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## IS AN AVERAGE POLISH STUDENT PROFICIENT IN SOLVING DIFFICULT AND NEW PROBLEMS?

**Marek Biernacki, Andrzej Misztal**

**Abstract.** Using selected assignments from admission tests for the Wrocław University of Economics during 1993-2004 and the results of the examinations in mathematics, the authors show the need to implement the teaching of problems. The paper includes the solutions to problems from admission tests with comments regarding the most typical mistakes made by students. Our findings agree with the results of the PISA 2012 results in Creative Problem Solving, where Polish 15-year-old students scored below the median, ranked 27<sup>th</sup> among 40 countries, in spite of being ranked 8<sup>th</sup> in mathematics among 34 OECD countries.

**Keywords:** admission test and course assignments in mathematics, effectiveness of math education, PISA – Programme for International Student Assessment.

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### 1. Introduction

The transformation of the Polish education system is reflected in the results Polish students obtain in the Programme for International Student Assessment (PISA). The analysis of the results should help to increase schools' effectiveness and consequently ensure the optimal human capital growth which is the basis of a modern economy.

In every edition of the PISA exam, the results of the Polish students have been improving. There were 27 OECD countries which took part in the reading for the PISA exam in 2000. Among these countries Poland was ranked 22<sup>nd</sup>. In 2009, Polish students were much better and were ranked 13<sup>th</sup>. In the most recent edition in 2012 the success of Polish students was spectacular. They were ranked 5<sup>th</sup> among all 34 OECD countries. The data are presented in Table 1.

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Table 1. Rank of Poland among all participating OECD countries in the subsequent editions of the PISA test when comparing mean scores

Year	Rank of Poland in reading	Rank of Poland in science	Rank of Poland in mathematics
2000	22	–	–
2003	13	–	21
2006	8	19	21
2009	13	13	19
2012	5	5	8

Source: own study based on the OECD data [Biernacki, Czesak 2013].

This success has been strongly publicized in the Polish media and by the Polish government. But no one mentioned another aspect of the PISA survey, which is the Creative Problem Solving test. “The problem-solving assessment in PISA 2012 focuses on students’ general reasoning skills, their ability to regulate problem-solving processes, and their willingness to do so, by confronting students with problems that do not require expert knowledge to solve” [OECD 2014b, p. 28]. There were 40 countries participating in the Creative Problem Solving test. Taking into account the mean score of Polish students (481), they were ranked 27<sup>th</sup>, i.e. their result was much worse than the average of all the OECD countries.

An analysis of the differences between the performance of pupils in computer-based problem solving, and their expected achievements, estimated from the results of tests in mathematics, in reading with interpretation, and in science, established in PISA traditional written tests indicates that Polish pupils ranked the 3<sup>rd</sup> worst, with a negative score difference (–44) which was a disappointing surprise to us.

In 1996, i.e. three years before the reform of education in Poland, the World Bank suggested recommendations which were supposed to support the education in countries of the Eastern bloc so as to adjust the process of education according to the requirements of a market economy:

1. Preserve the old system of general education.
2. Strengthen the ability to solve difficult and new problems.
3. Foster attitudes towards innovativeness.
4. Teach how to take responsibility for oneself.

In 2009 the Council of the European Union adopted a strategic framework for European cooperation in educating and training, addressing the following four strategic objectives:

1. Making lifelong learning and mobility a reality.
2. Improving the quality and efficiency of education and training.
3. Promoting equality, social cohesion and active citizenship.
4. Enhancing creativity and innovation, including entrepreneurship, at all levels of education and training.

Problem-solving skills (where a method of the solution is not immediately obvious) are regarded as a key competence in all courses of study. College graduates in economics will operate in a very unstable environment and therefore they should acquire those skills to some degree. In the following sections, we will present example problems from the admission tests and from course examinations at the Academy (now University) of Economics in Wrocław.

## **2. Selected assignments from admission tests to the University of Economics, 1993-2004**

Between 1993 and 2004, the numbers of candidates eager to enroll were several times greater than our teaching capacities would allow, therefore entrance examinations were conducted. At least one out of five examination tasks was a problem assignment, requiring an unconventional approach. A few tasks of this type with solutions and comments are given below.

**1.** (1993) Two workers need twelve days to perform a certain task. After eight days of the joint work, one worker completed the remainder in seven days. How many days does each of them need to carry out the task?

