KEPLER AS A STATISTICIAN

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Summary: Drawing on my previous publications (see Bibliography), I describe Kepler's work on the mathematical treatment of observations and astrology. In particular, I investigate how he rejected the Ptolemaic system of the world and note that his astrology had the features of qualitative correlation.

Keywords: reformation of astronomy, astrology, qualitative correlation, minimax method, Monte Carlo method.

1. Mathematical treatment of observations

This is my main subject. Modern astronomers are not interested in it anymore, and even historians of astronomy are ignorant of it. William H. Donahue, who translated Kepler's great work [1609] into English and thus made an excellent contribution to the history of astronomy, did not comment on Kepler's treatment of observations. This, however, is just what I will do in this section; and I quote Kepler [1609] by only mentioning the page numbers of its translation.

1.1. The arithmetic mean

Kepler (p. 200) collected four astronomical observations of the right ascension of Mars and, without any explanation, remarked: *The mean, treating the observations impartially* (*medium ex aequo et bono*), *is …*

Actually [Eisenhart 1976, p. 356], Kepler had chosen a weighted arithmetic mean (and had to assign subjectively the weights). But the main point here is that his Latin expression had occurred in Cicero (*Pro A. Caecina oratio*, § 65) whom Kepler likely read. It connoted *rather than according to the letter of the law*. (I have found this connotation in a Russian textbook of the Latin language for student lawyers.)

So now we know that at the very beginning of the 17th century or, somewhat earlier, the arithmetic mean became *the letter of the law*.

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1.2. The Monte Carlo method

When adjusting observations, Kepler sometimes corrupted them by small arbitrary magnitudes. Thus (p. 334) one might hold suspect such licence since then we will be able to change whatever we do not like in the observations. He reasonably added that the changes ought to remain within the limits of observational precision, and he certainly had to take into account the properties of usual random errors: an approximate equality of those changes of both signs and a larger number of changes smaller in absolute value.

Actually, Kepler applied elements of the Monte Carlo method.

1.3. Reformation of astronomy

Now the main point, Kepler's rejection of the Ptolemaic system of the world (p. 286):

Since the divine benevolence has vouchsafed us Tycho Brahe, a most diligent observer, from whose observations the 8′ error in this Ptolemaic computations is shown, it is fitting that we … acknowledge and honour this benefit of God … They could not be ignored, these eight minutes alone will have led the way to the reformation of all the astronomy.

This passage has been quoted a thousand times, but no one has thought of investigating it. Two questions have to be answered: why was Kepler sure that the error of Tycho's observations was less than 8′; and how did he arrive at that estimate?

Kepler gave an indirect answer to the first question by stating that the error of his own observations was of the order of two or three minutes (pp. 215, 621 and 611). But the main question is the second one, and I ought to go into detail about the adjustment of observations.

Given, a system of n equations with k unknowns, $n > k$

$$a_i x + b_i y + ... + w_i = 0, \; i = 1, 2, …, n . \quad (1)$$

Here, the coefficients are provided by the appropriate theory and the free terms are the observations or their functions. The observations, and therefore the equations, are mutually physically independent (linear independence was not yet known) and systems (1) had no solutions. Astronomers (and geodesists) had to be satisfied by any set of numbers $\hat{x}, \hat{y}, ...$ approximately satisfying (1), i.e. such that the residual free terms, call them v_i, were small enough and more or less satisfying the properties of *usual* random errors, cf. § 1.2. In other words, an additional restriction had to be imposed on those residuals. One of those restrictions was the condition of least squares

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$$v_1^2 + v_2^2 + \ldots + v_n^2 = \min.$$

Petrov [1954], is apparently still the best investigation of the optimal properties of the method of least squares.

Other methods had been earlier introduced, and, among them, the minimax method or rather its elements since only Laplace offered an algorithm for applying it properly. This method meant that the v_i, maximal in absolute value, is minimal among all the possible "solutions" of system (1); in the period before Laplace, minimal only among some reasonable "solutions".

The method of minimax is not optimal in any sense but it answers an important question: if the derived maximal v_i is unacceptably large, then either the theory justifying the system (1) was wrong, or the observations (the w_i) were too bad.

I believe that Kepler had indeed applied elements of the minimax method to a system corresponding to the Ptolemaic system of the world and decided as stated above. This, however, was not enough! I also believe that Kepler had then repeated such calculations for the Copernican system and likely arrived at a maximal v_i of the order of 3′, see above the estimation of the precision of his own observations. He had not regrettably said anything about that likely second calculation, but in principle it can be repeated now.

