Vibration diagnostic method for planetary gearboxes under varying external load with regard to cyclostationary analysis

Edited by
Walter Bartelmus

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1. INTRODUCTION

WALTER BARTELMUS

The monograph shows the results of research undertaken within a research grant financially supported by State Committee for Scientific Research in years 2005–2008 as research project no. 4T07B 009 29. The polish title of the project is: Metoda diagnostyki drganowej przekładni planetarnych w zmiennych warunkach obciążenia i z uwzględnieniem cyklostacjonarnej analizy sygnałów.

The monograph is the collection of some publications, which have been extended to give the full account of the research undertaken within the research project. Some collected papers go beyond the scope of research done within the project what gives the wider view of further developments for some other gearboxes. The chapters in the monograph mostly are equivalent to the paper titles, which are here collected. Second chapter and paper Root cause analysis of vibration signals for gearbox condition monitoring presents a root cause analysis of gearbox vibration signals, which takes into consideration design, technology, operation, and change of condition factors. It should be stated that the factor analysis is the clue of the success, which is presented in the monograph. The factor analysis is the base of all actions-research planning, inferring analysis and conclusions, which were undertaken within the project. The second chapter is an introduction to diagnostic methods, which are developed in following chapters.

The third chapter titled: Modelling planetary gearbox dynamics gives inside into design factors, which have influence the vibration signals generated by planetary gearboxes.

Fourth chapter titled: Influence of random varying load to vibration signal generated by planetary gearboxes driving bucked wheel in excavators gives inside into operation factors which have influence the vibration signal generated by planetary gearboxes. The basic problem of vibration condition monitoring is influence of an operation factor as random varying external load (RVEL). RVEL cause amplitude and frequency modulation (AFM). The first step for developing sound method for condition monitoring is RVEL identification and investigation on its influence to vibration signal.
Fifth chapter is titled: *Vibration condition monitoring of planetary gearbox under random varying external load*. In the chapter, systematic consideration has been taken of the influence of many factors on the vibration signals generated by a system in which a planetary gearbox is included. These considerations give the basis for vibration signal interpretation, development of the means of condition monitoring, and for the scenario of the degradation of the planetary gearbox.

Sixth chapter *Data mining from non-stationary vibration signals for machine condition recognition*. The paper is presented the way of finding the new diagnostic robust measure of machine condition that is the load yielding/susceptibility characteristic (LYCH). LYCH is developed on the introductory factor analysis influencing vibration signal and further data mining using a regression and adaptive process to obtain converged regression parameters.

Seventh chapter titled: *Cyclostationary analysis* shows the use of cyclostationary vibration signal properties, namely: Application of spectral correlation techniques on mining machine signals: Identification of faulty components, application of spectral correlation techniques on mining machine signals: Extraction of fault signatures. The chapter was developed in cooperation with Laboratoire Roberval de Mécanique Université de Technologie de Compiègne Centre de Recherche de Royallieu BP 20529 – 60205 Compiègne, France.

Eighth chapter *Modelling of gearbox dynamics under time varying non-stationary operation for distributed fault detection and diagnosis* by W. Bartelmus, F. Chaari, R. Zimroz shows the use of mathematical modelling and computer simulation for the investigation of load yielding/susceptibility characteristic of gearboxes. The chapter was developed in cooperation with University of Sfax, Tunisia.

Ninth chapter *Implementation of planetary gearbox condition monitoring* there is given the principles of an electronic system, in which a diagnostic method is implemented.

Tenth chapter gives conclusions and the overview of future developments.
2. ROOT CAUSE ANALYSIS OF VIBRATION SIGNALS FOR GEARBOX CONDITION MONITORING

WALTER BARTELMUS

The paper presents a root cause analysis of gearbox vibration signals which takes into consideration design, technology, operation, and change of condition factors. A survey of vibration signal analysis methods for gearbox condition monitoring is provided. The methods are broadly divided depending on the load factors influencing the vibration signal. The vibration signals are generated under a steady or almost steady level of external load and under randomly variable cyclic loads. The load conditions determine the ways in which the signals are analyzed. Signals generated by mathematical modelling and computer simulations and signals measured in industrial conditions during the normal operation of gearboxes are used in the analysis. Signals received from the housings of cylindrical, bevel and planetary gearboxes forming the trains of different systems are analysed to monitor the condition of gears and rolling-element bearings and other specified elements depending on the design factors of a given driving/transmission system train. The interpretation of the signals is based on (supported by) the analysis of design, technology, operation and change of condition factors.

Keywords: factor analysis, design, production technology, operation, change of condition, vibration, diagnostic, inference, signal analysis

2.1. INTRODUCTION

Proper vibration-based diagnostic inference should take into consideration all factors having an influence on the vibration signal. Diagnostic inference is described in [1]–[5]. The factors having an influence on the vibration signal are divided into: design, production technology, operation, and change of condition factors (DPTOCC). This paper deals with DPTOCC, analyzing the diagnosed object in more detail. It examines the object’s anatomy, (architecture/structure) and its genetics relating to its origin and development/change during its life. Engineering description leads to a DPTOCC factors analysis and diagnostic inference.

The relationship between the DPTOCC factors and the vibration signal is analyzed, using mathematical modelling and computer simulation (MM and CS), by the
author in many publications [1]–[14]. MM and CS are employed also by other authors, e.g. [15]–[17]. Papers [18]–[22] provide some ideas for using artificial intelligence techniques for gearbox condition monitoring. The results from MM and CS are compared with results from field and rig measurements. Most of the results are for constant values of the operational factors, i.e. the external gearbox load and the associated rotational speed. The actual load and rotation conditions are in most cases variable and load and rotation speed are usually negatively correlated, which means that as load increases, rotational speed decreases. Paper [23] presents the influence of load on the vibration signal, discussing elements of diagnostic vibration gearbox methods for variable load conditions. Many authors indicate the need to develop techniques of condition monitoring under varying load [24]–[30], presenting some such techniques for simplified load conditions. The papers review condition monitoring techniques for simplified operational factors (load and rotational speed). Cases of load identification from vibration measurements in industrial conditions are described in [31], [32]. Paper [33] deals with vibration signal generation by rolling-element bearings. Considering the design and operation factors having an influence on the signals, one can conclude that bearing fault signals are connected with shaft speeds, but there is no strict phase lock because of the variable slip between the bearing components, which can be explained by the fact that the kinematic formulas for various fault frequencies (determined by the design factors) contain the term \( \cos \varphi \), where \( \varphi \) is the load angle from the radial direction, which affects the rolling radius. Since the axial to radial load ratio for the individual rolling elements varies with their position in the bearing, they try to roll at different speeds but the cage forces them to maintain a uniform mean separation, causing slippage. This is an example of how design and operational factors influence the vibration signals generated by rolling-element bearings.

The present paper provides a comprehensive analysis of the influence of design, production technology, operation and change of condition factors on vibration signals. A factor analysis is carried out to determine the ways of analyzing the signals. Then a degradation scenario and prognosis are made and inference is performed. All of this makes up what may be called a diagnostic method.

### 2.2. FACTOR ANALYSIS

The factor analysis is shown in Fig.1. The diagram can be used to analyze the influence of different factors on the vibration signals generated by gearboxes. As shown in Fig. 1, the factors can be divided into primary and secondary factors. The primary factors are related to design and production technology.

The factors apply to new gearboxes characterized by geometrical and material factors. Macro-geometrical factors lead to a structural form and material factors re-
result in material moduli. The factors describe the properties of a gearbox (object). When the object is excited by a force impulse described by the delta function, the excitation reveals the structure’s dynamic properties. When expressing the macro-geometrical factors through the response of the structural form one ought to take into account stiffness factors, which may be constant or variable. For example, the outer structural form (housing) is considered to have constant stiffness properties, but when the gearing and the bearings are taken into account, variable stiffness properties appear and the system changes from linear to nonlinear. Structural forms are characterized by imperfections such as dimension and shape deviations. Important factors describing the gearbox design are: the number of gear teeth and rolling-element bearing parameters (the number of rolling elements and the bearing’s mean dimension). Micro-geometrical factors are described by surface roughness parameters. The material factors include constant and variable material properties. The constant properties are described by material constants such as stiffness and E and G moduli. Among other material properties, one should mention lubricating oil properties. The production technology factors resulting from nonadherence to the design specifications during production and from improper assembly influence the primary factors.

All the above mentioned factors have an influence on the dynamic properties when the analyzed new system is under the influence of secondary factors: operation, change of condition and environmental factors. The latter include dustiness, humidity and temperature. Load and rotational speed characterize the operation factors. Load is a periodic or non-periodic variable or a constant or slowly changing variable. Also rotational speed can be constant or variable. The change of condition factors include

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**Fig. 1. Factors affecting diagnostic signal**
frictional wear and distributed or local faults. Frictional wear causes misalignment and impulse interaction of the parts. Distributed faults are caused by pitting, scuffing, and erosion. Cracking, breakage, spalling or chipping results in local faults.

The above factor analysis reveals the vibration signal properties on the basis of which one can choose a proper way of analyzing the vibration signal. Then a degradation scenario is determined and the condition inferring process is carried out.

2.2.1. MATHEMATICAL MODELLING AND COMPUTER SIMULATION REVEALING VIBRATION SIGNAL PROPERTIES

In order to investigate the relationship between the factors having an influence on the vibration signal and its form [1]–[9], mathematical modelling and computer simulation (MM and CS) are employed and different gearbox models are analyzed. The models can be roughly divided into ones where only torsional vibration is taken into consideration and ones where both torsional and lateral vibration are taken into account. Figure 2 shows a model where only torsional vibration is considered.

Fig. 2. Two-stage gearing system with six torsional degrees of freedom, electric motor moment $M_e(\phi_1)$ and external load moment $M_r$; system consists of: rotor inertia $I_r$, first stage gear inertias $I_{1p}$, $I_{2p}$, second stage gear inertias $I_{3p}$, $I_{4p}$, driven machine inertia $I_m$, gearing stiffnesses $k_1$, $k_2$ and dampings $C_{1z}$, $C_{12z}$, gearing stiffness forces $F_1$, $F_2$ and damping forces $F_{1t}$, $F_{2t}$, first shaft internal moments $M_1$, $M_{1t}$ ($M_1$, coupling damping moment), shaft stiffnesses $k_1$, $k_2$, $k_3$ and second and third shaft internal moments $M_2$ and $M_3$.
If the model shown in Fig. 2 is to be used for computer simulations, it needs to be written as equations of motion [2]. The physical motion quantities shown in Fig. 2, expressing design and operation factors, are included in the equations. The design and operation factors are represented by electric motor moment $M_e(\phi_e)$ and external load moment $M_s$. The system consists of: rotor inertia $I_r$, first stage gear inertias $I_{1p}$, $I_{2p}$, second stage gear inertias $I_{3p}$, $I_{4p}$, driven machine inertia $I_{m}$, gearing stiffnesses $k_{1z}$, $k_{2z}$ and dampings $C_{1z}$, $C_{2z}$, internal forces $F_1$, $F_2$, and damping internal forces $F_{1i}$, $F_{2i}$, first shaft internal moments $M_1$; $M_4$, ($M_1$ – a coupling damping moment), shaft stiffnesses $k_1$, $k_2$, $k_3$ and second and third shaft internal moments $M_2$ and $M_3$. The design and operation factors are described by expressions (1)–(5).

$$F_1 = k_{1z}(a_{u1}, g_1)(\max(r_1 \phi_2 - r_2 \phi_3 - l_{a1} + E(a_{u1}, a, e, r_a, r_e),$$

$$\min(r_1 \phi_2 - r_2 \phi_3 + l_{a1} + E(a_{u1}, a, e, r_a, r_e), 0))$$  \hspace{1cm} (1)

$$F_2 = k_{2z}(a_{u2}, g_2)(\max(r_2 \phi_4 - r_4 \phi_5 - l_{a2} + E(a_{u2}, a, e, r_a, r_e),$$

$$\min(r_2 \phi_4 - r_4 \phi_5 + l_{a2} + E(a_{u2}, a, e, r_a, r_e), 0))$$  \hspace{1cm} (2)

$$a_{u1} = frac(\phi_{2z}/(2\pi))$$  \hspace{1cm} (3)

$$a_{u2} = frac(\phi_{4z}/(2\pi))$$  \hspace{1cm} (4)

In expressions (1) and (2), functions $k_{1z}(a_{u1}, g_1)$ and $k_{2z}(a_{u2}, g_2)$ have different $g$ values. Stiffness functions $k_{1z}(a_{u1}, g_1)$ and $k_{2z}(a_{u2}, g_2)$ and error functions $E(a_{u1}, a, e, r_a, r_e)$ and $E(a_{u2}, a, e, r_a, r_e)$ have different values of parameters $a$, $e$, $r_a$, $r_e$. Using the parameters one can describe imperfections in the form of dimension and shape deviations. The functions can also be used to describe distributed faults. Taking into account wheel eccentricities (shape deviations), error function $E(a_{u1}, a, e, r_a, r_e)$ should be replaced by

$$E_{b1} = E(a_{u1}, a, e, r_a, r_e) + b_1\sin(\phi_2) + b_2\sin(\phi_3)$$  \hspace{1cm} (5)

$$E_{b2} = E(a_{u1}, a, e, r_a, r_e) + b_1\sin(\phi_4) + b_2\sin(\phi_5)$$

where:

- $b_1$, $b_2$, $b_3$, $b_4$, wheel eccentricities [m],
- $\phi_2$, $\phi_3$, $\phi_4$, $\phi_5$, rotation angles [rad],
- $l_{a1}$, $l_{a2}$, intertooth backlash.

Functions max and min in (1) and (2) are defined as follows

- $\min(a, b)$ for which if $a < b$ then $\min = a$ else $\min = b$
- $\max(a, b)$ for which if $a > b$ then $\max = a$ else $\max = b$. 
The angle corresponding to the meshing period is
\[
\alpha_p = 2\pi/z_1 \tag{6}
\]
where:
- \(\alpha_p\) – an angle corresponding to one meshing period, rad;
- \(z_1\) or \(z_3\) – the number of driving gear teeth.

The place of contact in terms of relative length (0-1) is defined as
\[
\text{frac}(\phi/\alpha_p) = a_{ux} \tag{7}
\]
where:
- \(\text{frac}\) – the fraction part of ratio \(\phi/\alpha_p\),
- \(a_{ux}\) – a variable in a range of 0–1 (the relative place of tooth contact),
- \(\phi\) – the wheel’s complete angle of rotation [radians].

The damping forces are defined as follows
\[
F_{1t} = C_z(r_1 \phi_2 - r_2 \phi_3) \tag{8}
\]
\[
F_{2t} = C_z(r_1 \phi_4 - r_3 \phi_5)
\]

Damping coefficient \(C_z\) describes the lubricating oil’s damping properties.

In general, \(\phi, \dot{\phi}, \ddot{\phi}\) stand respectively for rotation angle, angular velocity and angular acceleration; \(M_i(\phi)\) is the electric motor driving moment characteristic; \(M_1, M_2, M_3\) – internal moments transmitted by the shaft stiffness, depending on the operation factors; \(I_1, I_m\) – moments of inertia for the electric motor and the driven machine, depending on the design factors. The other design and operation factors are as follows: \(M_{i1}\) – the coupling’s damping moment; \(C_1\) – the coupling’s damping coefficient; \(F_1, F_{1t}, F_2, F_{2t}\) – respectively stiffness and damping intertooth forces; \(k_1, k_2, k_3\) – shaft stiffnesses; \(M_zt_{11}, M_zt_{21}, M_zt_{22}\) – intertooth moments of friction, \(M_{i1} = T_1 \rho_{11}; M_{i12} = T_1 \rho_{12}; M_{i21} = T_2 \rho_{21}; M_{i22} = T_2 \rho_{22}\), where \(T_1, T_2\) are intertooth friction forces, \(\rho\) determines the place of action of the friction forces along the line of action; \(r_1, r_2, r_3, r_4\) – gear base radii. For a given pair of teeth the value of the error is random and it can be expressed as
\[
e(\text{random}) = [1 - r_e(1 - l_i)]e \tag{9}
\]
where:
- \(e\) – the maximum error value;
- \(r_e\) – an error scope coefficient, range (0–1);
- \(l_i\) – a random value, range (0–1).

Hence the value of \(e\) in error characteristic (error mode) \(E(a, e, r_e)\) is replaced by \(e(\text{random})\). The value of \(a\) has a range of (0–1), and it indicates the position of the maximum error value on the line of action. It can also be randomised by the equation
\[ a(random) = [1 - r_a(1 - l_i)]a \]  

(10)

where:

- \( a \) – the relative place of the maximum error, range (0–1);
- \( r_a \) – a relative place scope coefficient, range (0–1);
- \( l_i \) – a random value, range (0–1).

Hence the value of \( a \) in error characteristic (error mode) \( E(a, e, r_a) \) is replaced by \( a(random) \).

The above background to gearbox modelling gives one an idea of the influence of the different design and operation factors on the vibration generated by gearboxes.

2.2.2. VIBRATION SIGNAL PROPERTIES, WAYS OF SIGNAL ANALYSIS, DEGRADATION SCENARIO AND PROGNOSIS, DIAGNOSTIC INFERENCE PROCESS

Taking into consideration the vibration signal properties, the ways of signal analysis, the degradation scenario and prognosis and the diagnostic inference process, one can develop a diagnostic method for gearbox condition monitoring. The general aim of gearbox condition monitoring is to assess the change in the condition on the basis of the vibration signal. Condition monitoring can be based on different starting assumptions. The latter can be described as in this paper’s chapter on factor analysis. But usually only some factors, shown in Figs. 1 and 2, are taken into account.

![Fig. 3. Vectors of gear condition for modified, unmodified and pitted gears: W, 1, 2 – modified gears (class 6 PS); 3, 4, 5 – unmodified gears (class 8, PS); 6, 7, 8 – unmodified gears (class 7, PS); 9 – unmodified gears (class 6, PS); 10 – pitted gear (PS – Polish Standard similar to ISO)](image-url)
Referring to book [4] and paper [21], the aim of the presented diagnostic method is the diagnostic assessment of gearbox condition described by \textit{imperfections} solely in the form of \textit{dimension} and \textit{shape deviations} (according to the Polish Standard (PS) similar to ISO). Figure 3 shows gear condition vectors for modified, unmodified and pitted gears: W, 1, 2 – modified gears (class 6 PS); 3, 4, 5 – unmodified gears (class 8, PS); 6, 7, 8 – unmodified gears (class 7, PS); 9 – unmodified gears (class 6, PS); 10 – a pitted gear. For the condition evaluation the coherence gearbox condition monitoring method (CGCMM) is used as described in [4]. Gear condition is assessed by measuring six coherence components for the gearing frequency (900Hz) and its harmonics (1800–5400 Hz). Signals are received from four points on the gearbox housing. From the four points one can obtain six independent coherence function measurements. The values of the squared coherence function components are averaged and treated as the vector components of a six-dimensional space. The magnitudes of the condition vectors are given in Fig. 3. CGCMM is based on the assumption that signal vibration meshing spectrum components are the measure of gear condition.

Having a \textit{design factor} such as the number of gear teeth and an \textit{operation factor} such as the shaft rotation frequency one could calculate the meshing frequency and multiplying it by constants (1–6) one could obtain the meshing frequency components. But gear condition evaluation through the measurement of the vibration spectrum components proved to be unsuccessful. It was not possible to make such an evaluation for the different gear condition states described above. To overcome this problem new factors having an influence on the diagnostic signal had to be taken into account. Rolling-element bearings when running generate a vibration signal in the form of white noise. This follows from the \textit{design factor} represented by \textit{surface roughness parameters} and from the discussion in [33]. Since the gearbox vibration signal includes both vibration meshing components and their harmonics and white noise it provides the basis for CGCMM. Using the coherence function one can calculate the share of white noise and that of meshing signal in the diagnostic signal. The energy of the white noise is assumed to be constant but as the gear \textit{dimensional deviations} increase and \textit{distributed faults} occur, the energy of the meshing components increases and the coherence components become the measure of the gearing condition. Taking the first 6 components as vector components in a 6-dimensional space, one calculates the length of the vector and the relative angles between the reference vector. The vector magnitudes for the gearing in different condition are shown in Fig. 3. One of the problems is the number of components to be taken into account in a gearing condition evaluation. This problem is discussed in [21] where a principle component analysis (PCA) is made. The results of the PCA are shown in Fig. 4.
It follows from Fig. 4 that all the six feature components should be used whereby 100% of the information will be obtained. Only some factors described in Fig. 1 were taken into consideration when the diagnostic method was developed. But one should be aware that then only some condition parameters can be evaluated. Actually, only distributed imperfections described by dimension deviations and the change in condition caused by distributed faults (e.g. pitting, scuffing or erosion) can be then assessed. One limitation of CGCMM is that local faults cannot be evaluated. Since the evaluation of local faults is very important another diagnostic method should be developed for this purpose. Another limitation of CGCMM is that the method can only be used when an operation factor such as load is constant.

