fig. 3**b**, when $\Delta_3 = 3$, the OB threshold increases monotonously with the increasing of $\Delta_2$, which also can be explained from Fig. 4 by the same change trend between Im $\rho_{21}$ and $\Delta_2$. In Figs. 3**c** and 3**d**, we can find that the OB threshold changes conversely with the increasing of $\Delta_3$ which also can be explained from Fig. 4. When $\Delta_2 = 1$ ($\Delta_2 = 0.1$), Im $\rho_{21}$ increases (decreases) with the enlargement of $\Delta_3$. However, when $\Delta_2 > 3$, we find that Im $\rho_{21}$ changes very slowly for the alteration of $\Delta_3$ in Fig. 4. So, we also can conclude that $\Delta_3$ will be insensitive to the change in the OB threshold for $\Delta_2 > 3$.

Finally, in Figs. 5**a** and 5**b**, we show the dependence of the OB threshold on the coupling laser field $\Omega_c$ and the tunneling strength $T_e$. We can find that, with the increasing of $\Omega_c$ ($T_e$), the threshold increases (decreases). The different change trend of

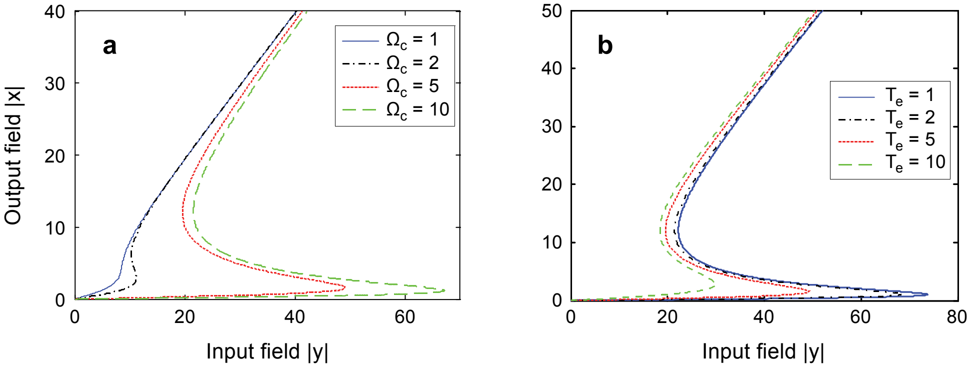

Fig. 5. Output field $|x|$ *vs*. input field $|y|$ for different values of $\Omega_c$ and $T_e = 5$ (**a**), and for different values of $T_e$ and $\Omega_c = 5$ (**b**). The other parameters are: $\Delta_2 = \Delta_3 = 1$, and $C = 150$.

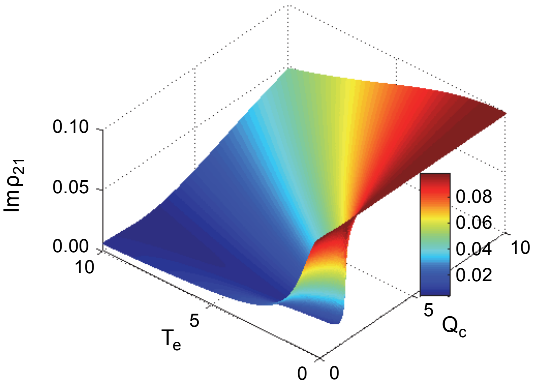

Fig. 6. Im $\rho_{21}$ as a function of $\Omega_c$ and $T_e$. The other parameters are: $\Omega_p = 10$, $\Delta_2 = \Delta_3 = 1$, and $C = 150$.

the OB threshold can be interpreted by the opposite change trend of Im $\rho_{21}$ as the function of $\Omega_c$ and $T_e$ in Fig. 6.

## 4. Conclusions

In general, we have theoretically examined OB behaviors under an external electrical field and a coupling laser field in a double QDM system. We find that the electronic cooperation parameter $C$, the tunneling strength $T_e$, the coupling laser field $\Omega_c$, the probe laser detuning $\Delta_2$ and the coupling laser detuning $\Delta_3$ all show a considerable effect on the OB. These results may provide new types of all-optical switches.

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## References

[1] Walls D.F., Zoller P., *A coherent nonlinear mechanism for optical bistability from three level atoms*, Optics Communications **34**(2), 1980, pp. 260–264, DOI: 10.1016/0030-4018(80)90029-2.

[2] Harshawardhan W., Agarwal G.S., *Controlling optical bistability using electromagnetic-field -induced transparency and quantum interferences*, Physical Review A **53**(3), 1996, p. 1812, DOI: 10.1103/PhysRevA.53.1812.

