EKONOMETRIA ECONOMETRICS 4(54) • 2016

ISSN 1507-3866
e-ISSN 2449-9994

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EXPLORATORY FACTOR ANALYSIS IN THE MEASUREMENT OF THE COMPETENCIES OF OLDER PEOPLE

EKSPLORACYJNA ANALIZA CZYNNIKOWA W OCENIE KOMPETENCJI OSÓB STARSZYCH

DOI: 10.15611/ekt.2016.4.03
JEL Classification: J14, J24, C38

Summary: Competencies are a crucial factor of professional position and career development. The aim of this paper is the assessment of the competencies of people in late productive age using exploratory factor analysis. The second point is the critical review of the theory and practice on exploratory factor analysis. The empirical analysis is based on the Study of Human Capital data. The survey results confirm the necessity of the factor analysis in research in the area of human capital in the context of ageing. The constructed synthetic indicators allowed for a synthetic assessment of the competencies of Poles aged 50-59/64. The results of the conducted analysis confirm the large significance of all the 24 analysed competencies. The competencies of Poles aged 50-59/64 were decomposed into three groups: (1) soft competencies and physical fitness (2) computer skills and (3) availability and technical competencies.

Keywords: exploratory factor analysis, competencies, ageing.

Streszczenie: Kompetencje stanowią kluczowy czynnik determinujący pozycję zawodową i rozwój kariery. Celem pracy jest ocena kompetencji osób w późnej fazie wieku produkcyjnego z wykorzystaniem eksploracyjnej analizy czynnikowej. Kolejnym zadaniem jest krytyczny przegląd teoretycznych i praktycznych prac w zakresie eksploracyjnej analizy czynnikowej. Analizę empiryczną oparto na danych Bilansu Kapitału Ludzkiego. Wyniki badań potwierdzają użyteczność analizy czynnikowej w analizach kapitału ludzkiego w kontekście starzenia się populacji. Skonstruowane wskaźniki pozwalają na syntetyczną ocenę kompetencji Polaków w wieku 50-59/64 lata. Analizy wskazują także duże znaczenie wszystkich badanych 24 kompetencji. Kompetencje Polaków w wieku 50-59/64 lata mogą zostać zdekomponowane w trzy grupy: (1) kompetencje miękkie i sprawność fizyczna, (2) kompetencje informatyczne, (3) dyspozycyjność i kompetencje techniczne.

Słowa kluczowe: eksploracyjna analiza czynnikowa, kompetencje, starzenie się.

1. Introduction

The issue of extending working life is vital in the context of one of the most important challenges of contemporary developed economies, i.e. the ageing of societies. This phenomenon is usually perceived as a threat to public finances, the healthcare system, the social protection system and the stable functioning of business entities. However, the concept of active ageing, developed in the last 20 years, is taking root in our awareness and is orientated towards the best use of the potential of people nearing retirement and already retired. Most important are the different actions involving active ageing supposed to delay economic deactivation. The most important factor in this area is the human capital of older workers, including health, level of education and competencies. This paper focused on the competencies. The methods of measurement of competencies may be as varied as the definitions used to describe them. In the light of one of the first definitions [Boyatzis 1982], competence is the potential, existing in the human, leading to such a behaviour that helps to satisfy the requirements at a given post within the organisational environment, which in turn provides the desired results. Filipowicz [2004], defines competencies as the predispositions in the area of knowledge, skills, and attitudes that allow professional tasks at an appropriate level to be conducted. In this paper, as in the Study of Human Capital, competencies are defined similarly to the definition by Filipowicz – as the knowledge, skills, and attitudes associated with the performance of specific actions, independent of the mode in which they were acquired, and whether they have been corroborated with a validation procedure [Górniak et al. 2011]. Starting from the single competencies, in the paper was proposed synthetic measurement of them using exploratory factor analysis.

The conceptual and theoretical rationale for factor analysis (including principal components analysis) was provided by Pearson [1901] and Spearman [1904]. The practical application of this approach facilitated the research of Thurston [1945] and Lawley [1940]. Exploratory factor analysis (EFA) became a broadly applied statistical technique in the social and experimental sciences and – nowadays – in economics. In Poland there are also many publications on EFA, and many of them relate to social sciences (including economic) research. Factor analysis is a multivariate statistical procedure used especially to: (i) reduce the large number of variables into a smaller set of variables, (ii) establish the underlying dimensions between measured variables and latent constructs, and (iii) provide construct validity evidence of self-reporting scales [Sztemberg-Lewandowska 2008, p. 7]. In this paper the considerations were focused on the application of exploratory factor analysis (confirmatory factor analysis was omitted).