A diagnostic method which allows one to evaluate distributed and local gearing faults is described in [11]–[13], [31] and [34]. A simplified version of the factors presented in Fig. 1 is shown (according to [24]) in Fig. 5. The diagnostic method proposed in [2], whose scheme is shown in Fig. 5, is limited to one operation factor represented by a constant load. In [23] there is an analysis of a gearbox loaded with different constant levels of load and it is shown how this operation factor is used for gearbox condition evaluation. According to Fig. 5, a gearbox should be considered as a system consisting of the following elements: {electric engine}, gearing, bearings, shafts, coupling, {driven machine}. When considering the system’s elements, factors influencing vibration, i.e. design, production technology, operation and change of condition factors, should be taken into account. Then a scenario of degradation caused by the action of the environment (e.g. dustiness or humidity), which may result in frictional wear and in the play of the elements, is determined. Then change in condition, described by such physical factors as unbalance, misalignment, pitting, scuffing and fracture, should be taken into account. It is important to carry out the mental transformation of faults and distinguish local and distributed faults.
For the considered factors signal analysis tools such as: the spectrum, the cepstrum and the time-frequency spectrogram are selected. One can diagnose distributed faults and local faults using respectively the spectrum and the cepstrum. It should be mentioned that local faults produce impulse perturbations in the signal, whose frequencies correspond to the gear shaft rotations. Papers [11], [13] show that local faults can be identified using the cepstrum analysis but the ultimate discriminator for this type of faults is the time-frequency spectrogram. The latter is indispensable since
one can obtain the same cepstrum for different (distributed or local) faults. Also the
demodulation procedure can be used to identify local faults, but then the load factors,
as mentioned earlier, should be constant at least during data acquisition/measurement.
Another ultimate discriminator for this type of fault is the time-frequency spectro-
gram.

In certain operating conditions a varying periodic load negatively correlated with
rotation speed is the main operation factor. The first step in the development of the
condition monitoring method was varying load identification [31], [32].

The diagnostic procedure presented below is for planetary gearboxes (PG) used
for driving bucket wheels in excavators. If one compares the plots of the forces acting
on a bucket wheel it becomes apparent that a gearbox in bad condition is more suscep-
tible to randomly varying external loads (RVEL), as shown in Fig. 6.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig6.png}
\caption{Envelopes proportional to RVEL: a) signal from gearbox before replacement, bad condition; b) signal from replaced gearbox, good condition}
\end{figure}

Four planetary gearboxes were selected for condition assessment. The main design
factor affecting the operation factor is the design of the bucket wheel with 11 buckets
mounted on it. When the wheel operates under the operation factors: varying rotatio-
nal speed (rpm) and load (determined by the properties of the dug ground) one may
describe the load condition as RVEL. On the basis of the rotation speed and the num-
ber of buckets one can calculate the bucket digging period, which in the considered
case is $T_b = 1.8$ s. In Fig. 6 approximately 3 periods of digging can be distinguished.
Two parameters, i.e. the mean value and the standard deviation, are used to describe
the variability of RVEL (an operation factor). There are three planetary gearboxes in
the bucket wheel excavator. One gearbox which is in bad condition has been replaced.
The adopted diagnostic parameters are shown in Fig. 7. A band-pass filter with its
midband frequency corresponding to the planetary gearbox meshing frequency filters the rough diagnostic vibration acceleration signal. The band-pass filter’s width also corresponds to the meshing frequency. The relative value of the filtered signal RMS (Fig. 7a)) and the relative mean value (Fig. 7b)) and standard deviation (Fig. 7c)) of RVEL are used for the diagnostic evaluation. The replaced gearbox G1 is compared with the one before replacement (also marked as G1). Also the condition of gearboxes G2 and G3 is examined as it evolves over time. Fig. 7d shows the relative amplitude of the arm rotational frequency and its harmonics.

Fig. 7. Relative condition change values: a) change of RMS, b) change of relative mean RVEL estimated from vibration c) change of relative standard deviation of RVEL estimated from vibration, d) change in amplitude of first (h1) and second (h2) harmonic of arm rotational frequency

Figure 8 shows the relation between the sum of vibration signal RMS amplitude components and rotational speed. The latter is assumed to be negatively correlated with load. The sum of the RMS amplitude components is calculated for ten signal harmonics of the planetary stage and for three harmonics of the bevel stage. The choice of ten and three harmonics was based on the examination of the measured vibration spectrum and it could also be deduced from the design factor, i.e. the difference in the design features between planetary gearboxes with spur gears and bevel gears.
In figs. 8a) and b) one can observe the relation according to [23] but the rotation speed-vibration signal correlation is negative if the gearbox is in bad condition. The figures show that this relation holds only for a certain rotational speed range when the gearbox is loaded during operation. It follows from figs. 8a, b and c that the relation is mainly determined by the condition of the planetary stage. Therefore the signals from the planetary stage and the bevel stage should be separated. If one looks only at fig. 8c, it is difficult to explain (interpret) the obtained result. But if one takes into consideration the factors having an influence on the vibration signal, one can diagnose the condition of the two considered bevel gearboxes. Firstly, one can notice that the bevel gear (which should be at in good condition) shows a higher vibration RMS sum, but at
a high load the RMS values decrease. Hence one can conclude that the bevel stage was improperly assembled. This means that the gear meshing trace is probably shifted. A proper gear meshing trace is highly important for bevel gears. If one looks carefully at the mean value curves for well and badly assembled bevel gears they have a similar shape but different parameters. It follows from the above analysis that the technology factors have an influence on the generated vibration. Therefore assembly should be treated as a technology factor. A similar situation may also occur when there are some teeth line imperfections due to the improper manufacture of the gear wheels, which is also a technology factor. If one examines the design structure of the bevel gearbox one can conclude that in the case of proper manufacture and assembly the measurement results should be represented by the curve with a lower vibration level (Fig. 8c).

3. CONCLUSIONS

The author’s experience relating to the development of condition monitoring methods has been presented. It should be stressed that such methods are developed bearing in mind that many factors have an influence on diagnostic vibration signals. The factors are divided into four groups: design, production technology, operation and change of condition factors. The presented methods take into account the factors having an influence on diagnostic vibration signals whereby one can properly interpret the signals and assess the condition of machines.

The first presented diagnostic method, called the coherence gearbox condition monitoring method (CGCMM), allows one to precisely evaluate gear condition described by design factors and a change in gear condition caused by pitting. The method is valid only for constant operation factors. The method was developed for one-stage gearboxes operating in laboratory conditions. The method can be used for the evaluation of distributed gear faults.

The second diagnostic method was developed for multistage bevel and cylindrical gearboxes operating in industrial conditions. The method is recommended for cases when the parameters representing operation factors are constant or almost constant. A scheme of the method is shown in Fig. 5. All the factors having an influence on vibration signals are taken into consideration in the condition inferring process. The method is based on an analysis of signals measured in industrial conditions.

The third method of diagnosing gearbox condition was developed for planetary gearboxes driving the bucket wheels of excavators. Special attention is focused on randomly varying cyclic loads (an operation factor).

The fourth diagnostic method was developed for the bucket wheels of compact driving systems with a planetary gearbox. Besides the design factors also the operation factors are taken into account which are characterised by the random varying load.
The paper does not provide a full description of the presented methods. More details can be found in the cited publications.

REFERENCES


3. MODELLING PLANETARY GEARBOX DYNAMIC

WALTER BARTELMUS

The papers gives physical models for a system with one stage planetary gearbox and system with double stage gearbox. The systems also includes: electric motor, damping coupling, driven machine. There are also given mathematical models in the form of equations of motion.

Keywords: planetary gearbox, system, physical model, mathematical model

3.1. INTRODUCTION TO PLANETARY GEARBOX DYNAMIC

First step to planetary gearbox dynamic modelling is given for the system presented in Fig. 1. The system consists of an electric engine with a rotor inertia $I$, [Nm$^2$]
and a damping coupling with its inertia parts $I_{sp1}$ and $I_{sp2}$ and stiffness and damping properties $k_1$ [Nm/rad] $C_s$ [Nms/rad]. The joining shafts have stiffness: $k_1$, $k_2$, $k_3$ [Nm/rad]. A planetary gearbox consists of 3 gears with radiiuses $r_1$, $r_2$, $r_3$ [m]. The planetary gearbox has 3 satellites presented in Fig. 2. More details are given in Figs. 1 and 2. The rotation and lateral motion is considered and is described by rotation motion by variables $\phi_1$ to $\phi_9$ [rad] and lateral motion by variables $x_{10}$, $y_{11}$ [m].

![Fig. 2. Planetary gearbox system cross-section view](image)

To describe the dynamic motion of the system given in Figs. 1 and 2 equations of motion according to the second Newton’s law can be written as follow

$$I_1 \ddot{\phi}_1 = M_1(\phi_1) - M_1$$

$$I_{sp1} \ddot{\phi}_2 = M_1 - (M_2 + M_2)$$

$$I_{sp2} \ddot{\phi}_3 = M_2 + M_2 - M_3$$

$$I_{fl} \ddot{\phi}_4 = M_3 - r_1(F_1 + F_{1t} + F_2 + F_{2t} + F_3 + F_{3t})$$
According to Fig. 2, equation of motion for the first satellite

$$I_{p2c} \ddot{\phi}_5 = 2r_2(F_1 + F_{11}) - ra(F_{a1} + F_{a11})$$

(moment of inertia $I_{p2c}$ calculated in reference to the point of instantaneous rotation)

equation of motion for the second satellite

$$I_{p2c} \ddot{\phi}_6 = 2r_2(F_2 + F_{23}) - ra(F_{a2} + F_{a23})$$

$$ra = (r_1 + r_2)/2 \rightarrow \text{arm/carrier radius}$$

equation of motion for the third satellite

$$I_{p2c} \ddot{\phi}_7 := 2r_2(F_3 + F_{35}) - ra(F_{a3} + F_{a35})$$

equation of motion for the arm/carrier

$$I_a \ddot{\phi}_7 := ra(F_{a1} + F_{a2} + F_{a3} + F_{a11} + F_{a23} + F_{a37}) - M_4$$

equation of motion for the driven machine

$$I_w \ddot{\phi}_9 := (M_4 - M_L)$$

equation of motion describing lateral vibration

$$m_x \ddot{x}_1 := (F_2 + F_{23})\sin(PI/6) + (F_3 + F_{35})\sin(PI/6) - (F_1 + F_{11}) + (F_4 + F_{45}) - (F_5 + F_{57})\sin(PI/6) - (F_6 + F_{67})\sin(PI/6) - (F_7 + F_{77})$$

$$m_y \ddot{y}_1 := -(F_2 + F_{23})\cos(PI/6) + (F_3 + F_{35})\cos(PI/6) + (F_5 + F_{57})\cos(PI/6) - (F_6 + F_{67})\cos(PI/6) - (F_7 + F_{77})$$

Other relations

$$ra := r_1 + r_2$$

Outer moment load conditions

if $t < 3$ then $M_L := 0.05M_n$ else $M_L := 2M_n$

Inner moments/toques and forces

$$M_1 := k_1(\phi_1 - \phi_2)$$

$$M_2 := k_2(\phi_2 - \phi_3)$$

$$M_3 := k_3(\phi_3 - \phi_4)$$

$$M_4 := k_4(\phi_4 - \phi_5)$$
Inter-tooth force (sun – first satellite)

\[ F_1 := k_{12}(r_1 \varphi_4 - 2r_2 \varphi_s) \]

\( c := k_{12}/k_{23} \) tooth stiffness ratio

\[ a = \frac{2cr_2 F_4}{F_1} \]

\[ F_4 := k_{23}(\varphi_4 - \varphi_s)a \]

\[ F_{1t} = C_{212}(r_1 \varphi_4 - 2r_2 \varphi_s) \]

\[ F_{4t} := C_{223}(\varphi_4 - \varphi_s)a \]

\[ F_2 := k_{12}(r_1 \varphi_4 - 2r_2 \varphi_s) \]

\[ F_{2t} := C_{212}(r_1 \varphi_4 - 2r_2 \varphi_s) \]

\[ F_5 := k_{23}(\varphi_4 - \varphi_s)a \]
\[ F_{3t} := C_{223}(\psi_4 - \psi_b) a \]
\[ F_3 := k_{12}(r_1 \psi_4 - 2r_2 \psi) \]
\[ F_{3t} := C_{212}(r_1 \psi_4 - 2r_2 \psi_b) \]
\[ F_6 := k_{23}(\psi_4 - \psi_7) a \]
\[ F_{5t} := C_{23}(\psi_4 - \psi_7) a \]
\[ F_{a1} := k_a(r_2 \psi_5 - r_a \psi_8) \]
\[ F_{a2} := k_a(r_2 \psi_6 - r_a \psi_8) \]
\[ F_{a3} := k_a(r_2 \psi_7 - r_a \psi_8) \]
\[ F_{a1t} := k_a(r_2 \psi_5 - r_a \psi_b) \]
\[ F_{a2t} := k_a(r_2 \psi_6 - r_a \psi_b) \]
\[ F_{a3t} := k_a(r_2 \psi_7 - r_a \psi_b) \]
\[ F_x := k_{xsp} x_{10} \]
\[ F_{xt} := C_{xsp} x_{10} \]
\[ F_y := k_{ysp} y_{11} \]
\[ F_{yt} := C_{ysp} y_{11} \]

3.2. THE MODEL WITH TWO PLANETARY STAGES

Two stage planetary gearbox is given in Fig. 4. Cross section of the first planetary gearbox is given in Fig. 5 for the second planetary gearbox in Fig. 6.

The system consists of an electric engine with a rotor inertia \( I_s \) [Nm^2] and a damping coupling with its inertia parts \( I_{sp1} \) and \( I_{sp2} \) and stiffness and damping properties \( k_s \) [Nm/rad], \( C_s \) [Nms/rad]. The joining shafts have stiffnesses: \( k_1, k_2, k_3, k_4 \) [Nm/rad]. A planetary gearbox consists of 6 gears with radiiuses \( r_1, r_2, r_3 \) [m] for the first stage and \( r_4, r_5, r_6 \) for the second stage. More details are given in Figs. 4 to 6. The inertia of the elements are marked in Fig. 4. The rotation motion is considered and is described by rotation motion by variables \( \psi_1 \) to \( \psi_{14} \) [rad]. The further details are given in equations of motion.

Equations of motions

\[ I_s \dot{\psi}_1 = M_i(\dot{\psi}_1) - M_1 \quad \text{equation of an electric motor motion} \]
\[ I_{sp1} \dot{\psi}_2 = M_1 - (M_2 + M_{2t}) \quad \text{equation of a driving part of flexible coupling motion} \]
\[ I_{sp2} \dot{\psi}_3 = M_2 + M_{2t} - M_3 \quad \text{equation of a driven part of flexible coupling motion} \]
\[ I_p \dot{\phi}_1 = M_3 - r_1(F_1 + F_{1y} + F_2 + F_{2y} + F_3 + F_{3y}) \] equation of a sun motion

\[ I_p \dot{\phi}_2 = 2r_2(F_1 + F_{1y}) - r_{a1}(F_{a11} + F_{a111}) \] (moment of inertia \( I_p \) calculated in ratio to the point of instantaneous rotation)

\[ I_p \dot{\phi}_5 = 2r_2(F_1 + F_{2y}) - r_{a1}(F_{a13} + F_{a133}) \]

\[ I_a \dot{\phi}_6 := r_{a1}(F_{a1} + F_{a2} + F_{a3} + F_{a12} + F_{a3}) - M_4 \]

\[ I_p \dot{\phi}_9 := M_4 - r_4(F_1 + F_7 + F_8 + F_9 + F_{9y}) \]

\[ I_p \dot{\phi}_{10} := 2r_5(F_1 + F_7) - r_{a2}(F_{a21} + F_{a211}) \]

\[ I_p \dot{\phi}_{11} := 2r_5(F_8 + F_{9y}) - r_{a2}(F_{a22} + F_{a222}) \]

\[ I_p \dot{\phi}_{12} := 2r_5(F_0 + F_{9y}) - r_{a2}(F_{a23} + F_{a233}) \]

\[ I_a \dot{\phi}_{13} = r_{a2}(F_{a21} + F_{a211} + F_{a22} + F_{a222} + F_{a23} + F_{a233}) \]

\[ I_a \dot{\phi}_{13} = M_5 - M_L \]

Other relations

\[ r_{a1} := r_1 + r_2, \quad r_{a2} := r_4 + r_5 \]

![Fig. 4. System with two planetary gearboxes](image)

The cross section of first stage for the two stage planetary gearbox is similar to the cross section given in Fig. 2.
Inner moments/torques and forces

\[ M_1 := k_1 (\phi_1 - \phi_2) \]
\[ M_2 := k_2 (\phi_2 - \phi_3) \]
\[ M_{2l} := C_s (\phi_2 - \phi_3) \]
\[ M_3 := k_3 (\phi_3 - \phi_4) \]
\[ M_4 := k_4 (\phi_4 - \phi_3) \]
\[ F_1 := k_{12} (r_1 \phi_4 - 2r_2 \phi_3) \]
\[ c_1 := k_{123}/k_{23} \]
\[ a_1 = \frac{2c_{r_2} F_4}{F_1} \]
\[ F_4 := k_{23} (\phi_4 - \phi_3) a_1 \]
\[ F_{1r} = C_{121}(r_1 \dot{\phi}_4 - 2r_2 \dot{\phi}_8) \]
\[ F_{4t} := C_{213}(\dot{\phi}_4 - \dot{\phi}_8)a_1 \]
\[ F_2 := k_{122}(r_1 \phi_4 - 2r_2 \phi_6) \]
\[ F_{2s} = C_{212}(r_1 \dot{\phi}_4 - 2r_2 \dot{\phi}_8) \]
\[ F_{5} := k_{223}(\phi_4 - \phi_6)a_1 \]
\[ F_{3t} = C_{233}((\dot{\phi}_4 - \dot{\phi}_6)a_1 \]
\[ F_3 := k_{122}(r_1 \phi_4 - 2r_2 \phi_7) \]
\[ F_{3s} = C_{212}(r_1 \dot{\phi}_4 - 2r_2 \dot{\phi}_8) \]
\[ F_{6} := k_{233}(\phi_4 - \phi_7)a_1 \]
\[ F_{6t} = C_{233}(\phi_4 - \phi_7)a_1 \]
\[ F_{a1} := k_a(r_2 \phi_5 - r_6 \phi_8) \]
\[ F_{a2} := k_a(r_2 \phi_6 - r_6 \phi_8) \]
\[ F_{a3} := k_a(r_2 \phi_7 - r_6 \phi_8) \]
\[ F_{a1t} := k_a(r_2 \dot{\phi}_5 - r_6 \dot{\phi}_8) \]
\[ F_{a2t} := k_a(r_2 \dot{\phi}_6 - r_6 \dot{\phi}_8) \]
\[ F_{a3t} := k_a(r_2 \dot{\phi}_7 - r_6 \dot{\phi}_8) \]
\[ M_5 := k_5(\phi_8 - \phi_9) \]
\[ F_{7} := k_{345}(r_4 \phi_9 - 2r_5 \phi_{10}) \]
\[ c_2 := k_{345}/k_{56} \]
\[ a_2 = \frac{2c_{r_2} \frac{F_{10}}{F_7}}{1 + c \frac{F_{10}}{F_7}} \]
\[ F_{10} := k_{56}(\phi_9 - \phi_{10})a_2 \]
\[ F_{7t} = C_{245}(r_1 \phi_9 - 2r_5 \phi_{10}) \]
\[ F_{10t} := C_{45}(\phi_9 - \phi_{10})a_2 \]
\[\begin{align*}
F_8 & := k_{c45}(r_4 \varphi_9 - 2r_5 \varphi_{11}) \\
F_{8t} & = C_{c45}(r_4 \varphi_9 - 2r_5 \varphi_{11}) \\
F_{11} & := k_{c56}(\varphi_9 - \varphi_{11})a_2 \\
F_{11t} & = C_{c56}(\varphi_9 - \varphi_{11})a_2 \\
F_9 & := k_{c45}(r_4 \varphi_4 - 2r_5 \varphi_{12}) \\
F_{9t} & = C_{c45}(r_4 \varphi_4 - 2r_5 \varphi_{12}) \\
F_{12} & := k_{c56}(\varphi_9 - \varphi_{12})a_2 \\
F_{12t} & = C_{c56}(\varphi_9 - \varphi_{12})a_2 \\
F_{a21} & := k_{a}(r_5 \varphi_{10} - r_{a2} \varphi_{13}) \\
F_{a22} & := k_{a}(r_5 \varphi_{11} - r_{a2} \varphi_{13}) \\
F_{a23} & := k_{a}(r_5 \varphi_{12} - r_{a2} \varphi_{13}) \\
F_{a21t} & := k_{a}(r_5 \varphi_{10} - r_{a2} \varphi_{13}) \\
F_{a22t} & := k_{a}(r_5 \varphi_{11} - r_{a2} \varphi_{8}) \\
F_{a23t} & := k_{a}(r_5 \varphi_{12} - r_{a2} \varphi_{8})
\end{align*}\]

3.3. CONCLUSIONS

In the chapter are given two models first for the one stage planetary gearbox second for the two stage planetary gearbox. The models gives the back ground for preparing the models for computing which can be used for computer simulations. The models for computing should take into consideration discussion given in [1] to [4]. The use of the papers [1]–[4] for gearbox modelling is also given in papers [5]–[10].