**Solution:** We assume that the 1<sup>st</sup> worker needs  $x$  days to perform the entire work, while the 2<sup>nd</sup> employee needs  $y$  days. Hence, within one day the 1<sup>st</sup> employee carries out  $1/x$  of the whole work and the 2<sup>nd</sup> employee  $1/y$  of the whole work. Thus, we get the set of equations:

$$\begin{cases} 12 \cdot \frac{1}{x} + 12 \cdot \frac{1}{y} = 1 \\ 8 \cdot \left( \frac{1}{x} + \frac{1}{y} \right) + 7 \cdot \frac{1}{x} = 1 \end{cases}.$$

Let  $a$  denote  $1/x$ , and  $b$  denote  $1/y$ . Then we obtain the set of linear equations:

$$\begin{cases} 12a + 12b = 1 \\ 15a + 8b = 1 \end{cases}.$$

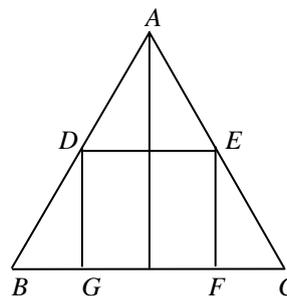
We easily compute that  $a = \frac{1}{21}, b = \frac{1}{28}$  and thus,  $x = 21, y = 28$ .

**Answer:** The 1<sup>st</sup> worker needs 21 days to perform the entire work, the 2<sup>nd</sup> worker – 28 days.

**Comment:** Adding  $x$  and  $y$ , instead of their reciprocals, was the most frequent mistake. Some students failed to comprehend that workers need less days to accomplish the task if they work together than if each works individually. Moreover, some students who obtained the correct set of equations did not use intermediate variables, and failed to achieve the final solution.

2. (1994) A cylinder with height  $h$  was inscribed into a cone with radius  $r$  and angle  $x$  between a generatrix and the base. What is the height of the cylinder with the greatest volume? Find this greatest volume.

**Solution:** Let  $H$  denote the height of the cone,  $R$  – its radius, and  $V$  – the volume of the cylinder. Joint axial cross section of both solid figures is represented by the rectangle  $DEFG$  inscribed to an isosceles triangle  $ABC$ . Then,  $BG = r - R, DG = h, \sphericalangle DBG = x$ , therefore:  $h = (r - R)\text{tg } x$ . Moreover,  $H = r \text{tg } x$  and  $V = \pi R^2 h$ . Hence,  $V = \pi R^2 (r - R)\text{tg } x = \pi \text{tg } x (rR^2 - R^3)$ . We treat  $V$  as a function of a positive variable  $R$  and compute its derivative:



$V' = \pi \text{tg } x (2rR - 3R^2) = \pi \text{tg } x R (2r - 3R)$ . Thus,  $V' = 0 \Leftrightarrow R = \frac{2r}{3}$ . Addi-

tionally, the sign of  $V'$  implies that for  $R \in \left(0, \frac{2r}{3}\right)$  function  $V$  is increasing,

and for  $R \in \left(\frac{2r}{3}, r\right)$  it is decreasing, that is,  $\max V = V\left(\frac{2r}{3}\right) = \frac{4}{27} \pi r^3 \text{tg } x$ .

Also,  $h = \frac{r}{3} \text{tg } x$ .

**Answer:**  $h = \frac{r}{3} \text{tg } x, \max V = \frac{4}{27} \pi r^3 \text{tg } x$ .

**Comment:** The solutions of the above task demonstrated students' incompetence when developing a system of equations for a word problem. Also curious errors emerged due to the presumption that  $x$  usually denotes a variable. As a consequence, the derivative of a function of the volume was calculated with respect to the constant  $x$  instead of the variable  $R$  or  $h$ . Another type of wrong answers was provided by students writing down any relevant equations concerning the triangle, e.g. the Pythagorean theorem and various trigonometric relations. Substituting the ones into the other yielded irrelevant identities.