The aim of this paper is the assessment of competencies of people in late productive age. The empirical study was limited to the population of Poles in the later stage of productive age, i.e. women aged 50-59 and men aged 50-64. The theoretical framework for this elaboration was the critical review of theory and practice on exploratory factor analysis (with a focus on the most important aspects).

2. Exploratory factor analysis – literature overview

2.1. Factor analysis versus principal components analysis

There is no consensus as to whether PCA is better or worse than classical EFA. Some authors (see for example: [Snook, Gorsuch 1989; Costello, Osborne 2005]) place an emphasis on the severely restricted use of components analysis in favour of a "true" factor analysis method. Others point out that there is almost no difference between principal components and factor analysis, or that PCA is preferable (see for example: [Steiger 1990; Velicer, Jackson 1990]). The choice between factor analysis depends on the number of variables and the magnitude of the factor loadings [Rietveld, van Hout 1993, p. 268]. In the principal components analysis there is no assumption existing of hypothetical factors [Zeliaś 1980, pp. 6-17], the components are geometrical abstractions, in factor analysis they are conceptualized as "real world" entities such as depression, anxiety, and disturbed thought [Tucker, MacCallum 1997, pp. 50-52]. Additionally, in PCA, all of the observed variance is analyzed, while in factor analysis it is only the shared variances that are analyzed. In the framework EFA, maximum likelihood (MLFA) and principle axis factoring (PAF) methods are the most popular [Walesiak, Gatnar (eds.) 2009, p. 328; De Winter, Dodou 2012], Rousson and Gasser [2003], proposed another method – simple component analysis (SCA). It should be noted that principal components analysis is usually used in "typical" EFA – it is one of calculating techniques leading to the calculate of factor loadings. In this meaning PCA is the adaptation of the classic Hotelling's principle components analysis [Hotelling 1933] for factor analysis and in practice the most popular [Walesiak, Bąk 1997; Malarska 2005; Czopek 2013].

2.2. Adequacy of data set

The sample size is important for these methods, but there are varying opinions in this area. Tabachnick and Fidell [2007], suggest a sample with at least 300 cases, Cattell [1966] – 250, Hair et al. [1998] – 100, Sapnas, Zeller [2002] – even 50 cases. Most authors cite the work of Comrey and Lee [1992] – they recommend that a sample with 100 or less cases is poor, 200 is fair, 300 – good, 500 – very good and 1000 or more – excellent. Another set of recommendations taking into consideration also the number of variables (p) – the most restricted proposition of the ratio $N:p$ (STV ratio) is 20:1, but more popular are the ratios 5:1 or 10:1 [Costello, Osborne 2005]. MacCallum et al. [1999] proved that the necessary N is in fact highly dependent on several specific aspects of a given study. Most importantly is the level of communality (interpreted as the proportion of variation in that variable explained by factors). When communalities are high (over 0.6), the factors are well determined, and computations converge to a proper solution, then sample size could be small (below 100). Izquierdo, Olea and Abad [2014], pointed out that 100 or 200 subjects are usually sufficient if the communalities are higher than 0.5 and each factor is defined by a minimum 7 variables. Under the worst conditions large samples (over 500) are required. The size of loading is the next important issue. Generally speaking, the sample-to-population pattern fit was very good for the high (0.80) loading condition, moderate for the middle (0.60) loading condition, and very poor (0.40) for the low loading condition [Velicer, Fava 1998]. If components possess four or more variables with loadings above 0.60, the pattern may be interpreted whatever the sample size used. Similarly, a pattern composed of many variables per component (10 to 12), but low loadings (0.40) should be an accurate solution if $n < 150$. Guadagnoli and Velicer [1988, p. 274] recommend in such a situation samples of over 300 elements.