LITERATURE


4. INFLUENCE OF RANDOM VARYING LOAD
TO VIBRATION SIGNAL GENERATED BY PLANETARY GEARBOXES DRIVING BUCKET WHEEL IN EXCAVATORS

WALTER BARTELMUS, RADOSŁAW ZIMROZ

Analysed planetary gearboxes are included into a driving system of bucket wheel excavators. The whole system may consist of a planetary gearbox as a first stage and a three stage cylinder gearbox. The basic problem of vibration condition monitoring is influence of an operation factor as random varying outer load (RVOL). RVOL cause amplitude and frequency modulation (AFM). The first step for developing sound method for condition monitoring is RVOL identification and investigation on its influence to vibration signal. For the identification the properties of the whole driving system should be considered as it is postulated by the authors in some published papers. All factors having influence to vibration signal should be considered. One may expect after study of the mentioned factors that for the frequency modulation beside of RVOL have influence an electric motor characteristic. So to vibration signal have influence the operation factor – RVOL and design factor – property of the electric motor given by it characteristic. The paper show influence of design and operation factors that cause AFM. The RVOL will be identified by filtration and demodulation process.

Keywords: planetary gearbox, bucket wheel, driving system load identification, vibration, condition monitoring, diagnostics, modulation, demodulation

4.1. INTRODUCTION

False alarms of professional diagnostic systems for vibration condition monitoring of gearboxes in a bucket wheel driving system (BWDS) give motivation for investigation of problems. BWDS system is used in bucket wheel excavators (BWE) in open cast mines. The nature of a digging process in BWE causes the random varying load of BWDS. Considering the tools of vibration signal analysis used for condition monitoring as a vibration spectrum and an envelope vibration spectrum it would be stated that these tools do not fulfil the tasks. The nature of the digging process is not taken into consideration. It seems that the factors that have influence to vibration diagnostic signal are not considered. As it is stated in (Bartelmus 1992) and after some developments in (Bartelmus 2001) when con-
Considering vibration signals the four group of factors should be taken namely: design, production technology, operation, and change of condition factors. In the presented papers special attention is directed to an operation factor that is varying outer load that come from the nature of digging process. Some authors see the problems connected with fluctuating/varying load. Some models of the load variation are taken into consideration like is given by (Baydar, Ball 2000), (Standen, Heyns 2002; 2005). In laboratory rig investigations are taken some simplified models of varying loads like sinusoidal and square trace. In the presented paper is undertaken an effort of the varying load identification from measured vibration signals in industrial condition.

4.2. OBJECT DESCRIPTION

In Figure 1 is given a general over view of BWDS. The system is driving by three independent electric motors and three gearbox systems which sketch is given in Fig. 2.

![Fig. 1. View of bucket wheel driving system](image)

![Fig. 2. Part of driving system for bucket wheel with planetary gearbox](image)

(gears: $z_1$ – sun, $z_2$ – planet, $z_3$ – standstill rim, $z_4$–$z_9$ – three stage cylindrical gearbox)
Figure 3 gives the general view of BWE and the scheme of one driving system, and an arrangement of three independent drives. In Figure 2 it is seen a part of BWDS a gearbox for which a first stage is a planetary gearbox with a standstill rim. Considered planetary gearboxes consist of: a sun gear $z_1$, planetary gear $z_2$ and standstill rim gear $z_3$. Taking into consideration some design factors one can calculate meshing and other characteristic frequencies.

Considering the system given in Fig. 2 and using notation $f_{12}$ as meshing frequency for a pair of gear wheels marked in Fig. 1 as $z_1$, $z_2$, $z_3$

$$f_{12} = f_{23} = \frac{n_1 z_1 z_3}{60(z_1 + z_3)} = \frac{950 \cdot 39 \cdot 93}{60(39 + 93)} = 435.067 \text{ Hz}$$

(1)

where $n_1$ – input rotation velocity RPM

The arm frequency is

$$f_a = \frac{n_1 z_1}{60(z_1 + z_3)} = \frac{950 \cdot 39}{60(39 + 93)} = 4.67 \text{ Hz}$$

(2)
The rotation frequency of second gear \( z_2 \) is

\[
f_2 = \frac{n_1 z_1}{2 \cdot 60 z_2} = \frac{950 \cdot 39}{2 \cdot 60 \cdot 27} = 11.43 \text{ Hz} \tag{3}
\]

Meshing frequencies for three stage cylindrical gearbox are as follow

\[
f_{45} = f_a z_4 = 4.67 \cdot 34 = 158.78 \text{ Hz} \tag{4}
\]

\[
f_{67} = f_a \frac{z_4 z_6}{z_5} = 4.67 \cdot \frac{34}{117} \cdot 42 = 57 \text{ Hz} \tag{5}
\]

\[
f_{89} = f_a \frac{z_4 z_6 z_8}{z_5 z_7} = 4.67 \frac{34 \cdot 42}{117 \cdot 145} \cdot 35 = 13.75 \text{ Hz} \tag{6}
\]

The bucket digging period \( T_b \) is obtained from a bucket wheel rotation period \( T_w = 20 \text{ s} \) by division of a bucket number which is in the considered case \( b_n = 11 \). The bucket frequency is

\[
f_b = \frac{b_n}{T_w} = \frac{11}{20} = 0.55 \text{ Hz} \tag{7}
\]

The spectrum of vibration presented by frequencies (1) to (6) and its harmonics together with inter-stage modulation components (Bartelmus 2001) and modulation caused by an arm rotation gives the gearbox vibration spectrum of gearboxes in BWDS. If one take further into consideration distributed faults which origin may come from gear transmission errors in the vibration spectrum one can see other components. These gear transmission errors depends of design factors and can be identify as side band components in a gearbox vibration spectrum. They occur as result of the amplitude modulation. The same effect give distributed faults caused by gear pitting or scuffing. Beside of the distributed faults may occur local faults that originate from a tooth foot crack, tooth breakage, a spall on a tooth flank. The local faults give the same effect in gear spectrum as distributed faults. There is possibility of identification of these two types of faults following the procedures given in (Bartelmus 2005). Taking into use the above consideration there is a need to identify the local faults frequency

\[
f_{y1} = \frac{n_1 z_3}{60(z_1 + z_3)} = \frac{950 \cdot 3.93}{60 \cdot (39 + 93)} = 33.4 \text{ Hz} \tag{8}
\]

\[
f_{21} = \frac{4 n_1 z_1 z_3}{60(z_3^2 - z_1^2)} = \frac{4 \cdot 950 \cdot 39 \cdot 93}{60 \cdot (93^2 - 39^2)} = 32.2 \text{ Hz} \tag{9}
\]

\[
f_{y3} = \frac{n_1 z_1}{60(z_1 + z_3)} = \frac{950 \cdot 3.39}{60 \cdot (39 + 93)} = 14 \text{ Hz} \tag{10}
\]
In a paper (Bartelmus, Zimroz 2005) there are given some results on vibration signal analysis that where measured in an industry environment for considered case of the planetary gearbox. As it is pointed out in (Bartelmus 1992) one should take into consideration interaction between gearbox components as it is given in Fig. 4 and influence of an environment.

Fig. 4. Interaction of gearbox elements condition and influence of environment

4.3. THEORETICAL DESCRIPTION OF VARYING LOAD IN BUCKET WHEEL EXCAVATORS

A bucket wheel during an operation is seen in Fig. 5. Theoretical description of varying load in a bucket wheel excavator is started showing periodic variability as is given in Fig. 5b. In Figure 6 is seen influence of a random parameter to the load variation and it gives a random varying outer load (RVOL). Figure 6 shows also an electric motor angular velocity fluctuation. The consideration on the load theoretical description and time traces of the load variation are given after (Bartelmus 1998).

Fig. 5. a) view of bucket wheel during operation, b) theoretical periodic load variation
The operation factors include the effect of an external load, particularly that of changes in external/outer load $M_r\ [Nm]$. The function of changes in the external load is shown in Fig. 5. The function is periodic. A relative value of the period was assumed in an interval of $[0, 1]$. Function $M_r$ (Fig. 5) can be written in the form of linear functions in two intervals. According to Fig. 5, in interval $[0, p_w]$

$$M_r = M_w/p_w\ aux_1 + M_n = M_n(w/p_w\ aux_1 + 1) \quad (11)$$

where:
- $w$ – an overload factor for the external load, (0–1);
- $p_w$ – a maximum load entrance coefficient,
- $aux_1$ – an auxiliary value.

The auxiliary value can be determined in the following way:

$$aux_1 = \frac{\varphi}{\text{period}} \quad (12)$$

$$\text{period} = \frac{2\pi}{b_n} \quad (13)$$

where:
- $b_n$ – number of buckets;
- $\varphi$ – angle of wheel rotation, rad.

In interval $(p_w, 1]$

$$M_r = M_w(aux_1 - 1)/(1 - p_w) + M_n = M_n(1 - w(aux_1 - 1)/(1 - p_w)) \quad (14)$$

Coefficients $w$ describing the variation of the load can be given a random character. The variation of coefficient $w$ can be defined as follows

$$w(\text{variable}) = [1 - r(1 - l)]w \quad (15)$$
where:

- \( w \) – maximum variation value, range \( w(0–1) \);
- \( r \) – variation range coefficient, range \( r(0–1) \);
- \( l_i \) – random value, range \( 0–1 \).

For example, if \( l_i = 1 \) and \( r = 1 \), then \( w \) (variable) = \([1 – 1(1 – 1)]w\); when \( l_i = 0 \) and \( r = 1 \), then \( w \) (variable) = \([1 – 1(1 – 0)]w = 0\). This means that \( w \) (variable) varies from 0 to 1 for \( r = 1 \); in the case when \( r = 0.5 \), \( w \) (variable) assumes values from \( w \) (variable) = 0.5 to 1, when \( w = 1 \). Value \( l_i \) is selected once per a load variation period. More on the load randomization modeling is given in (Bartelmus 1998).

4.4. CONSIDERATION ON FACTORS INFLUENCING VIBRATION SIGNAL

Figure 7 shows the scheme of some factors influencing vibration signals. RVOL can cause an amplitude and frequency modulation. This influence is possible to identify taking into consideration factors that come from design factors of a planetary gearbox and an electric motor design factors. The planetary gearbox design factor comes from influence of different teeth deflection under RVOL. The electric motor design factor is given by a motor characteristic presented in Fig. 8.

\[
\frac{M_s}{M_n} = \frac{n_s \omega_s}{\phi_1 \omega} = \frac{n}{\phi_1} \omega \text{ rad/s}
\]

Fig. 7. Factors influencing amplitude and frequency modulation of vibration signal like RVOL and gearbox housing structure

Fig. 8. Electric motor characteristic. \( M_s \) – current motor torque, \( M_n \) – rated/nominal motor torque, \( n_s \) – synchronized RPM, \( \phi_1 \) synchronized angular rotation rad/s
Using the spectral signal analysis one can discover that for most of measured signals there are problems with identification of spectral components, Fig. 10a. However, it has been found that for the same gearbox has been found one case when a signal does not reveal speed fluctuation. It is obvious that only in this case one can use the spectral analysis without problem and make detection of sidebands and mesh components.

![Figure 9. Frequency response function of gearbox structure obtained by impulse excitation](image)

Figure 9 gives frequency response function (FRF) from which one can see that the amplitude frequency variation can cause an amplitude increase or drop of amplitude components of the vibration spectrum. Summing up one can come to conclusion that in the diagnostic inferring process factors that are coming from the design should be taken into consideration together with a gearbox housing vibration properties.

![Figure 10. Limited spectrums (zoom) around first harmonic of mesh frequency:](image)

a) with varying load, b) with constant load, gearbox in the same condition
For further investigation a time-frequency (t-f) signal analysis technique has been applied. The time frequency analysis is called short time Fourier transform (STFT). In Fig. 11a STFT time frequency map for a raw vibration signal is presented. By dashed vertical lines the mesh frequency and its harmonics are marked. The fundamental mesh frequency is estimated as 447.9 Hz, (1). It is visible that for higher harmonics some frequency fluctuation of harmonics is seen. In Fig. 11b a part of t-f map of the same signal is presented. It shows 6, 7 and 8 harmonic of the meshing frequency. It is very interesting that there is the clear periodic fluctuation of the mesh frequency and component amplitude (MFCA). When frequency value of 7th harmonics is going left means that the outer load is going up and gearbox is slowing down. The amplitude of 6th harmonics is increasing with the same period but the meshing frequency variability is not so clear seen as for 7th harmonic. The MFCA fluctuations are due to a change of the working point on electric motor characteristic (see Fig. 8). In the case is seen influence of operation and design factors.
Figure 11 shows that for different frequency components one can see some differences caused by RVOL. It is also seen that frequency fluctuations can be referred to a bucket digging period that is \( T_b = 1.8 \text{ [s]} \).

4.5. IDENTIFICATION OF RANDOM VARYING OUTER LOAD

The procedures of load identification is connected with filtration, enveloping and envelope frequency analysis. The first step RVOL identification is filtration of an original vibration signal with the band of a filter which central frequency is one of planetary gearbox meshing frequency (1). The time signal course after the band filtration is given in Fig. 12 together with the time frequency analysis that shows the band of the signal around the harmonic of the mesh frequency.

Fig. 12. a) time signal trace of filtered signal around \( x \)-harmonic
b) time-frequency map/spectrogram of signal in a)

Fig. 13 gives an envelope of the filtered signal given in Fig. 12a. As it is stated before low frequency of vibration signal caused by RVOL (fundamental frequency \( 0.55 \text{Hz (7)} \)) and arm rotation frequency \( f_a = 4.67 \text{ Hz (2)} \). To identify load variation there is a need of a separation of the two different vibration sources the arm and RVOL. It is done by a low pass filter for the load vibration components separation and a high pass filter for the arm vibration components. The envelopes and its spectrums for separated signals are given in Fig. 14. Fig. 14a shows the envelope of a acceleration signal caused by RVOL and its spectrum. Because between acceleration and load there is the proportion so the course of the acceleration envelope is proportional to the RVOL.
Fig. 13. Envelope of the signal a) and its spectrum b)

Fig. 14. Envelopes for separated signals and its spectrums: a) load varying envelope and its spectrum, b) separated signal that depicts influence of arm rotation

Because there are two sources that cause a vibration signal amplitude modulation: RVOL and improper arm run. Their basic frequencies are $f_b = 0.55$ Hz (7) and $f_a = 4.67$ Hz (2). After filtration for a source separation there is only possible to evaluate RVPL trace with a limited accuracy. To investigate the problem the numerical experiment has been done. The results of the experiment are given in Fig. 15. In Figure 15 there is seen how the scope of spectrum components influenced the shape of RVOL time trace.
4.6. CONCLUSIONS

It has been shown some factors that have influence to the vibration signals generated by a planetary gearbox. It is stressed that all the factors should be taken into consideration these factors are gathered into four groups namely: design, production technology, operation, change of condition. Some of the factors for cylindrical and bevel gears where examined by the authors in former publications given in proceedings of COMADEM conferences and other conferences and journals which some of them are listed in literature attached to the paper. In the paper special attention is directed to the random varying outer load. It has been shown the identification possibility of the random varying outer load from the vibration signal. The identification process is based on signal filtration and demodulation procedure. It has been also shown that the signals that come from the random varying outer load and arm rotation run should be separated to obtain proper load identification which may be identify with limited accuracy.

LITERATURE

5. VIBRATION CONDITION MONITORING OF PLANETARY GEARBOX UNDER RANDOM VARYING EXTERNAL LOAD

WALTER BARTELMUS, RADOSŁAW ZIMROZ

The paper shows that for condition monitoring of planetary gearboxes it is important to identify the external varying load condition. In the paper, systematic consideration has been taken of the influence of many factors on the vibration signals generated by a system in which a planetary gearbox is included. These considerations give the basis for vibration signal interpretation, development of the means of condition monitoring, and for the scenario of the degradation of the planetary gearbox. Real measured vibration signals obtained in the industrial environment are processed. The signals are recorded during normal operation of the diagnosed objects, namely planetary gearboxes which are a part of the driving system used in a bucket wheel excavator, used in lignite mines. It is found that a planetary gearbox in bad condition is more susceptible to load than a gearbox in good condition. The estimated load time traces obtained by a demodulation process of the vibration acceleration signal, for a planetary gearbox in good and bad condition are given. It has been found that the most important factor of the proper planetary gearbox condition is connected with perturbation of arm rotation, where an arm rotation gives rise to a specific vibration signal whose properties are depicted by a short time Fourier transform (STFT) presented as a time-frequency spectrogram. The paper gives evidence that there are two dominant low frequency causes that have influence on vibration signal modulation, i.e. the varying load, that comes from the nature of the bucket wheel digging process, and the arm/carrier rotation. These two causes determine the condition of the planetary gearboxes considered. Typical local faults such as cracking or breakage of a gear tooth, or local faults in rolling element bearings have not been found in the cases considered. In real practice, local faults of planetary gearboxes have not occurred, but have been noticed heavy destruction of planetary gearboxes which are caused by prolonged run of a planetary gearbox at condition of the arm run perturbation. It may be stated that the paper gives new approach to the condition monitoring of planetary gearboxes. It has been shown that only a root cause analysis based on factors having an influence on the vibration solves the problem of planetary gearbox condition monitoring.