[3] Joshi A., Min Xiao, *Optical multistability in three-level atoms inside an optical ring cavity*, Physical Review Letters **91**(14), 2003, article ID 143904, DOI: 10.1103/PhysRevLett.91.143904.

[4] Joshi A., Brown A., Hai Wang, Min Xiao, *Controlling optical bistability in a three-level atomic system*, Physical Review A **67**(4), 2003, article ID 041801(R), DOI: 10.1103/PhysRevA.67.041801.

[5] Joshi A., Wenge Yang, Min Xiao, *Effect of quantum interference on optical bistability in the three-level V-type atomic system*, Physical Review A **68**(1), 2003, article ID 015806, DOI: 10.1103/PhysRevA.68.015806.

[6] JOSHI A., WENGE YANG, MIN XIAO, *Effect of spontaneously generated coherence on optical bistability in three-level Λ-type atomic system*, Physics Letters A **315**(3–4), 2003, pp. 203–207, DOI: 10.1016/S0375-9601(03)01046-6.

[7] LITCHINITSER N.M., GABITOV I.R., MAIMISTOV A.I., *Optical bistability in a nonlinear optical coupler with a negative index channel*, Physical Review Letters **99**(11), 2007, article ID 113902, DOI: 10.1103/PhysRevLett.99.113902.

[8] JIA-HUA LI, XIN-YOU LÜ, JING-MIN LUO, QIU-JUN HUANG, *Optical bistability and multistability via atomic coherence in an N-type atomic medium*, Physical Review A **74**(3), 2006, article ID 035801, DOI: 10.1103/PhysRevA.74.035801.

[9] ZHIPING WANG, MIAOCUN XU, *Control of the switch between optical multistability and bistability in three-level V-type atoms*, Optics Communications **282**(8), 2009, pp. 1574–1578, DOI: 10.1016/j.optcom.2008.12.063.

[10] ZHI-HONG XIAO, KISIK KIM, *Optical bistability using quantum coherence in a microwave-driven four-level atomic system*, Optics Communications **283**(10), 2010, pp. 2178–2181, DOI: 10.1016/j.optcom.2010.01.049.

[11] ASADPOUR S.H., ESLAMI-MAJD A., *Controlling the optical bistability and transmission coefficient in a four-level atomic medium*, Journal of Luminescence **132**(6), 2012, pp. 1477–1482, DOI: 10.1016/j.jlumin.2012.01.018.

[12] ZHEN WANG, AI-XI CHEN, YANFENG BAI, WEN-XING YANG, RAY-KUANG LEE, *Coherent control of optical bistability in an open Λ-type three-level atomic system*, Journal of the Optical Society of America B **29**(10), 2012, pp. 2891–2896, DOI: 10.1364/JOSAB.29.002891.

[13] ASADPOUR S.H., JABERI M., SOLEIMANI H.R., *Phase control of optical bistability and multistability via spin coherence in a quantum well waveguide*, Journal of the Optical Society of America B **30**(7), 2013, pp. 1815–1820, DOI: 10.1364/JOSAB.30.001815.

[14] JAFARI A., NADERALI R., BAKKESHIZADEH S., *Effect of spontaneously generated coherence in controlling the optical bistability and multistability in a left-handed four-level atomic system*, Optical and Quantum Electronics **48**(1), 2016, article ID 55, DOI: 10.1007/s11082-015-0322-z.

[15] HAMEDI H.R., *Optical switching, bistability and pulse propagation in five-level quantum schemes*, Laser Physics **27**(6), 2017, article ID 066002, DOI: 10.1088/1555-6611/aa6993.

[16] ASGHARI NEJAD A., ASKARI H.R., BAGHSHAHI H.R., *Optical bistability in coupled optomechanical cavities in the presence of Kerr effect*, Applied Optics **56**(10), 2017, pp. 2816–2820, DOI: 10.1364/AO.56.002816.

[17] SOLOOKINEJAD G., PAYRAVI M., JABBARI M., NAFAR M., SANGACHIN E.A., *Optical multistability in 1D photonic crystal doped with carbon-nanotube quantum dot nanostructures*, Laser Physics **27**(12), 2017, article ID 125202, DOI: 10.1088/1555-6611/aa9142.

[18] KARABULUT I., *Nonlinear optical response in intersubband transitions of a symmetric quantum well: role of electron–electron interactions*, Superlattices and Microstructures **111**, 2017, pp. 181–187, DOI: 10.1016/j.spmi.2017.06.034.