3. (1996) Solve the set of equations:

$$\begin{cases} x + y + z = 2 & (1) \\ x^2 + y^2 + z^2 = 6 & (2) \\ x^3 + y^3 + z^3 = 8 & (3) \end{cases}$$

**Solution:** Squaring the equation (1) and subtracting the sides of (2) yields:  $2(xy + yz + zx) = -2$ , i.e. (4):  $xy + yz + zx = -1$ . Next, multiplying the sides of equations (1) and (2) and subtracting the sides of (3) yields:  $xy(x + y) + yz(y + z) + zx(z + x) = 4$ . By (1), the following equation is obtained:  $xy(2 - z) + yz(2 - x) + zx(2 - y) = 4$ , while leaving out brackets and rearranging  $2(xy + yz + zx) - 3xyz = 4$ , i.e. by (4), yields (5):  $xyz = -2$ . Equations (1), (4) and (5) are Vieta's formulas with respect to third degree equation implying that  $x, y, z$  are the only roots of an equation  $t^3 - 2t^2 - t + 2 = 0$ , which can be written in the form  $(t - 2)(t^2 - 1) = 0$ .

**Answer:**  $x = -1, y = 1, z = 2$  and any permutations of this solution.

**Comment:** Problem 3 was solved by a few students who were familiar with Vieta's formulas for third degree equation. A few persons worked out the problem without using Vieta's formulas, thus proving a general incompetence when dealing with systems of non-linear equations.

4. (1996) For what values of parameter  $a$  does the equation  $ax^2 - (a^2 + 3)x + 2 = 0$  have two real roots of different signs?

**Solution:** If  $a = 0$ , then  $x = \frac{2}{3}$  would be a single root. Therefore, we must assume: (1):  $a \neq 0$ ; (2):  $\Delta > 0$ ; (3):  $x_1x_2 < 0$ . By Vieta's formulas and (3),

we obtain  $\frac{2}{a} < 0$ , that is (4):  $a < 0$ . Because  $\Delta = (a^2 + 3)^2 - 8a$ , thus (4) results in  $\Delta > 0$ , that is, a condition (2) holds. Of course (4) also results in (1), that is, (4) is a necessary and sufficient condition.

**Answer:**  $a \in (-\infty, 0)$ .

**Comment:** The task seems standard, but attempts to solve the inequality (2) resulted in a polynomial of degree four with no rational roots, therefore its factoring is not easy. The vast majority of students failed to see that inequalities could be solved in a reverse order (3), (2), as shown above. Blindly accepting the school scheme, where the discriminant of the quadratic function is “most important”, is the most likely explanation.

5. (1998) Examine the number of solutions to equation (\*) with respect to the parameter  $a$ .

$$\log(-x^2 + x) - \log(x^2 - ax - 2) = 0. \quad (*)$$

**Solution:** We should assume: (1):  $-x^2 + x > 0$  and (2):  $x^2 - ax - 2 > 0$ . From (1) we obtain the condition (1'):  $x \in (0, 1)$ . The initial equation yields successively: (3):  $\log(-x^2 + x) = \log(x^2 - ax - 2)$ , and (4):  $-x^2 + x = x^2 - ax - 2$ . Conditions (1) and (4) imply that assumption (2) is redundant, that is, (1') is the only relevant assumption. From the equation (4) we obtain the condition (5):  $2x^2 - (a+1)x - 2 = 0$ . Let  $f(x) = 2x^2 - (a+1)x - 2$ . We compute the discriminant of this quadratic trinomial:  $\Delta = (a+1)^2 + 16$ . Since  $\Delta > 0$ , the equation (5) has two real roots  $x_1 < x_2$ . Moreover,  $f(0) = -2$ , so  $x_1 < 0 < x_2$ , thus  $x_1$  does not satisfy the assumption (1'). Additionally,  $f(1) = -(a+1)$ , so there are two cases:

- I)  $a \geq -1 \Rightarrow f(1) \leq 0 \Rightarrow x_2 \geq 1$ , which contradicts the assumption (1').
- II)  $a < -1 \Rightarrow f(1) > 0 \Rightarrow x_2 \in (0, 1)$ , so the assumption (1') is satisfied.

**Answer:** For  $a < -1$  equation (\*) has one solution, and for  $a \geq -1$  there are no solutions.

**Comment:** Many candidates correctly wrote down assumptions (1) and (2), but yet the majority of them treated equations (\*) and (5) as equivalent. So after noticing that  $\Delta > 0$ , they concluded that equation (\*) always has two solutions. Among those who properly formulated assumptions, only a few

students made evident that the condition (2), as resulting from the equality (4), becomes irrelevant.

**6.** (2000) Nine people are randomly seated at one of three tables. Calculate the probability that at least one table is not selected.