The variables used in the exploratory factor analysis should be continuous. Some researchers incorporate dichotomously scored variables into their studies [Mislevy 1986]. But, as highlighted by Krzanowski [1982] and Stevens [2002], most factor extraction methods require multivariate normality in variables distributions, which cannot be maintained when studies rely on dichotomously scored observations. On the other hand, most studies used the EFA based on the ordinal data, especially measured on the Likert scale. Is this the correct approach? Mathematically, when the variable measured on the ordinal scale, estimators of mean, standard deviation etc. are biased. But, in the light of the results of many Monte Carlo simulations (see for example: [Baker, Hardyck, Petrinovich 1966; Borgatta, Bohrnstedt 1980]), for "typical data" differences between ordinal and interval measurement scales are in practice irrelevant (however, Velleman and Wilkinson [1993], while generally agreeing with this opinion, recommend caution when using parametrical tests for such data). A substantial number of studies has focused on the robustness of factor analysis models with respect to non-normality induced by ordered categorical outcomes. In the circumstances with a sufficiently large number of response categories (minimum 5), the absence of skewness, and equal thresholds across items, it seems possible to obtain reasonable EFA results even for ordinal data [Labovitz 1967; Olsson 1979; Lubke, Muthén 2004; Muthén, Kaplan 1985; Carifio, Perla 2007]. Understanding an ordinal scale equal with interval scale indicates the assumption of the existence of a latent continuous variable. Each category on the Likert scale is considered as means of intervals. It is assumed that the higher rating the greater intensity of the variable, and that the distance between each category of the Likert scale is equal [Pleśniak 2009]. This approach also has its opponents (see for example: [Jöreskog 2002; Jamieson 2004]), who, as a counter argument say that the assumption of equal distances between the different categories of scale could not be fulfilled in practice. Agreeing with the need for caution during the procedure of the EFA with ordinal data, the Monte Carlo simulations do not confirm such serious restrictions on the use of variables on the Likert scale.

Another thing is the quality of data – in the sense of their adequacy to factor analysis. This is usually evaluated by the Kaiser-Meyer-Olkin (KMO) Measure of Sampling Adequacy and Bartlett's test of sphericity. The KMO index is recommended when the cases to variable ratio are less than 1:5. The KMO index ranges from 0 to 1, with 0.50 considered suitable for factor analysis (Kaiser [1974] had suggested that anything in the 0.90s was marvellous, in the 0.80s – meritorious, in the 0.70s – middling, in the 0.60s – mediocre, in the 0.50s – miserable and below 0.5 – unacceptable). Bartlett's test of sphericity should be significant ($p<0.05$) for factor analysis to be suitable.

2.3. Number of factors to be retained

The next phase of factor analysis is the choice of a number of factors. The most popular criteria are: (i) the Kaiser rule (the Guttman-Kaiser rule) – retain only those factors with an eigenvalue larger than 1, (ii) the Cattell criterion – make a scree-plot, and (iii) keeping the factors which, in total, account for about 70-80% of the variance [Panek 2009, pp. 181-182]. If the communalities are low, the extracted factors account for only a little part of the variance, and more factors might be retained. Additionally, if the factor structure is not "cleanest", few item cross-loadings or there are factors with fewer than three items, the Kaiser rule is not decisive [Costello, Osborne 2005].

2.4. Rotation

The goal of rotation is to simplify and clarify the data structure. The most popular method of rotation is varimax, which – as quartimax and equamax – is the orthogonal method of rotation (oblimin, quartimin and promax are obligue and allow the factor to correlate) [Panek 2009, pp. 204-209; Walesiak, Gatnar 2009, pp. 332-335; Hair et al. 1998, pp. 110-111]. It should be stressed that if the factors are truly uncorrelated, orthogonal and oblique rotation produce nearly identical results [Tabachnick, Fidell 2007, p. 646; Fabrigar et al. 1999]. As Gorsuch [1983, p. 205] emphasised, if the simple structure is clear, rotating with varimax or promax is recommended. In the opinion of Tabachnick and Fidell [2007, p. 636], because the differences cannot be resolved by appeal to objective criteria, arguments over the best solution sometimes become vociferous. The final choice among alternatives depends on the researcher's assessment of its interpretability and scientific utility.