Interestingly, the minimax method is tantamount to generalized least squares:

$$\lim\,(v_1^{2k} + v_2^{2k} + \ldots + v_n^{2k}) = \min,\ k \to \infty.$$

An important circumstance here is that in astronomy, systems of equations are not linear and not even algebraic, but they can be linearized. Suppose that such a system involves x^2 (a similar conclusion will apply, for example, to $\sin x$). It is then possible to solve any subsystem with an equal number of unknowns and equations. The value x_0 will be calculated and

$$x = x_0 + \Delta x$$

with a comparatively small $|\Delta x|$. Then

$$ax^2 = a(x_0 + \Delta x)^2 \approx ax_0^2 + 2\, x_0 \Delta x$$

and the system will be linear in Δx.

Kepler had to linearize his systems, otherwise he would have been obliged to obtain reasonable solutions by solving non-algebraic systems many times over.

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1.4. Systematic influences

Kepler [1634/1967, p. 142], formulated recommendations for observers of solar eclipses. Actually, he insisted that systematic influences ought to be excluded (as far as possible).

2. Other topics

2.1. Randomness

Kepler [1606/2006, p. 163], rejected it: *What is chance*? *An idol* … Nevertheless, he had to find room for randomness, see Sheynin [2014, § 2]. There also, in § 3, I have followed the subsequent views of Kant and Laplace likely borrowed from Kepler. See also § 3 below.

2.2. An embryo of the law of large numbers

An embryo of the law of large numbers. Kepler [1627] stated that the total weight of many coins (more precisely, the mean weight of a coin selected from them) is constant.

3. Astrology

From a modern point of view, astrology is a pseudoscience. There were, however, astrologers, scholars of the highest calibre included, who strove to discover connections between heaven and earth. They sincerely believed in the existence of such connections, the more so since heaven does influence earth; thus, ocean tides are occasioned by the sun and the moon.

Astrologers singled out the *aspects*, i. e. remarkable mutual positions of the sun, the moon and the planets visible by the naked eye. Without any criteria they somehow separated randomness and regularity, a problem which still remains a fundamental challenge for modern mathematics. Kepler [1601/1979, p. 97], *added* three aspects to those recognized by ancient astrologers, so he also participated in the solution of that perennial problem.

Ptolemy [1956, I 2 and I 3], believed that the influence of heaven was a tendency rather than a fatal drive, and I understand his astrology as qualitative correlation. Indeed, ancient science was qualitative, witness Hippocrates [1952, vol. 10, no. 44]:

Persons who are naturally very fat, are apt (!) *to die earlier than those who are slender*.

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Kepler contributed to this direction of astrology. He stated that the influence of heaven at the moment of his birth was only a tendency, and, what is more interesting, he [1610/1941, p. 217; 1619/1939, pp. 256, 263], introduced intermediate causes (climate, geographical location, political structure of the land etc.) which were able to corrupt the influence of heaven. This was another step towards qualitative correlation since correlation analysis involves the isolation of the essential factors and a decision about the other influences (to disregard them, or to take them somehow into consideration). On the other hand, such intermediate causes pave the way for deception by quacks.

Kepler [1619/1997, book 4, chapter 6], considered himself the founder of a scientific astrology based on tendencies, but, even disregarding ancient scholars, Tycho Brahe had forestalled him [Hellman 1970, p. 410].

Kepler was mostly interested in studying the general destiny of nations according to the tendency of the prevailing aspects. As a *Landschaftsmathematiker*, he also had to compile yearly astrological almanacs, see M. Casper, p. 22 of his commentary on Kepler's *Welt-Harmonik* (1619/1939). He was dissatisfied by them since, as he [1610/1941, p. 253] stated, ordinary men were only interested in impossible predictions about their lives, and he decided to abandon those compilations (but had to continue owing to financial difficulties).

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KEPLER JAKO STATYSTYK

Streszczenie: Wykorzystując swoje poprzednie publikacje zamieszczone w bibliografii, autor w artykule opisał pracę Keplera stanowiącą matematyczne ujęcie obserwacji i astrologii. Przede wszystkim praca poświęcona jest kwestii odrzucenia ptolemeuszowskiego systemu świata oraz korelacji cech jakościowych w astrologii.

Słowa kluczowe: reforma astronomii, astrologia, korelacja cech jakościowych, metoda Monte Carlo.