Keywords: planetary gearbox, diagnostics, load variation, modulation, arm run perturbation
5.1. INTRODUCTION

The authors undertake investigations of false equipment deterioration warnings on a gear transmission that operates under conditions of fluctuating random load. The diagnostic warning is based on a commercial monitoring system. The presented paper shows the importance of taking into consideration external load varying conditions when gearbox condition is being evaluated. It should be also stated that most papers on condition monitoring developments for gearboxes are focused on specific faults (distributed or local) mostly for a single stage gearbox under laboratory conditions with constant external load or for some specified models of load fluctuation. One of the aims of the paper is the identification varying/fluctuating load in the real industrial situation. The issue of the influence of variable load (fluctuating load condition) is seen by many authors who have developed techniques that can be used for some specific condition in the industrial environment. Studying vibration signal properties Randall [1] states that the vibration amplitude of the gearbox casing, caused by the meshing of the gears is modulated by the fluctuations in the torque load. To examine the issue some computer simulation were done and presented by Bartelmus(2003) in [2]. To overcome the problem of the fluctuations in the torque load the assessment of the gearbox condition is sometimes done under free run of the diagnosed gearbox system. The condition assessment for this condition leads to improper condition evaluation. Smith [3] states that gearboxes should not be monitored under conditions where the angular acceleration multiplied by the effective moment of inertia of the system exceeds the steady load torque. This issue is the subject of the study in [4] Bartelmus (1999) where the problem of tooth separation is discussed. It is stated that when the so called dynamic factor exceeds 2 the tooth separation exists. This statement is equivalent to the statement given in [3]. But it should also be stated that for some very bad gear conditions the statement does not hold, see also the paper [5] Bartelmus (2001) where the issue of examination of tooth separation is presented and where the influence of a flexible coupling is taken into consideration. As presented in [1] Randall (1982) and [6] McFadden (1986), there are two aspects connected with a signal modulation in the vibration signal. One aspect is considered in [6] when the gearbox is loaded by constant external moment load. For this load condition in [6] a very effective technique for gear fault detection was developed by using band pass filtered time domain synchronous average (TSA) signal and Hilbert transform. In [6] it is shown that the phase modulation trace over an entire revolution of the target gear is particular powerful for detecting the presence of fault. This technique is thought to be independent of load level condition, since the amplitude of vibration signals cancelled out in the phase modulation function. The kurtosis was then calculated to give a quantitative description of the severity of a fault. This work was improved in [7] (McFadden1987), which is able to some extent to locate a fatigue crack in a gear. McFadden [6,7] mentions that changing load conditions that cause frequency modula-
tion influence the results he obtained for his time domain synchronized averaged vibration. Besides this, the effectiveness of this technique is subject to the selection of the bandwidth for band pass filtering which involves a subjective judgement. Such a disadvantage was noted by Dalpiaz in his study [8]. Kurtosis of the amplitude and phase modulation traces and time frequency representation of the residual signal by means of Morlet wavelets were calculated to illustrate the crack severity of a gear. There are also papers [9] by Baydar and [10] by Zahn which have rather misleading titles in which is the phrase “varying load conditions”. In the papers one can see condition monitoring evaluation under different constant level loads. The load is varying from one experiment to another experiment. In [11] the issue of the influence of load levels on the vibration signal is discussed and a simple normalization process is presented. Condition monitoring of the gearbox is directed to the evaluation of distributed faults. In the paper [12] by Stander 2002 consideration is directed to the problem of gearbox condition monitoring for detection local faults under fluctuating load conditions. Consequently, to find a methodology to deal with such conditions, experiments were conducted on a gearbox test rig with different severity levels of induced damage to the gear teeth (local fault) with the capability of applying fluctuating loads to the gear system. Different levels of constant load, as well as fluctuating sinusoidal, step and chirp loads were considered in these experiments. The test data were order tracked and time synchronously averaged with the rotation of the shaft in order to compensate for the variation in rotational speed produced by the fluctuating loads. This is also seen in [12] where load conditions are more precisely defined. But the notion of a step load is not precisely defined; it should rather be a square wave fluctuating load. The step load suggests load at two constant levels as was used in [9] or [10]. In paper [12], a vibration waveform normalization approach is presented, which enables the use of the pseudo-Wigner–Ville distribution to indicate deteriorating fault conditions (local fault) under fluctuating load conditions. The normalisation approach assumes that a local tooth defect will modulate the vibration signal with a wider frequency band amplitude modulation and higher modulation frequency content when compared with the smaller frequency band amplitude modulation caused by the load variation. The experimental data prove that this assumption is valid. Contour plots of the pseudo-Wigner–Ville distribution data are presented, which indicate that the normalisation approach enables the detection of local gear tooth faults under fluctuating load conditions.

In [13] by Stander 2005 a simplified mathematical model of a gear system was developed to illustrate the feasibility of monitoring the instantaneous angular speed (IAS) as a means of monitoring the condition of gears that are subjected to fluctuating load conditions. A distinction is made between cyclic stationary load modulation and non-cyclic stationary load modulation. It is shown that rotation domain averaging will suppress the modulation caused by non-cyclic stationary load conditions but will not suppress the modulation caused by cyclic stationary load conditions. An experimental
investigation on a test rig indicated that the IAS of a gear shaft could be monitored with a conventional shaft encoder to indicate a deteriorating gear fault condition. In [14] by Bonnardot a method to perform order tracking (with very limited speed fluctuation) or angular re-sampling by directly using the gearbox acceleration signal is described. In this paper the speed fluctuation as an operation factor is stressed.

Based on considerations presented in the cited papers, one can see that different definitions are given for varying condition loads (VCL) and different models are given for VCL. To give more information about VCL in the present paper real measured VCL are presented, which are recovered from vibrations that are further denoted as random varying external load (RVEL). The recovered RVEL is a specific case that is characteristic to the machine, which is a bucket wheel excavator. RVEL identification is one of the motivations for writing this paper. There are also others, such as systematic consideration of the influence of different factors on vibration signals generated by gearbox systems; the paper refers to Bartelmus [4], [5], [15]. The notion “factor” is understood as one that actively contributes a process, in this case vibration generated by a gearbox system. It should be stressed that the motivation is a trial of a systematisation of factors that should be taken into consideration when vibration is generated by a real system and not in the laboratory situation. One should be conscious that results obtained on rig investigations are difficult to transform to industrial use when factors that have influence on the vibration signal are not properly considered. So the results of these developments can only partly be transformed into the real industrial situation. One should also take into consideration several limitations that can be in the industrial situation they will be developed later.

In the paper is presented the investigation of a complex system with planetary gearbox (CSwPG) under RVEL. The vibration signals are received from an existing commercial system. It is one of the limitations associated with the investigations. The paper shows the difficulties and solutions for vibration condition monitoring (VCM) of (CSwPG) under (RVEL). In order to show the difficulties and solutions of VCM of CSwPG under RVEL there is a need to examine the problem using factor analysis (FA) which has an influence on the vibration signal, as given in [5], [11], [15]. This consideration leads to design (D), production technology (PT), operation (O), change of condition (CC) factors and can be summarized as “DPTOCC inferring diagnostic information of gearing system condition” [5]. When the influence of DPTOCC factors on vibration generated by single stage and two-stage gearboxes is given. Design factors include specified flexibility of the gear components, especially flexibility of each mesh, and specified machining imperfections of components. In the components one should also include the design features of the gearbox housing. Production technology factors include deviations from specified design factors during machining and assembly of the gearbox. Operational factors include the peripheral velocity (pitch line velocity) and its variation, load and its variation. Change of condition factors include all gear faults that may occur during a gearbox operation. The investigations presented in
this paper show the influence of mesh flexibility – (DF), amplitude modulation caused by the rotation of the planetary gearbox arm (DF and CCF) and RVEL – (OF) on the vibration signals. It should be noticed that in the CSwPG in the planetary gearbox cylindrical spur gears are used. The spur gears are very sensitive to the load level. That means that at the higher load levels they generate higher amplitude components in the vibration spectrum. Under conditions of RVEL also the amplitude modulation of vibration signals will occur. The mentioned design factors (DF) and operation factors (OF) cause difficulties in the evaluation of gear condition.

5.2. OBJECT DESCRIPTION AND SOME DPTOCC FACTOR CONSIDERATIONS

The diagnosed object/CSwPG is given in Fig. 1.

![Diagram of driving system for a bucket wheel with planetary gearbox](image)

Figure 1. Part of driving system for a bucket wheel with planetary gearbox (gears: \(z_1\) – sun, \(z_2\) – planet, \(z_3\) – stationary rim, \(z_4\)–\(z_9\) – three stage cylindrical gearbox)

In Figure 1 is seen a CSwPG/gearbox for which the first stage is a planetary gearbox with a stationary rim. The planetary gearboxes considered consist of: a sun gear \(z_1\), planetary gear \(z_2\) and rim gear \(z_3\). There are three cases of planetary gear element rotation. In the considered case where the sun gear is rotating, the planet gear makes planetary movement and the rim-gear is stationary as it is for the case given in Fig. 1. Taking into consideration some design factors one can calculate meshing and other characteristic frequencies.

Considering the system given in Fig. 1 and using notation \(f_{12}\) as meshing frequency for a pair of gear wheels marked in Fig. 1 as \(z_1, z_2, z_3\).
where $n_1$ – input rotation velocity RPM

The arm frequency is

$$f_a = \frac{n_1 z_1}{60(z_1 + z_3)} = \frac{950 \cdot 39 \cdot 93}{60(39 + 93)} = 435.067 \text{ Hz}$$

(1)

The rotation frequency of the second gear (planetary gear) $z_2$ is

$$f_2 = \frac{n_1 z_1}{2 \cdot 60 z_2} = \frac{950 \cdot 39}{2 \cdot 60 \cdot 27} = 11.43 \text{ Hz}$$

(2)

Meshing frequencies for the three stage cylindrical gearbox are as follows

$$f_{45} = f_a z_4 = 4.67 \cdot 34 = 158.78 \text{ Hz}$$

(4)

$$f_{67} = f_a \frac{z_4}{z_5} z_6 = 4.67 \cdot \frac{34}{117} \cdot 42 = 57 \text{ Hz}$$

(5)

$$f_{89} = f_a \frac{z_4 z_6}{z_5 z_7} z_8 = 4.67 \cdot \frac{34 \cdot 42}{117 \cdot 145} \cdot 35 = 13.75 \text{ Hz}$$

(6)

The spectrum of vibration presented by frequencies (1) to (6) and their harmonics together with inter-stage modulation components [5] and modulation caused by the arm rotation give the gearbox vibration spectrum of CSwPG. If one further takes into consideration distributed faults whose origin may be gear transmission errors/imperfections in the vibration spectrum one can see other components. These gear transmission errors depend on design factors and can be identified as side band components in a gearbox vibration spectrum. They occur as a result of amplitude modulation. The same effect is given by distributed faults caused by gear pitting or scuffing. Besides the distributed faults, local faults may occur that originate from a tooth root crack, tooth breakage, or a spall on a tooth flank. The local faults give similar effect in the gear spectrum as distributed faults. There is a possibility of identification of these two types of faults following the procedures given in [16]. Using the above considerations there is a need to identify the local fault frequencies

$$f_{u1} = \frac{n_1 z_3}{60(z_1 + z_3)} = \frac{950 \cdot 3 \cdot 93}{60(39 + 93)} = 33.4 \text{ Hz}$$

(7)

$$f_{u2} = \frac{4n_1 z_1 z_3}{60(z_1^2 - z_3^2)} = \frac{4 \cdot 950 \cdot 3 \cdot 93}{60(93^2 - 39^2)} = 32.2 \text{ Hz}$$

(8)
As is pointed out in [11] one should take into consideration the interaction between gearbox components as given in Fig. 2. Figure 2 shows a developed scheme as compared to the scheme given in [11].

\[
f_{sl} = \frac{n_s z_1}{60(z_i + z_3)} = \frac{950 \cdot 3 \cdot 39}{60 \cdot (39 + 93)} = 14 \text{ Hz}
\]  

(9)

The authors’ experience shows that interaction between the internal components of CSwPG and the external components in the system (electric motor – coupling – CSwPG – RVEL) is crucial to vibration diagnostic evaluation of the CSwPG condition. So the classical method of condition monitoring given in [16] is not always suitable. The influence of RVEL should be stressed. The high ratio of CSwPG determines the longer time acquisition of vibration signals. The described whole driving system in Fig. 3 of a bucket wheel excavator consists of three subsystems as in Fig. 1, and the system is used for driving a bucket wheel. The bucket wheel diameter is 17.5 m and has 11 evenly distributed buckets of capacity 4.6 m³. The rotation speed of the bucket wheel is about 3RPM which gives the rotation cycle, 20 s.
As was motioned at the beginning of the paper, the gearbox system is monitored by a commercial diagnostic system which gives the possibility to estimate the properties of the vibration signal. Some details on the commercial vibration monitoring system: it gives the possibility of estimation of vibration parameters as: Peak, RMS of vibration signal, vibration spectrum, envelope (time and spectrum) analysis, tools for spectrum component identification and sideband component identification, trend analysis – long term and short term trend, shock pulse method – for bearings.

The system supplier has adjusted some fixed parameters for the signal analysis as 1 s, 2.5 s acquisition time with the possibility of time extension to 5 s. Sequential signal acquisition is used in the system which means that only one signal from one measurement point is acquired on-line. The acquired signals, with the equivalent of 1; 2.5 or 5 s, digitalized time trace signals can be transmitted by an in-mine transmission system or by an internet for further signal analysis. The spectrum range of the signal analysis can also be adjusted to 0–9600 Hz with an envelope spectrum, frequency range 0–1000 Hz. The system supplier has also adjusted some fixed alarm limit values on which warning signals are activated. In the opinion of the user, the commercial vibration monitoring system gives false alarms. Now the system user in maintenance practice uses subjective planetary gearbox assessment, and the replacement criterion is a fluctuating heavy noise generated by the planetary gearboxes. To examine the condition monitoring problem of planetary gearboxes this paper presents a method for objective condition assessment based on the vibration signal analysis.

5.3. INTRODUCTORY VIBRATION SIGNAL ANALYSIS

For signal analysis typical procedures have been undertaken, as recommended in [16], which are good for the operation condition defined by constant or semi-constant gearbox
external load. The procedure recommends using: spectrum, cepstrum and time-frequency STFT. Using this procedure it is obvious that the condition monitoring evaluation will not be successful, taking into consideration the above discussion on factors which have an influence on the diagnostic signal. It is also obvious that signal parameters suggested by many results of investigations for gearbox vibration condition monitoring based on rig investigations with one-stage gearboxes, such as vibration time domain parameters: RMS, peak, mean, skewness, kurtosis etc will not be suitable for this case. In the case of simple one-stage gearboxes it is possible to estimate condition for very different cases such as advanced distributed pitting or advanced local faults (typically with periodic impulses in the signal). The mentioned parameters mostly do not have any direct practical use for systems such as those considered in this paper. For the raw vibration signals from complex gear systems, these mentioned signal estimators are not sufficient.

It is noted that in some operating conditions it is possible to apply spectral analysis as for stationary periodic signals. Application of the RMS spectrum for a vibration
signal as in Fig. 5 sometimes allows easy detection of characteristic frequencies related to shaft rotation speeds, mesh-frequencies as given in Fig. 6, where some characteristic frequencies can be identified as given by (1)–(9).

However, for many measured vibration signals obtained from the investigated CSwPG in the industrial environment one can meet problems with the component identification of the spectrum which are caused by RVEL. The consideration of the influence of RVEL on the vibration signal starts with the expected load variation whose course according to [17] is given in Figs. 7 and 8.

Figure 7a shows a bucket wheel excavator during an operation where buckets are seen. Figure 7b shows the constant cyclic variation of external load while Fig. 8a gives a randomly varying external load which causes fluctuation of the drive motor angular speed, Fig. 8b. The results have been obtained by mathematical modelling and computer simulation. The obtained results suggest that such a situation will happen in the real condition. The example shows how operation (varying load) and design (characteristics of the electric motor) factors have an influence on the diagnostic signal caused by the load and the angular speed variation.

Further for identification purposes, a band-pass filtered signal demodulation procedure, as in [6], has been applied. As seen from Fig. 9a the signal after filtration shows a strong amplitude modulation. Figure 9c shows the envelope of the filtered signal and the envelope spectrum. The spectrum of the envelope shows a component with arm frequency $f_a = 4.67$ Hz, (2). It is not possible to detect such sidebands directly in the vibration spectrum Fig. 9b) of a filtered vibration signal.
For further investigation a time-frequency (t-f) signal analysis technique has been applied. The time frequency analysis is called short time Fourier transformation (STFT). In Figure 10a the STFT time-frequency map for a raw vibration signal is presented. The mesh frequency and its harmonics are marked by dashed black vertical lines. The fundamental mesh frequency is estimated as 447.9 Hz, (1). It can be seen that for the higher harmonics some fluctuation of the harmonics is evident. In Fig. 10b a part of the t-f map of the same signal is presented. It shows harmonics 6, 7 and 8 of the meshing frequency. It is very interesting that there is a clear periodic fluctuation of the mesh frequency and the component amplitude (MFCA). When the frequency value of the 7th harmonic is going left it means that the external load is increasing, and the gearbox is slowing down. The amplitude of the 6th harmonic is increasing with the same period, but the meshing frequency variation is not so clearly seen as for the 7th harmonic. The MFCA fluctuations are due to a change of the working point on the electric motor characteristic (see Fig. 11). In this case the influence of OF and DF is seen.
Figure 10 shows that for the different frequency components one can see some differences which are going to be examined later. It is also seen that frequency fluctuations can be referred to a bucket digging period that is $T_{\text{bucket}} = 1.8$ [s].

![Fig. 10. a) Whole map of time frequency signal presentation, b) and c) restricted maps of time-frequency signal presentation](image)

Figure 11. Electric motor characteristic, $M_r$ – current motor torque, $M_n$ – rated/nominal motor torque, $n_s$ – synchronous RPM, $\omega_s$ – synchronous angular speed rad/s

![Fig. 11. Electric motor characteristic](image)
Using spectral signal analysis one can discover that for most of the measured signals there are problems with identification of spectral components, Fig. 12a. However, it has been found that for the same gearbox there is one case where a signal doesn’t reveal any speed fluctuation. It is obvious that only in this case one can use the spectral analysis without problem and make detection of sidebands and mesh components.

In most signal analysis cases for the gearbox with the same condition one can obtain a spectrum as given in Fig. 12a where the energy of a meshing component is smeared (Randall [19]), which is caused by the random frequency modulation. The issue of different factors/causes on the vibration signal generation and the connection with diagnostic problems can be summarized by the scheme given in Fig. 13.

In Figure 13 one can see that RVEL can cause amplitude and frequency modulation over and above which one can also see the influence of DF given by the gearbox housing structure. To consider more precisely the influence of the system transmis-
sion path, the frequency response function (FRF) does not have a constant influence on the vibration signal, nor the same amplification or suppression of the spectral components with speed variation of the gears.

![Frequency response function](image)

**Fig. 14. Frequency response function of a gearbox structure obtained by impulse excitation**

Figure 14 gives an FRF from which one can see that the frequency variation can cause an increase or decrease in the amplitude of components in the vibration spectrum. Summing up, one can come to the conclusion that in the diagnostic inference process, factors that come from design should also be taken into consideration, such as housing vibration properties, and gear resonance properties Wang [18]. Such properties of structure can be useful in local tooth fault detection (or generally in impulse detection in vibration signals caused by mechanical impacts), the idea being initially presented in [18] by Wang.

### 5.4. CONDITION MONITORING METHOD

The problem of the condition monitoring of planetary gearboxes is the subject of many publications which are reviewed in [22]. There, different diagnostic techniques are discussed and focused on a fatal fault for gearboxes, such as cracking of a tooth. In this paper different problems are discussed. The model of RVEL is presented in Fig. 8a). As was stated, it is important to identify RVEL from the vibration signal received from the gearbox housing. The symptoms of RVEL should be identified from the acceleration signal. The obtained result of RVEL obtained from acceleration signal is not directly the time trace of the load, but a suitable trace of acceleration which is proportional to the RVEL. The procedure of load identification is connected with the procedures of filtration, enveloping and envelope frequency analysis. The first step in RVEL identification is filtration of an original vibration signal with a band-pass filter whose central frequency is the first harmonic of the planetary gear-
box mesh frequency (1). For RVEL identification there two planetary gearboxes are taken, one before the decision was taken to make a repair, and the second gearbox when mounted in the system in place of a removed gearbox. Thus we get the possibility of comparing two gearboxes in quite different condition. The results of earlier measurements suggested that gearboxes in different condition have different susceptibility to RVEL. To investigate the load susceptibility, the RVEL identification was undertaken.

Fig. 15. Acceleration \([\text{m/s}^2]\) signal time traces [s]:
a) signal from gearbox before replacement b) signal from replaced gearbox

Figures 15 and 16 show signal time traces and time frequency spectrograms for two gearboxes in different conditions. At first glance one can see differences in the signals. Following the procedure of RVEL identification, the signals were filtered around the first mesh harmonic and the results are given in the form of time traces in Fig. 17, where in figure a) a strong amplitude modulation is seen. The next step of the signal analysis is to find the signal envelopes, which are given in Fig. 18.

Fig. 16. Time [s] – frequency [kHz] spectrograms:
a) signal from gearbox before replacement, b) signal from replacement gearbox
Fig. 17. Time traces [s] for filtered acceleration signal around first meshing harmonic: a) signal from gearbox before replacement, b) signal from replacement gearbox

Fig. 18. Signal envelopes from time traces given in Fig. 17: a) signal from gearbox before replacement, b) signal from replacement gearbox

Fig. 19. Envelope spectra: a) signal from gearbox before replacement, b) signal from replacement gearbox
Fig. 19 shows spectrum components connected with the arm rotation, that is \( f_a = 4.67 \) Hz (2).