2.5. Final steps

On the basis of loadings values we could indicate the variables with practical importance – it should be above ±0.5. Using orthogonal rotation we could indicate that if loading equals 0.6, the factor explains 36% of variance of this variable (this criterion needs a sample above 100) [Walesiak, Gatnar 2009, p. 336]. The analysis could be finished off with an interpretation of the factors (using loading for all variables). But the last step could be also the redefined procedures. The resulting factor scores are linear combinations of the observed variables which consider what is shared between the item and the factor (i.e. shared variance) and what is not measured (i.e. the uniqueness or error term variance) [Gorsuch 1983]. Using these methods we may obtain new variables which, in synthetic terms, measure the level of each factors. The most popular are the regression, the Bartlett method and the Anderson-Rubin method [DiStefano, Zhu, Mindrilă 2009]. Computing regression scores and multiple regression were used to estimate (predict) the factor scores. This procedure maximizes the validity of the estimates. The Bartlett method produces factor similar scores that are most likely to represent the true factor scores. The scores may be correlated even when the factors are orthogonal, but the procedure produces high validity estimates between the factor scores and factor. The method proposed by Anderson and Rubin [1956], is a variation of the Bartlett procedure in which the least squares formula is adjusted to produce factor scores that are not only uncorrelated with other factors, but also uncorrelated with each other. The resulting factor scores are orthogonal, with a mean of 0 and a standard deviation of 1.

3. Empirical results

3.1. Data and methods

The empirical analysis was performed on the basis of the data provided by Bilans Kapitału Ludzkiego (BKL, The Study of Human Capital) for 2013[1]. This multi-module study, performed annually in Poland in the period 2010-2014, aimed at a comprehensive diagnosis of human capital in Poland including, among others, the population of productive age. The study is representative for the whole population of Poles of productive age and for a cross section of sexes, age groups and voivodships. In all editions the study was conducted on a sample of 17,600 persons. In this paper the analysis was restricted to people aged 50-59/64 (further called 50+). The upper limit is related to the productive age, the lower was established according to the meaning of "older age" which is commonly used in Poland in context of the labour market. In 2013, people aged 50+ constituted ca. 28% of the sample (n = 4,999).

The measurement of competencies was determined by the methodology used in the in BKL study (2013). They are measured on the Likert scale through the self-assessment of the respondent (especially because of the idea to perform a holistic

[1] The study is conducted as part of a system project of the Polish Agency for Enterprise Development and the Jagiellonian University, co-financed by the EU. All databases of the BKL study are accessible to the public on the website http://bkl.parp.gov.pl/dane. The methodology was elaborated by the research group headed by Prof. J. Górniak. Detailed information is available on ttp://bkl.parp.gov.pl.

diagnosis of supply and demand for competencies on the labour market)[2]. The question was: "Now I'm going to read a list of different skills to you. For each of them, I will ask you to assess the level of your skill in this area on a 5-point scale, where: 1 denotes very low level, 2 – basic, 3 – medium, 4 – high, and 5 – very high". Respondents were asked about their self-assessment of 12 competencies and for 7 of them – their sub-dimensions (20 items). Finally, in this paper 24 variables listed in Table 1 were used (these are all the variables available on the lowest level of measurement in the BKL study). This means that in this study, skills which could be certified (as specific professional competencies, foreign language skills, certified by driver license etc.) were omitted (the method of their measurement was different, so because of difficulties with comparability, the analysis was limited to the 24 indicated variables).

The starting point of the analysis was the self-assessment of competencies. Then on the basis of a factor analysis: (i) a group of the most important competencies was distinguished, (ii) competencies were grouped in the homogenous classes, (iii) synthetic indicators of competencies were calculated. In all the calculations the standard level of significance ($\alpha = 0.05$) was adopted. The calculation was made in SPSS 22.0.

3.2. Results

In the light of BKL, in general, people aged 50+ assess their soft competencies as better than their hard competencies. The highest mark was given to interpersonal competencies – all skills included in this competency were assessed similarly: almost 90% consider them as at least "average", and almost 2/3 – at least "good". The median is highest for interpersonal and self-organisation competencies. People aged 50+ also highly assess their availability, cognitive and managerial competencies, especially flexible work hours and logical thinking, analysis of facts and – in contrast with common stereotypes – continuous learning of new things. The results of BKL confirm that the computer competencies of older Poles (still of productive age) remain low – only under half of people aged 50+ consider them as at least average and only 1/5 mark them as good or very good.