[19] SI-CONG TIAN, REN-GANG WAN, CUN-ZHU TONG, JIN-LONG ZHANG, XIAO-NAN SHAN, XI-HONG FU, YU-GANG ZENG, LI QIN, YONG-QIANG NING, *Control of optical bistability and third-order nonlinearity via tunneling induced quantum interference in triangular quantum dot molecules*, AIP Advances **5**(6), 2015, article ID 067144, DOI: 10.1063/1.4922959.

[20] TAHERZADEH S., NASEHI R., MAHMOUDI M., *Low threshold optical bistability and superluminal light propagation using a dielectric slab via inter-dot tunneling*, Laser Physics **25**(4), 2015, article ID 045901, DOI: 10.1088/1054-660X/25/4/045901.

[21] JIAHUA LI, XIANGYING HAO, JIBING LIU, XIAOXUE YANG, *Optical bistability in a triple semiconductor quantum well structure with tunnelling-induced interference*, Physics Letters A **372**(5), 2008, pp. 716–720, DOI: 10.1016/j.physleta.2007.08.011.

[22] ANTÓN M.A., CARREÑO F., CALDERÓN O.G., MELLE S., *Tunable all-optical bistability in a semiconductor quantum dot damped by a phase-dependent reservoir*, Optics Communications **281**(12), 2008, pp. 3301–3313, DOI: 10.1016/j.optcom.2008.02.027.

[23] Li J., Yu R., Liu J., Huang P., Yang X., *Voltage-controlled optical bistability of a tunable three -level system in a quantum-dot molecule*, Physica E: Low-dimensional Systems and Nanostructures **41**(1), 2008, pp. 70–73, DOI: 10.1016/j.physe.2008.06.002.

[24] Zhiping Wang, Hongyi Fan, *Phase-dependent optical bistability and multistability in a semiconductor quantum well system*, Journal of Luminescence **130**(11), 2010, pp. 2084–2088, DOI: 10.1016/j.jlumin.2010.05.031.

[25] Zhiping Wang, Benli Yu, *Switching from optical bistability to multistability in a coupled semiconductor double-quantum-dot nanostructure*, Journal of the Optical Society of America B **30**(11), 2013, pp. 2915–2920, DOI: 10.1364/JOSAB.30.002915.

[26] Zhiping Wang, Shenglai Zhen, Xuqiang Wu, Jun Zhu, Zhigang Cao, Benli Yu, *Controllable optical bistability via tunneling induced transparency in quantum dot molecules*, Optics Communications **304**, 2013, pp. 7–10, DOI: 10.1016/j.optcom.2013.04.024.

[27] Vafafard A., Goharshenasan S., Nozari N., Mortezapour A., Mahmoudi M., *Phase-dependent optical bistability in the quantum dot nanostructure molecules via inter-dot tunneling*, Journal of Luminescence **134**, 2013, pp. 900–905, DOI: 10.1016/j.jlumin.2012.06.016.

[28] Si-Cong Tian, Ren-Gang Wan, Cun-Zhu Tong, Yong-Qiang Ning, *Controlling optical bistability via interacting double dark resonances in linear quantum dot molecules*, Journal of the Optical Society of America B **31**(11), 2014, pp. 2681–2688, DOI: 10.1364/JOSAB.31.002681.

[29] Asadpour S.H., Soleimani H.R., *Optical bistability and multistability via biexciton coherence in semiconductor quantum well nanostructure*, Optics Communications **315**, 2014, pp. 347–351, DOI: 10.1016/j.optcom.2013.11.010.

[30] Hamedi H.R., *Inter-dot tunneling control of optical bistability in triple quantum dot molecules*, Physica B: Condensed Matter **449**, 2014, pp. 5–9, DOI: 10.1016/j.physb.2014.05.011.

[31] Asadpour S.H., Soleimani H.R., *Phase control of optical bistability based biexciton coherence in a quantum dot nanostructure*, Physica B: Condensed Matter **440**, 2014, pp. 124–129, DOI: 10.1016/j.physb.2014.01.033.

[32] Krause B., Metzger T.H., Rastelli A., Songmuang R., Kiravittaya S., Schmidt O.G., *Shape, strain, and ordering of lateral InAs quantum dot molecules*, Physical Review B **72**(8), 2005, article ID 085339, DOI: 10.1103/PhysRevB.72.085339.

[33] Yanchao She, Xuejun Zheng, Denglong Wang, Weixi Zhang, *Controllable double tunneling induced transparency and solitons formation in a quantum dot molecule*, Optics Express **21**(14), 2013, pp. 17392–17403, DOI: 10.1364/OE.21.017392.

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