Load variation is connected with a bucket period i.e. 1.8 s, which gives bucket frequency 0.55 Hz. Two causes have influence on the shape of the envelope, these being the load variation, and improper arm condition, with frequency \( f_a = 4.67 \) Hz. The next step of load identification is separation of these two sources of vibration. Fig. 20 shows separated signals from the load variation (solid line with stars) and from the arm (solid line).

Fig. 20. Spectrum source separation from load (solid line with stars) and arm (solid line)

Fig. 21. Arm signal envelopes after load signal separation:
   a) signal from gearbox before replacement, b) signal from replaced gearbox
For better visualisation two seconds of arm signal envelopes after the load signal separation are given in Fig. 22, with: a) being signal from the gearbox before replacement and b) being the signal from the replaced gearbox.

The Figures from 15 to 23 give a detailed analysis for two planetary gearboxes with two different conditions. One can draw the conclusion that there is evidence of a greater susceptibility to RVEL when the gearbox is in bad condition. In the planetary gearbox G1 after replacement there is also seen some susceptibility (Fig. 23b) to the load variation caused by design factors such as its tooth flexibility. This should especially be so for spur gears such as are used in the planetary gearbox considered. Besides the above considerations, the signal analysis has been made for another two gearboxes that run in the complete bucket drive system, Fig. 3. For condition assessment, measures of the load susceptibility are chosen to be the mean ($L_{\text{mean}}$) and standard deviation ($L_{\text{std}}$) of the RVEL estimated from the acceleration vibration signals.
That means that the RVEL is proportional to the estimated values of vibration. Figure 24 gives the values of relative changes of gearbox condition parameters.

a)  b)  c)  d)

Fig. 24. Relative values of parameters representing condition change for gearboxes marked G1, G2, G3, for G1 are considered two gearboxes before replacement and after replacement: a) change of RMS for filtered signals, values being compared for times \( t_1 \) (before replacement of G1) and \( t_2 \) (after replacement of G1), b) change of relative parameter values of mean (\( L_{\text{mean}} \)) of RVEL, estimated on vibration for times \( t_1 \) and \( t_2 \), c) change of relative values of standard deviation (\( L_{\text{std}} \)) of RVEL estimated on vibration, d) signal amplitude change of first (\( h_1 \)) and second (\( h_2 \)) harmonic of arm frequency rotation for times \( t_1 \) and \( t_2 \), relative values for gearboxes G2 and G3 are also given for times \( t_1 \) and \( t_2 \).

Besides the above-mentioned load susceptibility measures, which are also measures of planetary gearbox condition, another two condition measures are taken into consideration, these being the RMS values of the signals filtered around the first mesh harmonic \( f_{12} \pm 0.5 f_{12} \) (1) and first and second harmonic amplitudes of the arm frequency (2). For the evaluation of the condition change, relative values of the condition measures are considered. The considered relative values are given in Fig. 24. In the figure, the three planetary gearboxes are marked as G1, G2, G3 and compose a part of the bucket wheel drive system, Figs. 3 and 4. The signals were received from three planetary gearboxes some time before replacement (time \( t_1 \)) of gearbox G1 and some time after replacement, time marked as \( t_2 \). The decision to replace gearbox G1 was made on subjective grounds, not on the basis of measurements from the commercial monitoring system installed on the drive system, because the monitoring system
did not fulfil its task. As one can see for the planetary gearbox (PG) G1, the RMS value dropped about 14% (Fig. 24a) when comparing measurements for two times $t_1$ and $t_2$ (measurements for two gearboxes marked as G1 one before replacement and one after replacement) and for the PG marked G2 and G3 there were respectively increases of about 9% and 8% of RMS values (measurements at times $t_1$ and $t_2$ for two gearboxes G2 and G3). Similar tendencies for a decrease of the mean values of load parameter for G1, and increases for this parameter for G2 and G3, Fig. 24b). The mean value $L_{\text{mean}}$ of the load parameter was estimated from the vibration signals, so the estimated values are proportional to the values of load. The most important indication of the load susceptibility is the standard deviation of the load variation where in Fig. 24c) one can see a 50% drop when comparing measurements for $t_1$ and $t_2$. Important measures of PG condition are the amplitude value that are connected with arm rotation, as given in Fig. 24d). On the basis of the presented considerations, one can come to the conclusion that for condition evaluation the standard deviation of the load variability should be taken. See Figures 24c and 23a and compare them to 23b. Besides this, the changes of the harmonic amplitudes of the arm rotation are also important to consider, Fig. 24d).

![Fig. 25](image-url)

Fig. 25. a) Time trace of the signal for gearbox marked G1 before replacement, b) its time-frequency spectrogram, c) zoomed time trace of the signal
It should also be noted that in consideration of condition, the time-frequency spectrogram as given in Fig. 16a is useful, where one can see horizontal lines with the period $T_a/2 = 0.105$ s which is equivalent to the arm rotation period. The lines with the period of $T_a$ shows perturbation in arm rotation which gives vibration impacts. One can also see an other period connected with bucked period 1.8 s which separate groups of horizontal lines with the period $T_a/2$ so it corresponds to the second harmonic of the arm rotation frequency (2). Further considerations have been made of the gear condition by comparing the other results of the signal analysis. The starting point for the further discussion are the results given in Fig. 16 presenting the time frequency spectrograms, which give a clear picture of different gear condition (PG marked G1 and measured before replacement and after replacement) of the considered gearbox condition.

Figure 25 gives a detailed examination of the vibration signal acquired from the PG-G1 gearbox before replacement in bad condition. The average peak time separation is 0.105 s which is equivalent to a period $T_a/2$ of the arm rotation. A similar examination is presented in Fig. 26 for PG marked G3 where a time trace and a time-frequency spectrogram are given. In this case the time peak separation is equivalent to the arm rotation period $T_a$ so it corresponds to the first harmonic of the arm rotation frequency (2).

![Fig. 26. a) Time trace, b) time-frequency spectrogram for PG G3, signal measured at time $t_2$.](image)

A trial is also undertaken to use Wigner–Ville distribution analysis to evaluate the time peak separation, and the results are given in Fig. 27.

Figure 27 shows the evolution of the Wigner-Ville distribution with change in condition of the arm of the planetary gearbox where a) the arm is in good condition and b) there is an occurrence of distinct perturbations at the half period of an arm rotation $T_a/2$, the double arm rotation frequency, $2f_a$ and c) there is an occurrence of distinct perturbations at the arm period $T_a$, the arm rotation frequency $f_a$. 
Fig. 27. Evolution of Wigner–Ville distribution with change in condition of the arm of the planetary gearbox: a) arm in good condition, b) occurrence of distinct perturbations at second harmonic of arm rotation frequency (PG-G1 condition before replacement), c) occurrence of distinct perturbations at arm rotation frequency, (PG-G3)
At the end one should describe a typical scenario of change of condition of a planetary gearbox. For bevel and cylindrical gears in the open cast mining environment, the degradation scenario is given in [21]. The factors having an influence on the gearbox degradation in the open cast mining environment, should be taken into consideration in the case considered of a planetary gearbox, as pointed out in [20 and 21]. The influence of the open cast mining environment is manifested by the influence of impurities that come from the environment and get into the lubricating oil of a gearbox. The impurities cause bearing and tooth frictional wear. Especially important is rolling element bearing wear, that causes an increase of bearing backlash having an influence on the planetary gearbox run-out, as manifested by perturbations revealed in the vibration signal, Fig. 16a, 25b and 26b. The other manifestation of bearing backlash is the increased load susceptibility; compare Fig. 23a to b. The replacement limit condition of a planetary gearbox is the condition of a gearbox for which a time-frequency spectrogram is given in Fig. 16a). The time-frequency spectrogram shows periods \( T_{a}/2 = 0.105 \) s and \( T_{b} = 1.8 \) s connected with arm rotation and a load variation. The load variation, Fig. 23a) is characterised with 50% increase of the load variation in comparing to Fig. 23b for gearbox in good condition. In this way, objective rather than subjective measures of planetary gearbox condition have been obtained, which have been used in practice for replacement of a planetary gearbox.

5.5. CONCLUSIONS

The problems of planetary gearbox condition evaluation described in the paper represent quite new issues not considered before in the literature. The results of varying load evaluation show that the load trace is quite different from those considered in the literature [9], [10], [12], [13]. It can be stated that identification of the varying external load is crucial to the evaluation of planetary gearbox condition. It is expected that proper consideration of factors having an influence on the vibration diagnostic signal, together with the proper signal analysis methods, will lead to the proper choice of condition criteria for planetary gearboxes used in bucket wheel drive systems. It is expected that such design factors as the flexibility of spur teeth used in the considered planetary gearbox should cause amplitude modulation under the fluctuating load. It has been found that for a planetary gearbox in good condition, a vibration signal measured on the gearbox housing is modulated by a varying load. A time trace of the varying load obtained by a demodulation process is given in Fig. 23b. But it is also stated that a planetary gearbox in bad condition is more load susceptible than a gearbox in good condition. The time trace obtained by a demodulation process for a gearbox in bad condition is given in Fig. 23a. It can be further stated that the most important factor in proper planetary gearbox condition evaluation is connected with perturbation of the arm rotation, which gives the arm perturbed rotation which rising a specific vibration signal given in Fig. 16a, 25b, 26b as depicted by STFT and pre-
sented as a time-frequency spectrogram. The specific vibration signal Fig. 16a is characterised by two forms of perturbations first connected with the arm rotation, second connected with the bucket rotation period. The first kind of the perturbation shows horizontal lines in STFT with the time period $T_a/2$ or $T_a$ of arm rotation. The second kind of the perturbation shows perturbations with the period equivalent to the bucket digging period $T_b$ which also can be noticed in STFT, Fig. 16a. Figure 25b together with details of the time signal shows perturbations in the signal. Figure 26 shows only perturbations connected with arm rotation. These perturbations dose not show Fig. 16b. The paper gives evidence that there are two dominant low frequency causes that have an influence on the vibration signal modulation, namely the varying load, that comes from the bucket wheel digging process and the arm rotation. For an arm rotation in good condition one may expect sinusoidal modulation of the vibration signal at the arm rotation frequency (2). For an arm rotation in bad condition one may expect a periodic perturbation with the period of arm rotation or its second harmonic.

The expected scenario of changes in condition suggested in literature, such as faults in rolling element bearings, or a tooth cracking or breakage, have not occurred in the cases considered. The suggested diagnostic method for planetary gearbox condition evaluation is limited to values given in Fig. 24 including demodulation techniques, and by separation of the main causes of change in the planetary gearbox condition given by the load susceptibility, and the arm perturbed rotation depicted in time-frequency spectrograms. It may be concluded that an objective method for determining planetary gearbox condition has been presented in chapter 4 “Condition monitoring method” which may be used instead of the subjective method used now because the commercial diagnostic system did not fulfil its tasks. The objective diagnostic method is quite different from the method used for condition assessment by the commercial system. It should be also stated that the subjective method of condition assessment was done to avoid complete destruction of a planetary gearbox. Using the commercial diagnostic system this aim of avoiding planetary gearbox destruction has not been achieved, but many fault warnings have been noticed. The proposed diagnostic procedure and a means of condition assessment should avoid such confusing situations. It is expected that the occurrence of the above-mentioned local faults would be recognized using the signal filtration and time-frequency spectrograms. In actual practice, local faults of the planetary gearboxes have not occurred, but cases of severe destruction of planetary gearboxes have been noticed, which are caused by prolonged running of the gearbox under conditions of the arm perturbed rotation.

ACKNOWLEDGEMENT

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LITERATURE

6. DATA MINING FROM NON-STATIONARY VIBRATION SIGNALS FOR MACHINE CONDITION RECOGNITION

WALTER BARTELMUS, RADOSŁAW ZIMROZ

The paper is presented the way of finding the new diagnostic robust measure of machine condition the load yielding/susceptibility characteristic (LYCH). LYCH is developed on the introductory factor analysis influencing vibration signal and further data mining using a regression and adaptive process to obtain converged regression parameters.

**Keywords:** adaptive process, vibration signal, condition recognition, planetary gearbox

6.1. INTRODUCTION

For vibration condition monitoring there is gathered a lot of data in databases. From the data, machine condition should be evaluated because vibration is considered as signal of machine condition. To vibration signals have influence many factors [1]–[4]. It is the question if can be found some general or specific measure of the machine condition under condition of random varying external load. At this condition there is a need to develop a robust condition monitoring method and diagnostic inference procedure. There are many method of vibration condition monitoring, which at constant external load give good results of machine condition evaluation. The detailed analysis of machine condition using vibration signal can be obtained when spectrum or cepstrum analysis is done for stationary signals [2] and [4]. Many signals generated by machines are non-stationery. For rotating machines some generated signals are cyclostationery. For the cyclostationery signals there are also develop method for machine condition assessment [5] and [6]. There is also possibility to use wavelet analysis, which maybe used for some kind of non-stationary signals [7]. In this case non-stationary signal is caused by change of machine condition. The mentioned analysis could be rather used when the external load is constant o varying slowly. Generally one can say that there can be recognized two types of factors, which cause vibration signal non-stationary: change of machine condition and varying external
load. The paper will show that proper consideration on influence of different factors on vibration signal and data mining leads to robust condition monitoring method and inferring process. Some problems connected with varying load are described in papers [8]. In paper [8] there is identifying influence of random varying load to vibration generated by planetary gearboxes driving bucket wheel in excavators. In [9] there is presented a diagnostic method for planetary gearbox under a cyclic random amplitude varying external load where from the signal there is possibility to evaluate modulation parameters which are the measure of the gearbox condition.

There are also papers [10] by Baydar and [11] by Zahn, which have rather misleading titles in which is the phrase “varying load conditions”. In the papers one can see condition monitoring evaluation under different constant level loads. The load is varying from one experiment to another experiment. In [12] the issue of the influence of load levels on the vibration signal is discussed and a simple normalization process is presented. Condition monitoring of the gearbox is directed to the evaluation of distributed faults. In the paper [13] by Stander 2002 consideration is directed to the problem of gearbox condition monitoring for detection local faults under fluctuating load conditions. Consequently, to find a methodology to deal with such conditions, experiments were conducted on a gearbox test rig with different severity levels of induced damage to the gear teeth (local fault) with the capability of applying fluctuating loads to the gear system. Different levels of constant load, as well as fluctuating sinusoidal, step and chirp loads were considered in these experiments. The test data were order tracked and time synchronously averaged with the rotation of the shaft in order to compensate the variation in rotational speed produced by the fluctuating loads. This is also seen in [13] where load conditions are more precisely defined. But the notion of a step load is not precisely defined; it should rather be a square wave fluctuating load. The step load suggests load at two constant levels as was used in [10] or [11]. In paper [12], a vibration waveform normalization approach is presented, which enables the use of the pseudo-Wigner–Ville distribution to indicate deteriorating fault conditions (local fault) under fluctuating load conditions. The normalisation approach assumes that a local tooth defect will modulate the vibration signal with a wider frequency band amplitude modulation and higher modulation frequency content when compared with the smaller frequency band amplitude modulation caused by the load variation. The experimental data prove that this assumption is valid. Contour plots of the pseudo-Wigner–Ville distribution data are presented, which indicate that the normalisation approach enables the detection of local gear tooth faults under fluctuating load conditions.

In [13] by Stander 2005 a simplified mathematical model of a gear system was developed to illustrate the feasibility of monitoring the instantaneous angular speed (IAS) as a means of monitoring the condition of gears that are subjected to fluctuating load conditions. A distinction is made between cyclic stationary load modulation and non-cyclic stationary load modulation. It is shown that rotation domain averaging will
suppress the modulation caused by non-cyclic stationary load conditions but will not suppress the modulation caused by cyclic stationary load conditions. An experimental investigation on a test rig indicated that the IAS of a gear shaft could be monitored with a conventional shaft encoder to indicate a deteriorating gear fault condition. In paper [9] there has been stated that local faults may not occur but a gearbox condition may be in critical state and prolonged operation at this condition may lead to complete gearbox destruction. And as it was mentioned before an increase modulation depth of diagnostic signal with bucket digging frequency is one of the measures of gearbox condition. In the presented paper there is given a new diagnostic method, which may be used in the more complicated structure of a load variation.

6.2. INTRODUCTORY SIGNAL ANALYSIS

The vibration signals for condition monitoring may be picked from the machine housing. Examples for 60 s time record is given in Fig. 1c.

![Fig. 1. a) Input shaft rotation speed RPM variation, b) electric current consumption variation, c) vibration signal variation](image-url)
The lengths of time records for 60 s was decided on ground to obtain good frequency resolution of a vibration spectrum and order spectrum. The order spectrum is used to get no smeared spectrums of shaft orders. For the condition of varying load the raw signal after frequency signal analysis has smeared frequency components. So there is a need to use synchronous time signal from which one can get no smeared vibration spectrum. The synchronous time signal eliminates of influence of the frequency modulation of the signal. These spectrums are good when the vibration can be thought as stationary. Machines, as is presented in Fig. 1c, generate non-stationary vibration signals, when they are loaded by variable external load as it is given in Fig. 1b. When the external load is represented by electric current consumption, which is proportional to load. Figures 1b and 1c show that the time course of the signal is load dependent. With increase of load, amplitudes of acceleration vibration m/s² also increase. Figures 1a and 1b show negative correlation between a rotation speed RPM and an electric current consumption. When the load/current increase the rotation speed decrease. It comes directly from linear characteristic of an electric motor. Two diagnosed objects are considered which can be characterised by the length of an operation time. The first object was in operation for about 10 000 h and second 20 000 h. The second object after 20 000 h service was given for overhaul. As it was mentioned vibration signals were received and recorded for condition monitoring assessment. For further analysis, variation of a first shaft rotation frequency [Hz] was analysed presented the variation in the form of distributions of occurrence number of instantaneous value of input shaft frequency, as it is presented in Fig. 2.

![Figure 2](image_url)

**Fig. 2. Distributions of instantaneous value of frequency occurrence for 60 s time courses**

Figure 2a shows the distributions of instantaneous value of frequency occurrence for the time course given in Fig. 1a. Properly loaded object should have the distribution with one maximum of an envelope curve, as it is presented in Fig. 2c. Figure 2a shows two maximums. All the vibrations values, which are generated when the frequency rotation is around the second maximum, should be rejected from signal before further signal analysis, which should lead to diagnostic assessment of diagnosed objects. For diagnostic assessment some spectrum frequency components can be related to for specific machine elements of the diagnosed object/machine. Amplitudes values
of the frequency components are the measure of machine parts condition. The spectrum amplitudes were obtained by Fourier transformation for shot time segments, to avoid components smearing. The time scope of segments can be chosen from external load variation cycle. In the case where the load is not cycle related the time of segment should be many times shorter than rotation cycle of an output shaft. After rejection some segments from the vibration signal, according suggestion connected with Fig. 2 and described within the text, only these segments for which a diagnosed object was properly loaded are taken for further processing. In the following chapter a detail procedure for diagnostic feature extraction is given.

6.3. DIAGNOSTIC SIGNAL FEATURES EXTRACTION

Figure 3 shows time signals of frequency variations of an input shaft Fig. 3a and b where a shows the course of a raw signal b a smoothed signal. Figure 3c presents the segmented signals connected with a variable cycle of a machine.

![Fig. 3. Segments of tacho and signal segmentation: a) Raw signal of input shaft rotation speed, b) Smoothed signal of input shaft rotation speed, c) segments of vibration signal](image-url)
For the segments of time signals frequency analysis where done, which examples are given in Fig. 4. Comparing of two spectrums in bad and good condition is given in Fig. 5. The spectrum equivalent to good condition of the object is green in bad condition red. From the sum of 10 amplitude frequency components a measure of machine part condition (MPC) is defined.

Fig. 4. Examples of segment frequency analysis

Fig. 5. Spectrum comparing for good (green) and bad (red) object condition

For the measure of MPC, for all selected segments, data distribution curves have been prepared and presented in Fig. 6 for two diagnosed machine parts (kinematic pair) which are in good and bad condition.
The distributions presented in Fig. 6 overlap each other. In industry practice the results presented in Fig. 6 are the reason of confusion. A commercial diagnostic system sends ambiguous alarm signals. One can see the spread of data distributions is different for a good and bad condition. The Figure 6 shows that at variable external load the object in a bad condition is more yielding/susceptible to load than the object in good condition. There is a need to extract from the modified data, given in the form of distribution in Fig. 6, load yielding/susceptibility characteristics (LYCH).