**Solution:** Let  $\Omega$  denote the space of elementary events and  $A$  – the event that at least one table is not selected. Generally, as each person out of nine people can be seated at any of three tables, we deal with a 9-element variation on a 3-element set, that is  $|\Omega| = 3^9$ . Let  $A_{23}$  denote the event that only table one is not selected by any person. Similarly we define the events  $A_{31}$  and  $A_{12}$ . Let  $A_1$  denote the event that all people are seated at table one. Similarly we define events  $A_2, A_3$ . Then obviously,  $|A_1| = |A_2| = |A_3| = 1$ . The event  $A_{23}$  represents the 9-element variation on the 2-element set, therefore,  $|A_{23}| = 2^9$ . However, we have to exclude the outcomes that either all nine persons are seated at table two or at table three. Hence the correct formula is:  $|A_{23}| = 2^9 - 2$ . Likewise,  $|A_{31}| = 2^9 - 2$  and  $|A_{12}| = 2^9 - 2$ . Eventually we get

$$|A| = |A_1| + |A_2| + |A_3| + |A_{12}| + |A_{23}| + |A_{31}| = 3 + 3(2^9 - 2) = 3(2^9 - 1).$$

$$\text{Hence, } P(A) = \frac{|A|}{|\Omega|} = \frac{2^9 - 1}{3^8}.$$

$$\text{Answer: } P(A) = \frac{|A|}{|\Omega|} = \frac{2^9 - 1}{3^8}.$$

**Comment:** One may joke that the number of various solutions was equal to the number of students. The number of correct solutions was very small. In addition to completely pointless answers, typical errors were made of mistaking input data, computing e.g. that  $|\Omega| = 9^3$ . Less significant errors were about ignoring the fact that a  $k$ -element variation on a  $n$ -element set includes a  $k$ -element variation on a  $(n-1)$ -element set.

**7.** (2002) Evaluate  $x = 2\sqrt{40\sqrt{12}} + 3\sqrt{5\sqrt{48}} - 2\sqrt[4]{75} - 4\sqrt{15\sqrt{27}}$ .

**Solution:** Let  $a, b, c, d$  and  $e$  denote the terms, respectively:  $a = 2\sqrt{40\sqrt{12}}, b = 3\sqrt{5\sqrt{48}}, c = 2\sqrt[4]{75}, d = 4\sqrt{15\sqrt{27}}, e = \sqrt{5\sqrt{3}}$ . Then, we compute the respective values:

$$a = 2\sqrt{40\sqrt{4}\cdot 3} = 2\sqrt{80\sqrt{3}} = 2\sqrt{16\cdot 5\sqrt{3}} = 8\sqrt{5\sqrt{3}} = 8e;$$

$$b = 3\sqrt{5\sqrt{16}\cdot 3} = 3\sqrt{20\sqrt{3}} = 3\sqrt{4\cdot 5\sqrt{3}} = 6e;$$

$$c = 2\sqrt[4]{75} = 2\sqrt{\sqrt{25}\cdot 3} = 2\sqrt{5\sqrt{3}} = 2e;$$

$$d = 4\sqrt{15\sqrt{27}} = 4\sqrt{15\sqrt{9}\cdot 3} = 4\sqrt{45\sqrt{3}} = 4\sqrt{9\cdot 5\sqrt{3}} = 12\sqrt{5\sqrt{3}} = 12e.$$

$$\text{So } x = 8e + 6e - 2e - 12e = 14e - 14e = 0.$$

**Answer:**  $x = 0$ .

**Comment:** The problem was actually trivial, and yet many students failed because they did not recognize the similarity of expressions  $a$ ,  $b$ ,  $c$  and  $d$ . Mathematical ingenuity as regards the handling of radical expressions has never been satisfactory. Students at high schools have been allowed to use calculators even for trivial operations, and consequently the reinforcement of their mathematical skills was poor (using calculators was forbidden during admission tests).

8. (2004) Solve the system of equations:

$$\begin{cases} |x| + 3y = -6 \\ 2x + |y| = -2 \end{cases}.$$

**Solution:** It follows from the given system that  $3y = -6 - |x|$ , and  $2x = -2 - |y|$ , so  $x < 0$ ,  $y < 0$ . By noticing this fact, the system becomes simpler:

$$\begin{cases} -x + 3y = -6 \\ 2x - y = -2 \end{cases},$$

Then we can easily compute:  $x = -2, 4$ , and  $y = -2, 8$ .

**Answer:**  $x = -2, 4$ , and  $y = -2, 8$ .

**Comment:** The vast majority of students failed to notice that  $x < 0$ ,  $y < 0$ . Therefore they considered all four possible cases and performed unnecessary operations. Some of those students made errors in calculation, and also mismatched assumptions and operations in individual cases.