An exploratory factor analysis was conducted using the principal component analysis as the method of factors extraction, based on the correlation matrix (with 1 on the diagonal). Because of using the PCA, the orthogonal methods of rotation turned out to be optimal. In this study an important issue was the construction of a synthetic indicator (so the optimum structure of all factors was significant), so the Quartimax rotation was used. In this study the *STV* ratio is very high – about 200:1. All the variables are correlated positive and statistically significant. The correlation

[2] In the starting waves the respondents were asked two questions about skills and attitudes, but because of the very high correlation between these two components of competencies, in the last waves the second question was omitted. For a detailed explanation see: [Górniak et al. 2011, pp. 34-38].

is moderate – for most pairs of variables the coefficient of correlation (Spearman's rho) equals about 0.4-0.5; the relatively lowest correlated with other variables are the three competencies: "knowledge of specialist software, ability to write applications and author websites" "operating, assembling, and repairing devices" and "artistic and creative skills". Following Field [2000, p. 446], all the elements on the diagonal of anti-image matrix of covariances and correlations are greater than 0.5, the lowest value was marked for technical competency – 1.275, so significantly below the limit. It should be noted that the value of correlation matrix determinant is very low (close to 0). Consequently, the quality of data also is marvellous – *KMO* is close to 1, in Bartlett's sphericity test $p < 0.0001$. The Kaiser rule as well as the Cattell criterion indicate three factors among 24 competencies. For more than three factors communalities are admittedly a little higher but the solution is difficult to interpret factually and formally (the factors are ambiguous or/and the number of variables in the factors are very small). Additionally, for three factors the level of mismatching measured by the matrix of reproduced correlation coefficients is low (only for two variables these values are slightly higher than 0.10).

It should be noted that for some variables communalities are relatively low – they range from 0.352 (for "artistic and creative skills") to 0.732 (for competency "ease in establishing contacts with colleagues and/or clients"). This could be caused by the generally low significance of artistic skills for the position of people aged 50+ on the labour market. Analogous conclusions could be drawn for advanced computer and mathematical competencies.

The conducted analysis identified three components (Table 1), explaining in total the 60% variation of the latent variable (competencies of Poles aged 50+), with the first factor (component) making up 46%, the second – 8.5%, and the third – 5%. The first identified group encompasses most of the analyzed competencies, first and foremost the soft competencies, as well as artistic competencies, organising and running office work and physical fitness. They form the main component of competencies of Poles value 1 for aged 50+, and therefore may be considered the essential ones for the assessment of the human capital of members of this age group.

Most loadings are for the adequate factor over 0.5 (only for two competencies – artistic and technical – they are close to this value: 0.464 and 0.455), for most of the items they are over or close to 0.7. It should be noted that factors with the highest factor loading, in which there are soft competencies, especially creativity, entrepreneurship and showing initiative (with loadings above 0.8), as well as being communicative and independent making of decisions (with loadings close to 0.8), are the most significant. The first factor explains over 60-65% of the variance of these variables. The second factor is "computer skills", and the third is less homogenous, encompassing availability and technical competencies. It is worth noting that the values of all the factor loadings are considerably high, which confirms the large significance of all the analyzed competencies. Relatively least important are the less universal competencies, that is artistic and technical ones.