6.4. LOAD YIELDING/SUSCEPTIBILITY CHARACTERISTICS IDENTIFICATION

The presentation of LYCH characteristic is started from presenting the data in the form of relation given in Fig. 7.
Fig. 7. Load yielding/susceptibility characteristics for two kinematic pairs in good (green) and bad (red) condition: a) for all data, b) for restricted data
Figure 7 shows load yielding susceptibility characteristics for two kinematic pairs in good (green) and bad (red) condition. The characteristics are given as a function of rotation speed, which negatively correlated with load. Figure 7a is prepared for all data and Fig. 7b for restricted data that comes from the need to use data which are restricted by rule that vibration signal is the measure of object condition when the object is properly loaded. One may also find LYCH for spectrum components as it is given in Fig. 8. In Figure 8 one can see how individual spectrum components react to the external load. It is seen that under condition of decreased of rotation speed, that means that the external load increase, except of second harmonic, the amplitude of frequency components increase. For further data processing the second harmonic has been rejected. As it suggested in [1] and [2] the process of condition monitoring should be done when the object is properly loaded. Referring to the Fig. 2 there is only taken the data for further consideration for rotation speed is not higher than 985 RPM. The results presented are interpreted that two different phenomena occur that are described by two different straight lines, Fig. 7a. After data selection load yielding/susceptibility characteristics are presented in Fig. 7b. Further data analysis is based on regression to find the tendency for the expected value of one of two jointly correlated random variables to approach more closely the mean value of its set than other. For regression equation $y = ax + b$; $a$ and $b$ constant and $r^2$ goodness of fit were
evaluated. Additionally the adaptive process for data processing was done by finding the tendency of constant $a$ and $b$ together with the $r^2$ to reach the stabilisation of regression parameters. From each 60 s measured signal as it is given in Fig. 1c, after described data rejection, a set of data was prepared which were used for finding the load yielding/susceptibility characteristics. As a number of sets increased there were calculated regression parameters $a$, $b$, $r^2$.

![Graph showing variation of $r^2$, 'a', and 'b' as a function of data set number (good condition)](image)

**Fig. 9.** Variation of “$r^2$”, “a” and “b” as a function of data set number (good condition)

![Graphs showing relative variation of $r^2$: a), a and b (good condition) a) and b)](image)

**Fig. 10.** Relative variation of $r^2$: a), a and b (good condition) a) and b)

Figure 9 shows variation of $r^2$ and a and b parameters of regression characteristic as a function of number of considered sets of data. Further investigation is to find if with increase of number of sets the mentioned values have convergent properties. For the investigation the values are presented as relative values according to statement

$$P_{\text{relative}}(i) = \{[P(i) - P(i - 1)]/P(i - 1)\} \times 100\%$$  \hspace{1cm} (1)

where:

- $i$ – number of set,
$P(i), P(i - 1)$ – absolute value of parameters.
$P$ relative$(i)$ – relative value of parameters.

The results of the convergent property investigation are given in Figs. 10 and 11. The results presented in Figs. 9 and 10 are for the object in good condition. The same investigation has been done for an object in bad condition the results are given in Figs. 11 and 12.

![Graphs showing variation of $r^2$, $a$, and $b$ as a function of data set number](image)

Fig. 11. Variation of $r^2$, $a$ and $b$ as a function of data set number (bad condition)

![Graphs showing relative variation of $r^2$, $a$, and $b$](image)

Fig. 12. Relative variation of $r^2$: a), $a$ and $b$ (bad condition), b) and c)
For final data mining regression lines are presented in Figs. 13 and 14.

Fig. 13. Linear regression of data from gearbox in good condition: a) linear regression line with data scattering, b) expected regression lines and its fluctuation caused by increase of number $i$ of data sets

Fig. 14. Linear regression of data from gearbox in bad condition: a) linear regression line with data scattering, b) expected regression lines and its fluctuation caused by increase of number $i$ of data sets

6.5. LOAD YIELDING/SUSCEPTIBILITY

CHARACTERISTIC IDENTIFICATION PRACTICAL SOLUTION

Practical start for LYCH is realised by a computer system analyser which graphic interface (GI) is presented in Fig. 15 where 60 seconds time vibration courses are
analysed. Where one can see shaft frequency fluctuation and distribution of instantaneous shaft frequency, time course of vibration acceleration signal, LYCH for many sets of data for gearbox in good condition (green dotes), and for bad condition (red dotes). Results for a particular set of data which 60 is marked as blue dots. Figure 15 shows also columns, which gives the numbers of accepted time segments and classification percent for the good or bad condition calcification.

After using the data rejection system regression analysis of data is done including convergent data analysis. The final analysis led to the regression equation — equivalent to load yielding/susceptibility characteristics (LYCH) for the object in good condition

\[ y = -0.12x + 120 \] (2)

for bad condition

\[ y = -0.52x + 550 \] (3)
86

b) Fig. 15. Graphic interface for system of data rejection

The object in bad condition was dismantling and it was found that in all rolling element bearings over limit backlash have occurred. The over limit backlash is the result of influence of tough industry environment (dustiness) in which the investigated objects were used. The tough industry environment caused frictional wear, which resulted in the bearing backlash increase.

6.6. CONCLUSIONS

The paper shows how the data mining analysis technique leads to new robust measure of machine condition, which may be used for condition monitoring of objects, which are subjected to the random varying load. The new robust measure is called load yielding/susceptibility characteristics (LYCH). The paper also shows that using factor analysis one can find some introductory directions for the ways of signal analysis. The condition prove was done. The object in bad condition was dismantling and it was found that in all rolling element bearings the over limit backlash have oc-
curred. The over limit backlash is the result of influence of tough industry environment in which the investigated objects were used.

LITERATURE

7. CYCLOSTATIONARY ANALYSIS.
PART 1. APPLICATION OF SPECTRAL CORRELATION TECHNIQUES ON MINING MACHINE SIGNALS:
IDENTIFICATION OF FAULTY COMPONENTS

WALTER BARTELMUS*, ROGER BOUSTANY**, RADOSŁAW ZIMROZ*, JEROME ANTONI**

In complex mechanical systems like a driving system used in mining belt conveyor transportation or multistage gearboxes used in driving bucket wheel excavators, the local fault detection in crucial elements like rolling-element bearings or gears is a very important and difficult task. For the detection of (mainly local) defects, envelope analysis was classically used. However, vibration signals measured on complex systems are characterised by a very rich frequency structure, a small level of energy of the fault signal produced by mechanical impacts, and a high level of interferences having both periodic and wideband structures. For these reasons, the informative fault part of a measured signal is drowned and masked by the interfering components both in time and frequency domains.

This work exploits the cyclostationary nature of the fault signatures resulting from the rotation of the machine parts. It uses the spectral correlation density function as a tool to detect and identify faulty components in machines.

Experiments carried on mining machine vibration signals enabled the detection of faults in rolling-element bearings, gears and planetary gearbox arms.

Keywords: mining machinery, gearboxes, rolling-element bearings, condition monitoring, vibration analysis, cyclostationarity

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* Wrocław University of Technology, Vibration and Diagnostic Scientific Laboratory, pl. Teatralny, 50-051 Wrocław, Poland.
** Laboratoire Roberval de Mécanique, Université de Technologie de Compiègne, Centre de Recherche de Royallieu, BP 20529 – 60205 Compiègne, France.
7.1. INTRODUCTION

One can divide factors influencing vibration signals into four groups [1], namely: design, production technology, operation, change of condition. The design and production technology factors can be considered as primary factors, operation and change of condition factors can be thought as secondary factors. Simplified division of factors is given in Fig. 1; the developed consideration is provided in [2]. The design factors can be described by macro-geometrical and micro-geometrical factors given by the structural forms, the admissible tolerances, the shape errors and the element surface condition. Detailed examination, using mathematical modelling and computer simulations, of the influence of the mentioned factors for gearbox cases is given in papers [1], [3], [4]. Results of the computer investigations are used for a diagnostic method development given in [5]. The results also show that vibration signals present a very complicated structure of frequency components. These components can be identified taking into consideration some design factors as number of gearbox teeth and the design factors depicted by the schemes given in Fig. 7. The vibration spectrum components show meshing frequencies and its multiples, shaft rotation components and its multiples, arm rotation components in a planetary gearbox. Considering the structural form of a gearing one should come to conclusion that the system is non-linear and at this case the frequency components can be modulated. The non-linearity comes from variable gearing stiffness. The amplitude modulation may be caused by gearing imperfections given by admissible tolerances, tooth shape errors, details are given in [1], [3]–[5]. The frequency components can be counted for a rated electric
motor rotation frequency but they can fluctuate as the result of operation factors, load and it cyclic variation as is in the case of bucked wheel excavators. Considering rolling-element bearings one can calculate inner, outer and ball frequency of local faults. These frequencies are generated as a result of bearing condition change. Referring to the paper [6] one can see the consideration of vibration signal generation caused by rolling-element bearings. Considering design and operation factors that have influence to the signals it can be concluded that bearing fault signals are tied to shaft speeds, but not strictly phase-locked because of variable slip between the bearing components. The latter can be explained by the fact that the kinematics formulae for the various fault frequencies, defined on design factors, contain a term $\cos \phi$, where $\phi$ is the load angle from the radial and thus affects rolling radius. Since the ratio of axial to radial load for individual rolling elements varies with their position in the bearing, in general, they are trying to roll at different speeds, and the cage forces them to maintain a uniform mean separation by causing slippage. This description gives an example how the design and operation factors influence to the vibration signals generated by rolling-element bearings. Considering gearboxes the characteristic frequencies are influenced by the operation factor as an external load variation which causes frequency fluctuation and the signal amplitude and frequency modulation if the external load is characterised by cyclic variability [7]. Using demodulation procedure one can identify the course of the external load for a bucked wheel excavator as it is given in [7]. In [8] is presented another cause which shows simultaneously influence of condition change of rolling-element bearings caused by frictional wear and influence of external load to vibration signal. The frictional wear is caused by impurities which get into lubricated oil during gearbox operation; it is environment dependent operation factor. Paper [2] shows that the diagnosed bevel gearbox is load susceptible, and its susceptibility increases with a gearbox condition change. The susceptibility can be caused by frictional wear of rolling-element bearings or by developed local bearing faults influencing gearing cooperation. Deeper consideration of influence of different factors is given in [2]. One can concluded that vibration component identification connected with condition change connected with rolling-element bearing faults is at this situation of vary complex systems is difficult. To solve the problem spectral correlation techniques are given in the paper.

7.2. DESCRIPTION OF THE SYSTEMS

Figure 2 shows general view of a driving system which is used for driving belt conveyors in lignite mines. The system directly drives a conveyor pulley. in and it consists of an electric engine, a flexible coupling, a two stage gearbox (bevel and cylindrical gears), and a pulley as depicted by Figs. 3 and 4. Characteristic signature frequencies for the gearbox driving the pulley are given for the described system are given in Table 1.
Fig. 2. Driving system view which scheme is given in Figs. 3 and 4

Fig. 3. Scheme of the driving system

Fig. 4. Scheme of gearbox for belt conveyor with rolling-element bearings identification number
Table 1. Characteristic frequencies for two stage gearbox

<table>
<thead>
<tr>
<th>Name of component</th>
<th>Frequency, ratio</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{01}$ input shaft frequency [Hz]:</td>
<td>$f_{01} = n_1/60$</td>
<td>16.5</td>
</tr>
<tr>
<td>$f_{02}$ second shaft frequency [Hz]:</td>
<td>$f_{02} = n_2/60 = n_1/(60*{u_1})$</td>
<td>4.08</td>
</tr>
<tr>
<td>$f_{03}$ output shaft frequency [Hz]:</td>
<td>$f_{03} = n_3/60 = n_1/(60*{u_1*{u_2}})$</td>
<td>1.3</td>
</tr>
<tr>
<td>$f_z$ first stage mesh frequency [Hz]:</td>
<td>$f_z = n_1*{z_1}/60$</td>
<td>379.5</td>
</tr>
<tr>
<td>$f_z$ second stage mesh frequency [Hz]:</td>
<td>$f_z = n_2*{z_2}/60$</td>
<td>146</td>
</tr>
<tr>
<td>$u_p$ transmission ratio</td>
<td>$u_p = {z_2*{z_1}}/{z_3}$</td>
<td>12.69</td>
</tr>
</tbody>
</table>

The pulley is supported by rolling-element bearings for which the local fault characteristic frequencies are:
- Outer race fault frequency $f_0 = 12.35$ Hz,
- Inner race fault frequency $f_i = 16.1$ Hz,
- Rolling element frequency $f_r = 8.9$ Hz.

Driving systems of bucket wheels are also considered.
The first driving system is illustrated by Fig. 5 together with its view Fig. 6. Figure 5 shows three independent subsystems that are driven by independent electric motors. The scheme of these subsystems is given in Fig. 7(a); Fig. 7(b) gives another solution where only one electric motor is used. In Fig. 7(a), the first stage forms a planetary gearbox with a standstill ring, gear $z_3$, the sun $z_1$ and a planetary gear $z_2$. As it may be seen from Fig. 7, one can distinguish two types of structure forms. Analyzing generally the structure of planetary gearbox one can state that it is a system of two degrees of freedom. In some design as in Fig. 7(a) one degree of freedom is reduced by fixing the gear $z_3$. Kinematical analysis of planetary gearbox gives the possibility of developing relations for finding characteristic frequencies. These are listed in Table 2(a) for the system Fig. 7(a) and in Table 2(b) for the system Fig. 7(b).

### Table 2. Gear characteristic frequencies

#### a) For the system given in Fig. 7(a)

<table>
<thead>
<tr>
<th>Component description</th>
<th>Value [Hz]</th>
<th>Comments</th>
</tr>
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<tbody>
<tr>
<td>$f_{12} = f_{23}$</td>
<td>435</td>
<td>Planetary gear meshing frequency</td>
</tr>
<tr>
<td>$f_2$</td>
<td>11.43</td>
<td>Rotation frequency of gear marked as $z_2$</td>
</tr>
<tr>
<td>$f_3$</td>
<td>4.67</td>
<td>Arm frequency</td>
</tr>
<tr>
<td>$f_{45}$</td>
<td>158.7</td>
<td>Other meshing frequencies of cylindrical gear stages</td>
</tr>
<tr>
<td>$f_{67}$</td>
<td>57</td>
<td></td>
</tr>
<tr>
<td>$f_{89}$</td>
<td>13.75</td>
<td></td>
</tr>
</tbody>
</table>

Values are calculated for the input shaft rotation speed $n_1 = 950$ RPM

#### b) For the system given in Fig. 7(b)

<table>
<thead>
<tr>
<th>Component description</th>
<th>Value [Hz]</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{12}$</td>
<td>287</td>
<td>Meshing frequency of bevel stage</td>
</tr>
<tr>
<td>$f_{34}$</td>
<td>99.9</td>
<td>Planetary gear meshing frequency</td>
</tr>
<tr>
<td>$f_{57}$</td>
<td>25.6</td>
<td>Other meshing frequencies of cylindrical gear stages</td>
</tr>
<tr>
<td>$f_{69}$</td>
<td>8.76</td>
<td></td>
</tr>
<tr>
<td>$f_j$</td>
<td>0.365</td>
<td>Arm frequency</td>
</tr>
<tr>
<td>$f_1$</td>
<td>15.95</td>
<td>Input shaft rotation frequency</td>
</tr>
<tr>
<td>$f_2$</td>
<td>4.7</td>
<td>Second shaft rotation frequency</td>
</tr>
<tr>
<td>$f_3$</td>
<td>1.6</td>
<td>Satellite rotation frequency</td>
</tr>
<tr>
<td>$f_4$</td>
<td>0.43</td>
<td>Gears $z_5$ and $z_6$ rotation frequency</td>
</tr>
<tr>
<td>$f_{31}$</td>
<td>17.42</td>
<td>Local fault frequency for gear $z_3$</td>
</tr>
<tr>
<td>$f_{41}$</td>
<td></td>
<td>Local fault frequency for gear $z_4$</td>
</tr>
<tr>
<td>$f_{51}$</td>
<td></td>
<td>Local fault frequency for gear $z_5$</td>
</tr>
<tr>
<td>$f_6$</td>
<td>0.82 or 0.91</td>
<td>Bucked frequency for 10 and 11 buckets</td>
</tr>
<tr>
<td>$n_b$</td>
<td>–</td>
<td>5.5 RPM</td>
</tr>
</tbody>
</table>

Values are calculated for the input shaft rotation speed $n_1 = 950$ RPM

For instance, $f_{12}$ denotes the meshing frequency of the pair of gears $z_1$ and $z_2$. Table 2 also specifies other characteristic frequencies which are needed for fault identi-
fications. Considering the described systems from the point of view factors influencing vibration diagnostic signals a belt conveyer driving the system may be considered as one which is loaded with the constant load or slowly varying external load. So for the given vibration signal with time length of a few seconds signal can be considered as stationary or cyclostationary. More sophisticated load conditions occur when one is considered operation factors connected with load variability for such driving systems which are used for driving bucket wheels. The varying load is fixed to the bucket cycle but in each cycle shape of load variability is random. Taking in mind other design factors which are connected with an electric motor characteristic and characteristic of hydro coupling the vibration signals are amplitude and frequency modulated. Many faults like: bearing faults, gear faults, perturbation faults of the planetary gearbox arm generate repeated impulse signals. These types of faults are called local faults. As it is given in [4], [5] and [7] the mentioned faults can be roughly identified by in time frequency representation. For more accurate frequency identification and for weak signals, the cyclostationary properties of these signals should be taken into consideration and more specifically the spectral correlation density function and the cyclic spectra.

Fig. 8. Example of a cyclostationary signal and its second-order statistical descriptors

7.3. THE SPECTRAL CORRELATION OF CYCLOSTATIONARY SIGNALS

Due the rotating parts in systems like those described in the previous section, vibration signals reveal some cyclical behaviour. The so-called property of cyclostationarity characterises non-stationary random signals exhibiting some hidden perio-
icity. This periodicity may be detected in the signal statistics. For instance, gear meshing generates first-order cyclostationary vibration signals [9], i.e. signals whose mean is periodic (e.g. a sinusoidal signal corrupted by an additive random noise).

Local faults in bearings or gears are likely to produce second-order cyclostationarity [9]–[11]. Figure 8(a) illustrates a particular case of a second-order cyclostationary signal $x(t)$, namely a periodically amplitude-modulated white noise (with the period denoted by $T$). Its auto-correlation function $R_x(t, \tau)$ – displayed in Fig. 8(b) – is time-periodic with a fundamental frequency $\alpha_1 = 1/T$ referred to as the cyclic frequency. This means that it admits a Fourier expansion over a set $A$ of harmonics $\alpha_k = k \times \alpha_1$ of $\alpha_1$, where $k \in N$ ($N$ – a set of natural numbers), such that

$$R_x(t, \tau) = \sum_{\alpha_k \in A} R^{\alpha_k}_x(\tau) e^{j2\pi \alpha_k t}$$  

The Fourier coefficients $R^{\alpha_k}_x(\tau)$ are referred to as the cyclic correlation functions.

The extra-information carried by cyclostationarity is also observed in the frequency domain through the double Fourier transform of (1.1) with respect to both lag $\tau$ and time $t$, known as the spectral correlation density function (SCD) (Fig. 7c)

$$S(\alpha, f) = \sum_{\alpha_k \in A} S^{\alpha_k}_x(f) \delta(\alpha - \alpha_k)$$  

where $\delta(\alpha)$ is the Dirac function and $S^{\alpha_k}_x(f)$ – the Fourier transform of $R^{\alpha_k}_x(\tau)$ with respect to the lag $\tau$ – is the cyclic spectrum.

Equation (1.3) means that the spectral correlation of a cyclostationary signal shows not only the power spectral density line positioned at $\alpha = 0$ classically used to describe stationary signals, but also a family of cyclic spectral lines located at $\alpha = \alpha_k$. These additional spectral lines also mean that two frequency components $X(f)$ and $X(f - \alpha)$ of a cyclostationary signal, spaced by $\alpha = \alpha_k$, are correlated. Reference [12] applies an advanced signal extraction technique making use of this correlation [10], [11], in order to recover the cyclostationary vibration signal linked to a mechanical fault.

The present paper will only focus on the use of the spectral correlation and the cyclic spectra as a tool for the detection and diagnostics of faults in mining machines.

7.4. APPLICATION TO MINING MACHINES – PULLEY’S BEARINGS

Figure 9 displays two different representations of the spectral correlation of a vibration signal measured on the system of Fig. 3: the plane view and the integrated spectral correlation (the integration is performed with respect to the $f$-frequency). It enables the identification of a fault in the rolling-element bearing supporting the pulley at the outer race fault frequency $f_o = 12.35$ Hz.
7.5. APPLICATION TO MINING MACHINES – 2 STAGE GEARBOX

Figure 10 shows a fault frequency of \( \approx 4 \) Hz connected with the second shaft (Figs. 3 and 4) rotation frequency as given by Table 1. The identified signature frequency of 4 Hz was not identified by using the time-frequency representation (the spectrogram) suggested in [5] for local fault detection like a spall, chipping, crack or breakage. These types of faults may occur in a gear \( z_2 \) or \( z_3 \). It is an example when SCD analysis gives results of fault identification when a fault generated signal is very weak compared to the signals generated by another sources of vibration.

A multi-fault problem may be encountered in mining machines. Signals linked to different faults at various stages of evolution or location may appear. Application of SCD based technique allows identifying number and properties of modulating
sources. As an example, the spectral correlation of a signal measured on the 2 stage gearbox (Figs. 3 and 4) with two types of faults is displayed in Fig. 11.

Figure 11 permits the simultaneous detection of two faults which are connected with first end second shaft as given in Table 1. Moreover, different types of modulated signals are present (4 Hz is a narrow band; 16.5 Hz is a wideband excitation). Wideband excitation suggests that the excitation is caused by the local fault like a spall, chipping, crack or breakage in a gear $z_1$ of the double stage gearbox, which scheme is given in Fig. 3. The 4 Hz component linked to the second shaft rotation frequency can be identified as a shaft misalignment caused by influence of rolling-element bearing frictional wear as result of environment factor influence.

7.6. APPLICATION TO MINING MACHINES – MULTISTAGE GEARBOX WITH PLANETARY STAGE

Figures 12 and 13 show two results on the use of the spectral correlation density for perturbed faulty run of a planetary gearbox arm (Fig. 7(a)). The fault frequency is according Table 2(a) 4.57 Hz. Fig. 12 shows SCD for a planetary gearbox in bad condition. Fig. 13 shows a gearbox in good condition as a planetary gearbox arm is considered.

Figure 14 displays the spectral correlation density map and integrated SCD for vibration signals received from a system given in Fig. 7(b), whose characteristic frequencies are given in Table 2(b). The bucket rotation frequency of 0.9 Hz and the satellite rotation frequency 1.6 Hz were identified together with 6 Hz frequency which is local fault for the bearing supporting the system of two rotated gears $z_5$ and $z_6$ with frequency of $f_4 = 0.43$ Hz.
Fig. 12. Spectral Correlation Density map and integrated SCD for planetary gearbox of Fig. 7a in a bad condition

Fig. 13. Spectral Correlation Density map and integrated SCD for planetary gearbox of Fig. 7a in a good condition

Fig. 14. Spectral Correlation Density map and integrated SCD for planetary gearbox in bad condition Fig. 7b
7.7. CONCLUSIONS

This work exploited the property of cyclostationarity characterizing the fault signatures for condition monitoring purposes of mining machines. It uses the spectral correlation density function as a tool to detect and identify faulty component.

Experiments carried on vibration signals enabled the detection of faults in rolling-element bearings, gears and planetary gearbox arms for giant lignite machinery systems.

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The extraction of the information linked to a fault in mechanical systems is a hard task since due to the presence of temporally and spectrally overlapping interferences.

In order to solve the problem, a novel cyclostationarity-based blind signal extraction (BSE) technique developed by Antoni and Boustany is applied using the information obtained from the spectral correlation analysis.

It is found that for some cases, the results obtained by OFB and BSE based techniques are nearly the same. However, mainly for complex multistage gearbox with planetary stage involving spur gears that generate high energetic signal in a wide frequency range, simple filtering does not work due to the mentioned overlapping effect. Application of BSE achieved a significant improvement in the quality of the separation.

**Keywords:** mining machinery, planetary gearboxes, rolling-element bearings, condition monitoring, vibration analysis, cyclostationarity, signal separation

8.1. INTRODUCTION

In a previous work [1], [2], a signal processing technique based on an optimizing procedure was proposed. It consists of band-pass filtration of a measured signal at an optimal frequency range – the range for which the signal-to-interference ratio is the highest. In Figure 1 there are seen two ranges of frequencies for which the kurtosis is high.

* Laboratoire Roberval de Mécanique, Université de Technologie de Compiègne, Centre de Recherche de Royallieu, BP 20529 – 60205 Compiègne, France.

** Wroclaw University of Technology, Vibration and Diagnostic Scientific Laboratory, pl. Teatralny, 50-051 Wroclaw, Poland.
Fig. 1. Distribution of kurtosis as a function of band-pass filter frequencies for optimal frequency band choice

Fig. 2. Original time signal and result of filtering at OFB for 2 stage gearbox with local fault on second stage

However, this method is limited to the case where the components to be separated have non-overlapping frequency structures. It has been found that for complex multistage gearbox with planetary stage involving spur gears that generate high energetic signal in a wide
frequency range, simple filtering does not work due to the mentioned overlapping effect. Application of BSE achieved a significant improvement in the quality of the separation.

8.2. ON THE USE OF THE EXTRA-INFORMATION CARRIED BY THE SPECTRAL CORRELATION

Based on the brief review of the theory of cyclostationarity presented in [3], this section will show through an example how the spectral correlation provides a useful

Fig. 3. Demonstration of the extra-information carried by cyclostationarity
extra-information for the extraction of the (second-order) cyclostationary part of a signal. Figure 3a) displays 5000 samples of a stationary random signal whose power spectral density is obtained at the location $\alpha = 0$ from the three-dimensional plot of the spectral correlation depicted by Fig. 3d). As predicted by the theory for a stationary signal, this is the only significant spectral line exhibited by the spectral correlation. Similarly, Figures 3c) and 3d) show respectively a periodically amplitude-modulated noise – a particular case of cyclostationary signals – and its spectral correlation. Contrary to stationary signals, the spectral correlation exhibits a few additional significant cyclic spectral lines located at $\alpha = \pm 1/T$ and $\alpha = \pm 2/T$ characterising this type of modulation. Assume now that we receive on a sensor the sum of signals (a) and (b), whose time-waveform and spectral correlation are respectively displayed by Fig. 3c) and 3f). If the objective is to extract the cyclostationary signal from the mixture (c), Fig. 3c) clearly shows that time-filtering is not satisfactory solution since the two signals overlap temporally. The power spectral density of signal (c), obtained from Fig. 1d) for $\alpha = 0$, also demonstrates that classical spectral bandpass filtering does not achieve an efficient separation since the two added signals also overlap spectrally. However, in order to extract the signal of interest, one may make use of its extra-information embodied in the spectral lines at $\alpha = \pm 1/T$ and $\alpha = \pm 2/T$. This property is obviously discriminative against the stationary signal (a) or any other additive contribution provided that its spectral correlation does not exhibit cyclic spectral lines at $\alpha = \pm 1/T$ and $\alpha = \pm 2/T$.

8.3. SIGNAL EXTRACTION TECHNIQUE BASED ON THE CYCLOSTATIONARY FEATURE

This section briefly explains the procedure allowing the extraction of a second-order cyclostationary signal using the spectral correlation property.

The method first assumes that any measured signal is a linear combination of an unknown (and usually large) number of unobserved vibration signals called sources.
Also, it is reasonable to neglect the dimensions of the physical sources compared to the vibration wavelengths implicated in the frequency band of interest. The sources may thus be considered punctual. Hence, any measured signal $X_j(f)$ at a location $j$, may be expressed in terms of the $M$ source signals, the frequency response functions (FRF), and the additive noise signal $B_j(f)$ on sensor $j$, as depicted by Fig. 3, such as:

$$X_j(f) = \sum_{i=1}^{M} H_{ij}(f)S_i(f) + B_j(f)$$

or in matrix form,

$$X(f) = H(f)S(f) + B(f)$$

where

- $X(f) = (X_1(f),...,X_j(f),...,X_N(f))^T$ is the $N$-dimensional response vector,
- $S(f) = (S_1(f),...,S_j(f),...,S_M(f))^T$ is the $M$-dimensional source vector,
- $H(f)$ is the FRF matrix, and
- $B(f) = (B_1(f),...,B_j(f),...,B_N(f))^T$ is the $N$-dimensional noise vector.

When a fault (e.g. a spalling on the outer race of a bearing) takes place, it will contribute to the measurement but will still be masked by the higher contributions of the gear meshing, blade passing, etc. The reduced-rank cyclic regression (RRCR) method [5] can cope with such a situation. It makes use of the spectral redundancy exhibited by a second-order cyclostationary vibration source (e.g. a spalling on a bearing race) in order to extract its time-waveform contribution. It does not require the knowledge of the number of sources contributing to the measurements. This is very convenient since complex mechanical systems usually implicate a high and an unknown number of sources. RRCR aims at recovering the contributions\(^1\) $X/S_1(f)$ of one cyclostationary source denoted by $S_1(f)$, without loss of generality, solely from the measured signals $X(f)$. The property of cyclostationarity is very advantageous for being discriminative against all the stationary components contained in the mixture (e.g. the additive noise) and the quasi-periodical parts (e.g. the gear mesh contributions). Moreover, by tuning to the cyclic frequency of a specific cyclostationary source, the method also rejects any other cyclostationary contribution provided that the latter is characterised by other cyclic frequencies. The theoretical foundations of RRCR are developed in [4]. An interpretation of the method is discussed here and illustrated by Fig. 6. RRCR only requires measuring of at least one cyclic frequency $\alpha_i$ of the signal of interest $X/S_1(f)$. This can be performed for example by computing the spectral correlation of one sensor signal. An extended observation vector $R(f)$ is

\(^1\) It is meant by contribution of a source $S_i(f)$, the signal as it would be measured by the sensor if only source $S_i(f)$ was present.
then constructed using frequency-shifted versions of $X(f)$ such that $R(f) = (X(f - \alpha_1)^T, ..., X(f - k\alpha_1)^T, ..., X(f - K\alpha_1)^T$, where $K$ denotes the number of frequency shifts (i.e. the number of harmonics of $\alpha_i$). In [2], it was shown that the extracting filter $W(f)$ to be applied to $R(f)$ may be obtained by cascading a cyclic Wiener filter and a rank-one subspace projector [3]. The cyclic Wiener filter is first applied to $R(f)$ yielding a first estimation of the signal of interest $X/S_1(f)$ and improving significantly the signal-to-noise ratio. However, it does not provide satisfactory interference rejection. The rank-one subspace projector remedies to this problem since it insures that only the contribution of the cyclostationary source of interest is recovered at its output and thus increases considerably the signal-to-interference ratio.

![Fig. 5. block scheme of the RRCR method](image)

8.4. APPLICATION TO MINING MACHINES – PULLEY’S BEARINGS

Figure 6 shows the original signal received from the rolling element bearing which supporting a pulley in the system described in [3]. In Fig. 6 there is given a rough and an informative part of the signal after blind signal extraction. In the considered case the vibration signal is interrupted by the signal generated by a two stage gearbox meshing frequencies. The pulley is driven by the mentioned two stage gearbox, the gearbox and pulley consists one mechanical system.

![Fig. 6. Original signal and extracted informative signal (after separation) for outer race bearing fault](image)
8.5. APPLICATION TO MINING MACHINES – 2 STAGE GEARBOX

Figure 7 shows the original signal received from the two stage gearbox from the system described in [3]. In Fig. 7 there is given a rough and an informative part of the signal after blind signal extraction. The bearing condition informative signal is interrupted by signal generated by the gearing of the two stage gearbox.

![Figure 7](image)

Fig. 7. Original signal and extracted informative signal (after separation) for local fault on a gear of second stage of two stage gearbox

8.6. APPLICATION TO MINING MACHINES – MULTISTAGE GEARBOX WITH PLANETARY STAGE

Figure 8 shows the original signal received from the multistage stage gearbox with a planetary stage from the system described in [3]. In Fig. 8 there is given a rough and an informative part of the signal after blind signal extraction. In the rough signal there

![Figure 8](image)

Fig. 8. Original signal and extracted informative signal (after separation) – arm problem on planetary stage
many different components that come from meshing frequency components of the multistage gearbox. Fig. 8 shows that from the rough signal after the blind signal extraction gives the symptom of perturbed/improper run of an planetary gearbox arm. The period of the peaks in the extracted time signal is equivalent to the period of the planetary gearbox arm.

8.7. CONCLUSIONS

It has been shown that the blind signal extraction is a very effective techniques at signal extraction after signal identification done by the spectral correlation density given in [3], where SCD was used for amplitude modulation component identification for objects described in [3]. Application of BSE achieved a significant improvement in the quality of the extraction/separation.

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Fault detection and diagnosis in mechanical systems during time varying non-stationary operation is one of the most challenging issues. During over last two decades some authors have noticed that machines during normal operation work under non-stationary load/speed condition. Phenomena that occur in gearboxes cause that traditional diagnostic futures are load dependent. It was experimentally confirm by different authors that non-stationary operations affect the features, however most of them noticed smearing effect in spectrum only.

To understand phenomena we have decided to use models of system with gearboxes (fixed axis two stage gearbox and planetary gearbox). Based on experimental work done for gearboxes used in mining industry models of phenomena were proposed. We state that due to load variation shape of transmission error function is changing. We assume, for simplicity, linear dependency between load values and diagnostic features. As features we have used energy based parameters, i.e. RMS value of a signal or the sum amplitudes of spectral components. Diagnostic data, pairs of feature value-load indicator, were statistically analysed in two dimensional space.

Two models of load variation (one dedicated to conveyor belt system, another one referred to bucket wheel excavator – mechanical systems used in mining industry) have been used to assure comparability of simulation and experimental results.

Definition of the problem, phenomena in the geared system under non-stationary load/speed condition, models of load variation, models of change of condition proposed in the paper have been investigated for the first time in the literature. Importance of these results is crucial for condition monitoring. Usage of models allowed better understanding phenomena that have been noticed during vibration signals analysis captured from real machines

The main purpose of this paper – confirmation of our hypothesis we assumed based on experimental work (load variation influences transmission error function) has been fully achieved.

**Keywords:** planetary gearbox, modelling, varying load, distributed faults
9.1. INTRODUCTION

Fault detection and diagnosis in mechanical systems during operation is one of the most challenging issues. During over last two decades some authors have noticed that machines during normal operation work under non-stationary load/speed condition. The non-stationary operating conditions considered as both load and speed variation simultaneously influence diagnostic signal in crucial way. Performing diagnostics in such condition one must be aware that non-stationary, fluctuating speed will bury frequency structure of signal (frequency modulation) and in fact classical spectral based feature extraction will be useless due to smearing effect. Application of order analysis is suggested by many authors as an antidote for such circumstance however just a few scientist have considered that load variation influence amplitudes of signal i.e. amplitudes of components related to shaft, mesh, bearing etc. It will be shown in the paper that in our cases we have both speed and load variation and some basic conclusion regarding diagnostic relation value of the feature-operating condition will be formulated. First we will recall some work published in the field related to gearbox modelling and condition monitoring under non-stationary operating condition. As it was suggested by [1] (Bartelmus 2001) model based diagnostic inferring scheme is very useful so we have developed models of non-stationary operating condition recognised during experimental work done for mining industry.

Existing papers that may be found in literature [2]–[6] are mostly focused on dynamics of gear pair only. We state that in real word we can not isolate gear pair, their behaviour is influenced by other parts of mechanical system (i.e. other stages, external load, motor characteristic etc look for consideration in [1] and [7] (Bartelmus 2001, 2008)). We have extended models developed by [1] (Bartelmus 2001) (two stage fixed axis) and also by Chaari [8] (planetary gearbox) including into them non-stationary operations models and models of change of condition related to non-stationary operation. The main task of this paper is to propose model of condition change and to discover diagnostic relation condition of machine-operating condition-diagnostic feature for this model that will be similar to experimental results.

Initially we have found this relation experimentally, however to understand phenomena and confirm result we used model based approach. So in this paper we include both results obtained from models and we recall some experimental results for real objects diagnosed during normal operation [9] and [10] (Bartelmus 2009).

9.2. CONDITION MONITORING OF GEARBOX IN NON-STATIONARY OPERATION

Randall [11] (1982) was probably the first scientist discovered that cyclic load causes amplitude modulation. In fact, it was the basis for investigation done by other authors and it is the case for one of our machines [9]. The case that load variation is
systematic and can be modelled as a sine wave is not often in real world. For some machines like ships, mining machines, wind turbines etc we have quite complicated shape of load profile and its variation is large (from 20% to over loading). For bucket wheel excavator for example we have mixture of two effects – cyclic, approximately systematic load variation related to cyclic digging process and a random factor related to other factors like operator skills, environmental, geological impacts and so on.

However, for belt conveyor system load depends on a transported bulk material stream and it is completely random and varying slowly. So, the problem how to include time varying operation for non-cyclic load is opened.

The methodology how to discover the influence of load value to a diagnostic feature was proposed by Bartelmus [12] (1992, Machine Vibration). He claimed that relation between the load value and the value of acceleration of vibration signal at some range of frequency (100–3500 Hz) is linear. Signals were filtered by analogue band pass filters and then RMS value is used for shafts, meshes and bearings assessment. The example of vibration spectrum for a gearbox is presented in Fig. 1.

In fact this technology gives opportunity to classify data into 4 classes (A–D), where A – is good condition, D is bad condition. This was a base for a paper published by Bartelmus and Zimroz [9] that used susceptibility of a gearbox to load as a feature for condition monitoring. The conclusion from that paper is that machine in good condition is less sensitive to load changes than machine in bad condition. This was general conclusion and it required more advanced techniques for the condition–load–feature relation discovery.
To improve reliability of results and get more precise results in comparing to results given in [12] it is proposed the technology given in [10] using statistical analysis of data set – spectral feature calculated for signals captured under different values of load. Using regression analysis for pairs: \textit{sum of spectral components-value of load} it is obtained linear relation as it is suggested in [12]. Conclusion is the same – diagnostic relation becomes stronger if condition changes. In this paper we will use this approach to investigate phenomena using mathematical modelling and computer simulation.

9.3. CONSIDERING TWO STAGE GEARBOXES

9.3.1. BACKGROUND FOR MATHEMATICAL MODELLING AND COMPUTER SIMULATION – TWO STAGE GEARBOX

As it was mentioned before two gearbox models are used, one based on [1] for two stage gearboxes and second based on [8] for planetary gearboxes.

The system with two stage gearbox is given in Fig. 3 in which torsional vibration is considered.

If the model shown in Fig. 3 is to be used for computer simulations, equations of motion needs to be written as in [1]. The physical motion quantities shown in Fig. 3, expressing design and operation factors, are included in the equations. The details on factors influencing vibration signals one could found in [1] and [7]. The design and operation factors are represented by electric motor moment \( M_{e}(\phi) \) and external load moment \( M_{r} \). The system consists of: rotor inertia \( I_{r} \), first stage gear inertias \( I_{1p}, I_{2p} \), second stage gear inertias \( I_{3p}, I_{4p} \), driven machine inertia \( I_{m} \), gearing stiffness \( k_{z1}, k_{z2} \) and damping \( C_{z1}, C_{z2} \), internal forces \( F_{1}, F_{2} \), and damping internal forces \( F_{1t}, F_{2t} \), first shaft internal moments \( M_{1t}, M_{2t} \) \( (M_{1t} – a coupling damping moment) \), shaft stiffness \( k_{1} \),...
$k_2$, $k_3$ and second and third shaft internal moments $M_2$ and $M_3$. The design and operation factors are described by expressions (1)–(5).