Table 1. Distribution of competencies of people aged 50+ among the sub-scales

Competencies	*Me*	*W1*	*W2*	*W3*	*C*
Creativity	3.00	**0.813**	0.045	0.041	*0.664*
Entrepreneurship and showing initiative	3.00	**0.811**	0.006	0.051	*0.660*
Being communicative and sharing ideas clearly	4.00	**0.786**	–0.327	–0.015	*0.724*
Independent making of decisions	4.00	**0.786**	–0.165	0.024	*0.645*
Ease in establishing contacts with colleagues and/or clients	4.00	**0.773**	–0.365	0.026	*0.732*
Coordination of work of other staff	3.00	**0.756**	0.246	0.028	*0.632*
Logical thinking, analysis of facts	3.00	**0.749**	0.050	–0.109	*0.575*
Timely completion of planned actions	4.00	**0.747**	–0.280	0.066	*0.640*
Cooperation within the group	4.00	**0.742**	–0.379	0.047	*0.697*
Solving conflicts between people	3.00	**0.735**	0.019	0.032	*0.541*
Disciplining other staff – taking them to task	3.00	**0.732**	0.228	0.057	*0.590*
Continuous learning of new things	3.00	**0.732**	0.101	0.047	*0.548*
Performing simple calculations	3.00	**0.730**	0.098	–0.099	*0.552*
Quick summarising of large volumes of text	3.00	**0.698**	0.299	–0.125	*0.592*
Organising and running office work	3.00	**0.670**	0.405	–0.132	*0.631*
Resilience to stress	3.00	**0.664**	–0.067	0.225	*0.496*
Performing advanced mathematical computations	2.00	**0.565**	0.502	–0.021	*0.572*
Physical fitness	3.00	**0.508**	–0.041	0.425	*0.440*
Artistic and creative skills	2.00	**0.464**	0.370	–0.006	*0.352*
Knowledge of specialist software, ability to write applications and author websites	1.00	0.345	**0.702**	0.123	*0.627*
Basic knowledge of MS Office-type package	2.00	0.569	**0.587**	0.037	*0.670*
Readiness to travel frequently	3.00	0.499	0.089	**0.678**	*0.717*
Flexible working hours (no fixed slots)	3.00	0.532	–0.041	**0.622**	*0.672*
Operating, assembling, and repairing devices	3.00	0.361	0.085	**0.445**	*0.336*
% of total variance explained by factors		46.1	8.5	5.0	x

Me – median; *KMO* = 0.958; Bartlett's sphericity test: $p < 0.0001$*; in the table factor loadings are presented; *C* – communalities.

Source: own calculations on the basis on the BKL 2013 data.

In the next stage, the values of synthetic indicators *W1* (soft competencies and physical fitness), *W2* (computers skills) and *W3* (availability and technical competencies) were computed using the values of factor loadings, with the use of the Anderson-Rubin method. Comparing the synthetically measured competencies of people aged 50+ in these three groups we can notice that:

- women achieved a higher level of *W1* and *W2*, and lower – *W3*; this means that their soft, physical and computer competencies are higher than for men, but women have lower availability and technical competencies;
- people aged 50-54 have higher soft, physical and computer competencies than older groups; availability and technical competencies are the highest for people aged 50-54, the lowest – for aged 55-59;
- comparing people aged 50+ we can notice that soft and physical competencies (*W1*) as well as computer competencies (*W2*) are lowest for rural residents, and the highest – for residents of the biggest cities (over 500 thousands); other results are obtained for availability and technical competencies (*W3*) – they are lowest for residents of medium cities (50-99 thousand) as well as rural areas and the biggest cities;
- the higher the level of education the higher the soft and physical as well as computer competencies, but *W3* are highest for people with vocational education and lowest for people with higher education;
- in comparison with unemployed and economic inactive, working people aged 50+ have the highest all competencies; the lowest *W1* can be noted for economically inactive persons, *W2* and *W3* – for the unemployed.

We should remember that in the population there are working people as well as unemployed and inactive ones, but they are still of productive age. This could explain the results for computer competencies – higher for women than men.

4. Conclusions

Exploratory factor analysis is one of the multivariate statistical methods with wide use in the area of economic and social research. The selection of their appropriate procedures is not without impact on the final results of the analysis. The approach used by most researchers, i.e. use the method of principal components with varimax rotation is not always the best solution – especially when the sample is relatively small and the factors (or components) are correlated, the results may be incorrect. We should remember that if the factors or components are correlated, oblique rotation methods should be used. There was no such problem in the study presented in this paper. Additionally, it should be noted that in most research (including this study) principle components analysis turned out to be the best method of factor extraction, so its popularity is not surprising.

Summarizing the results from the conducted study confirms the necessity of the factor analysis in research in the area of competencies in the context of ageing. The constructed synthetic indicators allowed for a synthetic assessment of the competencies of Poles aged 50-59/64. The results of the conducted analysis confirm the large significance of all the analysed competencies. The conducted analysis allows to indicate that the competencies of Poles aged 50-59/64 should be decomposed into three groups: (1) soft competencies and physical fitness (2) computer skills and (3) availability and technical competencies. Loadings for each component were used for the calculation of the values of new variables – the synthetic measures of competencies of Poles aged 50+. Constructing variables could be used in the regressions models, for example as one of the independent variables describing extending working life, salaries, productivity, etc.

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