$F_1 = k_{z1}(a_{ux1}, g_1)(\max(r_1 \varphi_2 - r_2 \varphi_3 - l_{a1} + E(a_{ux1}, a, e, r_a, r_e)), \min(r_1 \varphi_2 - r_2 \varphi_3 + l_{a1} + E(a_{ux1}, a, e, r_a, r_e), 0)))$ (1)

$F_2 = k_{z2}(a_{ux2}, g_2)(\max(r_2 \varphi_4 - r_3 \varphi_5 - l_{a2} + E(a_{ux2}, a, e, r_a, r_e)), \min(r_2 \varphi_4 - r_3 \varphi_5 + l_{a2} + E(a_{ux2}, a, e, r_a, r_e), 0)))$ (2)

$a_{ux1} = \frac{\varphi_2}{2\pi}$ (3)

$a_{ux2} = \frac{\varphi_4}{2\pi}$ (4)

In expressions (1) and (2), functions $k_{z1}(a_{ux1}, g_1)$ and $k_{z2}(a_{ux2}, g_2)$ have different $g$ values. Stiffness functions $k_{z1}(a_{ux1}, g_1)$ and $k_{z2}(a_{ux2}, g_2)$ and error functions $E(a_{ux1}, a, e, r_a, r_e)$ and $E(a_{ux2}, a, e, r_a, r_e)$ have different values of parameters $a$, $e$, $r_a$, $r_e$. Using the parameters one can describe imperfections in the form of dimension and shape deviations. The functions can also be used to describe distributed faults. Taking into account wheel eccentricities (shape deviations), error function $E(a_{ux}, a, e, r_a, r_e)$ should be replaced by
\[ E_{b1} = E(a_u, a, e, r_u, r_e) + b_1\sin(\varphi_2) + b_2\sin(\varphi_3) \]  
\[ E_{b2} = E(a_u, a, e, r_u, r_e) + b_1\sin(\varphi_4) + b_2\sin(\varphi_5) \]  
where:

- \( b_1, b_2, b_3, b_4 \) – wheel eccentricities [m],
- \( \varphi_2, \varphi_3, \varphi_4, \varphi_5 \) – rotation angles [rad],
- \( l_{u1}, l_{u2} \) – intertooth backlash.

Functions \( \text{max} \) and \( \text{min} \) in (1) and (2) are defined as follows:

- Minima \( \text{min}(a, b) \) for which if \( a < b \) then \( \text{min} = a \) else \( \text{min} = b \)
- Maxima \( \text{max}(a, b) \) for which if \( a > b \) then \( \text{max} = a \) else \( \text{min} = b \).

The angle corresponding to the meshing period is

\[ \alpha_p = \frac{2\pi}{z_1} \]  

where:

- \( \alpha_p \) – an angle corresponding to one meshing period, rad;
- \( z_1 \) or \( z_3 \) – the number of driving gear teeth.

The place of contact in terms of relative length (0–1) is defined as

\[ \frac{\varphi}{\alpha_p} = a_{ux} \]  

where:

- \( \varphi \) – the wheel’s complete angle of rotation [radians].
- \( a_{ux} \) – a variable in a range of 0–1 (the relative place of tooth contact).

The damping forces are defined as follow

\[ F_{1t} = C_z(r_1 \varphi_2 - r_2 \varphi_3) \]  
\[ F_{2t} = C_z(r_1 \varphi_4 - r_2 \varphi_5) \]

Damping coefficient \( C_z \) describes the lubricating oil’s damping properties.

In general, \( \varphi, \dot{\varphi}, \ddot{\varphi} \) stand respectively for rotation angle, angular velocity and angular acceleration; \( M_i(\varphi) \) is the electric motor driving moment characteristic; \( M_1, M_2, M_3 \) – internal moments transmitted by the shaft stiffness, depending on the operation factors; \( I_1, I_2 \) – moments of inertia for the electric motor and the driven machine, depending on the design factors. The other design and operation factors are as follows:

- \( M_{1t} \) – the coupling’s damping moment; \( C_1 \) – the coupling’s damping coefficient; \( F_1, F_{1t}, F_2, F_{2t} \) – respectively stiffness and damping inter-tooth forces; \( k_1, k_2, k_3 \) – shaft stiffness; \( M_{d11}, M_{d12}, M_{d21}, M_{d22} \) – inter-tooth moments of friction, \( M_{st11} = T_1 \rho_{11}; M_{st12} = T_1 \rho_{12}; M_{st21} = T_2 \rho_{21}; M_{st22} = T_2 \rho_{22} \), where \( T_1, T_2 \) are inter-tooth friction forces, \( \rho \) determines the place of action of the friction forces along the line of action; \( r_1, r_2, r_3, r_4 \) –
gear base radii. For a given pair of teeth the value of the error is random and it can be expressed as

\[ e(random) = [1 - r_e(1 - l_i)]e \]  \hspace{1cm} (9)

where:
- \( e \) – the maximum error value;
- \( r_e \) – an error scope coefficient, range (0–1);
- \( l_i \) – a random value, range (0–1).

Hence the value of \( e \) in error characteristic (error mode) \( E(a, e, r_e) \) is replaced by \( e(random) \). The value of \( a \) has a range of (0–1), and it indicates the position of the maximum error value on the line of action. It can also be randomised by the equation

\[ a(random) = [1 - r_a(1 - l_i)]a \]  \hspace{1cm} (10)

where:
- \( a \) – the relative place of the maximum error, range (0–1);
- \( r_a \) – a relative place scope coefficient, range (0–1);
- \( l_i \) – a random value, range (0–1).

Hence the value of \( a \) in error characteristic (error mode) \( E(a, e, r_a) \) is replaced by \( a(random) \).

The above background to gearbox modelling gives one an idea of the influence of the different design and operation factors on the vibration generated by gearboxes.

Fig. 2. a) Stiffness function for gearing, b) Error function for error mode \( E(0.5, 10, 0.3) \), c) Error function for error mode \( E(0.1, 10, 0.3) \)
During the gearbox operation in hash environment lubricating oil contaminate. The contamination course the frictional wear of gearbox parts. The uneven frictional wear of rolling elements bearings cause the misaligned teeth cooperation. This misaligned teeth cooperation is modelled by change of “a” parameter. The course of error function from 0.5 to 0.1 is given in Fig. 2b) and c).

In Figure 2 the value of the stiffness $k(p_{om}, g)$ and the value of the error function $E(a; e; r)$ are given as a function of time from 3.0 to 3.005 s and a time range is divided into ten equal parts as are the value of $k(p_{om}, g)$ for a range of 1.2–1.3 E9 N/m and the value of $E(a; e; r)$ for the range 0–0.00001 m. The considered factors having influence on generation on vibration signal is frequently called by general term the transmission error. The above consideration shows how details factors influence the transmission error.

9.3.2. COMPUTER SIMULATION – TWO STAGE GEARBOXES

As it mentioned, during the gearbox operation in hash environment results in lubricating oil contamination. The contamination cause the frictional wear of gearbox parts. The frictional wear of rolling elements bearings cause the misaligned teeth cooperation. This misaligned teeth cooperation is modelled by change of “a” parameter. The course of error function for 0.5 and 0.1 is given in Figs. 2b) and c).

Load is changing from 40% to 180%, equivalent to notation $L = 1$ to 1.8. The parameter “a” is changing from 0.5 to 0.1 and is the function of external load. If the gearbox is in good condition $a = 0.5$ but as the condition of the gearbox is changing the “a” parameter is dropping to 0.1 as a cause of misalignment under influence of the external load. As results of simulation acceleration signal has been obtained (Fig. 3).

According [10] the diagnostic feature of gearbox condition is the sum of amplitudes components from acceleration vibration spectrum (Fig. 4).

![Fig. 3. Results of simulation: acceleration signal for: a) $e = 20$ um, $L = 1.0$, b) $e = 30$ um, $L = 1.0$](image)
The results of computer simulations showing the time course of the acceleration signals are given in Fig. 3.

![Fig. 4. Spectra of simulated signals: a) $e = 20$ um, $L = 1.0$, b) $e = 30$ um, $L = 1.0$](image)

Fig. 4. Spectra of simulated signals: a) $e = 20$ um, $L = 1.0$, b) $e = 30$ um, $L = 1.0$

![Fig. 5. Influence of gearbox condition and load on diagnostic feature; relation for the gearbox in good condition when $a = 0.5$ (♦), relation for gearbox in bad condition when “$a$” is varying from 0.5 to 0.1 (■) under influence of varying load from $L = 0.5$ to 1.5](image)

Fig. 5. Influence of gearbox condition and load on diagnostic feature; relation for the gearbox in good condition when $a = 0.5$ (♦), relation for gearbox in bad condition when “$a$” is varying from 0.5 to 0.1 (■) under influence of varying load from $L = 0.5$ to 1.5.

Figure 5 shows influence of gearbox condition and load on diagnostic feature; relation for the gearbox in good condition when $a = 0.5$, relation for gearbox in bad condition when $a$ is varying from 0.5 to 0.1 under influence of varying load from $L = 0.5$ to 1.5.
One may notice that for good condition mentioned relation is linear and rather “weak” in comparison with relation for bad condition, that is non linear and for load > 0.9 of nominal load – much stronger (value of feature increase rapidly when load is increased).

Figure 6 gives improved relation: gearbox condition and load on the diagnostic feature. The improvement is obtained by changing the scope of load variation from \( L = 0.9 \) to 1.5.

Because the non-linear relation analysis is much more complicated so in previous works Bartelmus [6] and also Cempel [7] used assumption that relation is linear for some range of load, in this work we limited load range, too as is given in Fig. 6.

The simulation of influence value of “\( e \)” parameter in the error function \( E(a_{ui}, a, e, r_a, r_e) \) is presented in Fig. 7. As it is seen in Fig. 7 the increase “\( e \)” cause almost a parallel shift of the line depicting relation the diagnostic feature as a function of load. One would expect such result.

After prolonged gear cooperation in the condition of the teeth misalignment one may expect change of the gearbox condition as pitting or other form, which cause distributed faults, as the result of over loading and this cause increase of “\( e \)” value. This phenomena is presented in Fig. 7 where computer simulation shows the influence of “\( e \)” parameter on diagnostic feature. One may expect that simultaneous effect of influence of two parameters may results in no-linear relation; the load – diagnostic feature. Some tendency to the no linear relation has noticed when investigating a planetary gearbox which is in bad condition in paper [10].
9.4. CONSIDERING PLANETARY GEARBOX

9.4.1. BACKGROUND FOR MATHEMATICAL MODELLING AND COMPUTER SIMULATION – PLANETARY GEARBOX

A one-stage planetary gear train [8] with $N = 4$ planets is modelled as presented on Fig. 8. Sun ($s$), ring ($r$), carrier ($c$) and planets ($p$) are considered as rigid bodies. Bearings are modelled by linear springs. The gearmesh stiffness is modelled by linear springs acting on the lines of action. Each component has three degrees of freedom: two translations $u_i$, $v_j$ and one rotation $w_j$, with $w_j = \theta_j (j = c, r, s, 1, ..., 4)$. $r_j$ is the base radius for the gears and the radius of the planets rotation centre; Damping is not considered here, it will be introduced later as a modal viscous damping. Radial and tangential coordinates $u_p$, $v_p$ describe planet deflections; $e_{sp}$ represents respectively the transmission error on sun-planets gear-mesh and planets-ring gear-mesh induced by wear which is detailed later. Translations are measured with respect to the frame $(O, \hat{i}, \hat{j}, \hat{k})$ fixed to the carrier and rotating with a constant angular speed $\Omega_c$ in relation to a stationary reference frame. Circumferential planet positions are specified by the fixed angles $\phi_j$ measured relatively to the rotating frame with $\phi_1 = 0$.

Applying Lagrange formulation allow us to recover the equations of motions of the 21 degrees of freedom system.

Assembling the equations in matrix forms leads to the global equation of motion of the system:
The gearmesh stiffness $K(t)$ matrix is time varying. Generally a square waveform is adopted to express this variation [8]. It is can be written by:

$$K(t) = \bar{K} + K_c(t)$$  \hspace{1cm} (12)

$M$ is the mass matrix, $G$ is the gyroscopic matrix, $K_b$ is the bearings stiffness matrix and $K_\Omega$ is the centripetal stiffness matrix. $\Omega(t)$ represents the time varying external load. $F(t)$ is the overload induced by the errors. All these matrices are given in [8].

Parameters of the studied planetary gear train are presented on Table 1. The ring is fixed, input element is the sun and output element is the carrier. The input rotational speed in the sun is $N_s = 950$ rpm so that the gear-mesh frequency is $f_g = 435$ Hz. A constant input torque of 150 Nm is applied on the sun gear.
Table 1. Parameters of the planetary gear model

<table>
<thead>
<tr>
<th></th>
<th>Sun</th>
<th>Ring</th>
<th>Carrier</th>
<th>Planet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teeth number</td>
<td>39</td>
<td>93</td>
<td>–</td>
<td>27</td>
</tr>
<tr>
<td>Mass (Kg)</td>
<td>2.3</td>
<td>2.94</td>
<td>15</td>
<td>0.885</td>
</tr>
<tr>
<td>( Il/r^2 ) (Kg)</td>
<td>1.36</td>
<td>3.795</td>
<td>7.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Base radius (m)</td>
<td>0.073</td>
<td>0.174</td>
<td>–</td>
<td>0.05</td>
</tr>
<tr>
<td>Gearmesh stiffness (N/m)</td>
<td>( k_{sp} = k_{rp} = 2.10^9 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bearing stiffness (N/m)</td>
<td>( k_p = k_r = 10^8 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Torsional stiffness (N/m)</td>
<td>( k_{sw} = k_{cw} = 0 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pressure angle</td>
<td>( \alpha_s = \alpha_r = 20^\circ )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

9.4.2. COMPUTER SIMULATION
– PLANETARY GEARBOX DYNAMIC RESPONSE

– Healthy case Figure 10 shows the acceleration registered on the ring over one load cycle given in Fig. 9.

![Fig. 9. Cycles of load for planetary gearbox](image)

It is noticed in Fig. 10 that the vibration level increases as the load increase.
Fig. 10. Acceleration on the ring gear for the healthy case

– Defected case Figure 11 shows the acceleration registered on the ring for the de-
lected case.

Fig. 11. Acceleration on the ring gear for the defected case

The response Fig. 11 shows the same evolution with much increased vibration levels as the load increase.

This result is confirmed by the plot of the evolution of the RMS value for each load step given in Figure 12 for each case.
It is well observed that the slope of the evolution of the RMS value is quite different from the healthy case to the defected case. This shows that the defect amplify the vibration level.

9.5. EXPERIMENTAL RESULTS

For experimental supporting of results of computer simulations, measurements were made of vibration signals received from double stage gearboxes. In Figure 13 there are given results for three gearboxes. The diagnostic features are presented as a function of input shaft rotation RPM. In this case the diagnostic features are negatively correlated with rotation.
It comes directly from an electric motor characteristic given in Fig. 14 in which one see that with increase of the load/moment decrease the rotation (RPM) and vice verse with the increase of rotation the load/moment decrease.

![Electric motor characteristic](image)

Fig. 14. Electric motor characteristic

In Figure 15 there is given example where diagnostic features are depicted for two gearboxes one in good condition described by linear relation $y = -1.6479x + 937.02$ and second in bad condition described by linear relation $y = -5.5391x + 3142.5$.

In [10] there are given results show that with the change of planetary gearbox condition the diagnostic features increased (Fig. 13) so is shown here using computer simulations for a double stage gearbox.
9.6. CONCLUSIONS

Models of behaviour of gearbox under time varying load condition have been presented in the paper. Investigation were done for simple two stage fixed axis gearbox as well as complex planetary gearbox. Based on experimental work done for gearboxes used in mining industry models of phenomena were proposed. We state that due to load variation shape of transmission error function is changing. We assume, for simplicity, linear dependency between load values and diagnostic features. As features we have used energy based parameters, i.e. RMS value of signal or sum of amplitudes of spectral components.

Diagnostic data, pairs of feature value-load indicator, were statistically analysed in two dimensional space.

One of main purpose of this paper is to confirm our hypothesis we assumed based on experimental work. In order to do this we used models developed by Bartelmus (two-stage gearbox) and Chaari (planetary gearbox). It needs to be underlined that model of gear-pair is not sufficient so we have modelled mechanical systems with gearbox (motor, gearbox, machine used as load). Two models of load variation (one dedicated to conveyor belt system, another one referred to bucket wheel excavator – mechanical systems used in mining industry) have been used to assure comparability of simulation and experimental results.

Definition of the problem, phenomena in the geared system under time varying load change TVLC, models of load variation, models of change of condition proposed
in the paper have been investigated for the first time in the literature. Importance of these results is crucial for condition monitoring. Usage of models allowed better understanding phenomena that have been noticed during vibration signals analysis captured from real machines.

LITERATURE

10. IMPLEMENTATION OF PLANETARY GEARBOX CONDITION MONITORING

WOJCIECH SAWICKI, WALTER BARTELMUS, RADOSŁAW ZIMROZ

The chapter describes the concept of a condition monitoring system in the LabView environment. Vibration transducers type IEPF attached to a gearbox housing measure acceleration signals. The tacho signal is acquired directly from a control box or indirectly from a tacho probe MM0024 with the suitable electric adapter/supplier. The vibration and tacho-signals through suitable analogue modules are conditioned and transmitted to a portable computer. Unprocessed signals are recorded in a hard-disc. Current signal analysis can be processed or recorded signals can be acquired from a hard-disc.

**Keywords:** planetary gearbox, condition monitoring, implementation

10.1. MEASUREMENT APPLICATION

Measurement application using the chassis controllers is written in LabView 8.6 language. The application defines the number of measurement channels, sampling frequency, and recording time. The diagram of measurement application in LabView 8.6 language is presented in Fig. 1.

Fig. 1. Block diagram for measurement application
The simultaneous recording of signals is done for all defined channels.

### 10.2. ANALYSING APPLICATION

The modules of an analysing application are presented in Fig. 2. The first module is described earlier in the measurement application module.

![Diagram of Analysing Application Modules](image)

**Fig. 2. Analysing application modules**

The analysing application is prepared according to chapter 6 and [1]. First of all, a tacho-signal is used for finding the speed profile which is next segmented. One segment is equivalent the bucket cycle period. The idea of segmentation is given in
Fig. 3. Next FFT spectrum is estimated for each segment period. The mean value of rotation velocity for each segment is also estimated. For the estimated mean value of rotation, meshing frequency and its components for gear condition estimation are valued.

Fig. 3. Idea of signal segmentation: a) load time course, b) input shaft frequency course segmentation
Fig. 4. Short time spectrums for segmented signals

Fig. 5. Operator application panel
The analysing application is also written in LabView language but some modules as (cyklosWS.m, tachospeed.m) are written in Matlab. The Matlab script is used for data transmission between two counting environments.

In Figure 5 the operator application panel is presented. In the left window the time course of the signal is given. The window of a velocity profile (upper yellow window) and the velocity histogram for the velocity profile is presented. Lower two windows show diagnostic features as the function of operation condition. The windows show two cases one (left) for a bevel stage second for a planetary stage gearbox. Under the lower two windows the parameters of estimated straight lines as a slope and intercept together with mean square value (msw) are given. The block diagram for the analysing application is given in Fig. 6.

![Block Diagram for Analysing Application](image)

**Fig. 6. Block diagram for analysing application**

**LITERATURE**

[1] BARTELmus W., ZIMROZ R., A new features for monitoring the condition of gearboxes in non-stationary operating conditions, Mechanical Systems and Signal Processing (accepted for publication).
11. CONCLUSIONS

WALTER BARTELMUS

The book shows that taking into account factors having influence to vibration signals generated by planetary gearboxes leads to the development of effective diagnostic methods. The background of the diagnostic method is published in the prestige journal as is Mechanical System and Signal Processing. The published there papers titled: “Vibration condition monitoring of planetary gearboxes under varying external load” and A new feature for monitoring of gearboxes in non-stationery operating condition shows important brake through in gearbox vibration condition monitoring. The book shows that the new look for gearbox degradation process is presented. The main reason of its condition change/degradation is the increasing backlash in rolling elements bearings. The increased backlash is caused by frictional wear of the rolling elements bearings as a result of dustiness in mining environment. The most danger is fine silica particles, which cause intensive frictional wear.

Talking about future research some phenomena connected with gearbox condition change should be investigated on:

– Bevel gear behaviour and its potential of self-adaptation to increased rolling element bearing backlash.
– Possibilities of use the proposed diagnostic method for cylindrical gearboxes.
– Vibro-acoustic environment severity caused by increased backlash due to bad gear cooperation/meshing, which cause excitation of a gearbox housing and a supporting structure on which gearboxes are fixed and cause increased noise generated by gearboxes.