9. (2004) Solve inequality:

$$\log_{x-7} \frac{x-6}{x-8} \geq 1.$$

**Solution:** We make the following assumptions: (1):  $x-7 > 0$ ; (2):  $x-7 \neq 1$ ; (3):  $\frac{x-6}{x-8} > 0$ . From (1) and (3) we get (4'):  $x-8 > 0$ , or equivalently (4):  $x-7 > 1$ . This condition replaces the assumptions (1)-(3). Moreover, if we write the given equation in the form  $\log_{x-7} \frac{x-6}{x-8} \geq \log_{x-7} (x-7)$ , then due to (4), we may leave out logarithms, receiving  $\frac{x-6}{x-8} \geq x-7$ . Applying (4'), we multiply the sides of this inequality by  $x-8$ . Then  $x-6 \geq (x-7)(x-8)$ , or after rearranging:

$$x^2 - 16x + 62 \leq 0.$$

After solving this inequality and considering the inequality (4), we obtain the answer.

**Answer:**  $x \in \left(8, 8 + \sqrt{2}\right]$ .

**Comment:** A considerable number of applicants accomplished the task of writing assumptions (1)-(3), but failed to apply them for solving the inequality in the form  $\log_{x-7} \frac{x-6}{x-8} \geq \log_{x-7} (x-7)$ . And so the two cases were considered:  $x-7 < 1$  and  $x-7 > 1$ , thus making unnecessarily more operations. However some applicants concluded from the logarithmic inequality that  $\frac{x-6}{x-8} \geq x-7$ , but they failed to demonstrate it. One may guess that if the assumptions implied that the base of the logarithm was less than 1, then some applicants would come up with the same inequality that was originally given. More errors and mistakes occurred when handling the inequality  $\frac{x-6}{x-8} \geq x-7$ . Some applicants multiplied its sides by  $x-8$  without asking whether  $x-8$  is positive. Another group unnecessarily attempted to determine the common denominator, thus prolonging the exercise.

### 3. Course examination assignments in 2014

The PISA Solving Problem Test in 2012 claimed that Polish students are not particularly proficient in solving problem tasks that require creative thinking. In order to look at this conjecture, we included two tasks requiring creative thinking into six-task sheets for 1<sup>st</sup> year students taking the examination in mathematics in 2014 at the University of Economics in Wrocław, Faculty of Management, Information Science and Finance, and one such task for the Faculty of Economic Sciences.

Group one (Faculty of Economic Sciences) included 101 students, with 62 persons (61%) who passed just a basic level high-school exit examination in mathematics. The tasks required at the course examination in mathematics are presented below.

1. For given linear transformations:

$$f(x, y, z) = (-7x + 4y - 2z, 3x - y + z, 4x - 2y + z);$$

$$g(x, y, z) = (x - 2y + z, -2x + 4y - 3z, x - y + z).$$

find  $f \circ g^{-1}(x, y, z)$ .

**Comment:** Basically, most students solved this problem correctly, even though it required the capability to apply three concepts: linear transformation matrix, matrix inverse and theorem on composition matrix of linear transformation.

2. Find the dimensions of a cuboid maximizing its volume, given that the sum of its edges is 12.

**Comment:** Only five students of the Faculty of Economic Sciences (5%) solved this problem correctly. The main difficulty was how to write the function of the volume of a cuboid in terms of its edges, although it seems incredible that 95 per cent of students are not familiar with the volume formula of a cuboid. Unfortunately, this fact validates the claim that we do not teach how to “solve difficult and new problems”, as recommended by the World Bank in 1996.

3. Find intervals of monotonicity, local extrema, intervals of convexity and inflection points of the function:  $f(x) = x^2 e^{-2x}$ .

**Comment:** One difficulty emerged when finding a derivative of a composite function (students ignored a derivative of innermost function), while the other one involved solving the inequality  $f'(x) > 0$  without using the set of values of the exponential function.

4. Using the first order differential estimate the value of the expression:

$$\ln(\sqrt{1,03} + \sqrt[3]{0,98} - 1).$$

**Comment:** Similarly as in Task 3, frequent errors resulted from the lack of skill to find derivatives of a composite function (as the consequences of shortcomings in Analysis 1 course), and from being unfamiliar with the formula for the first order differential of the function of two variables.

5. Examine the linear independence of the set of vectors:

$$[1, 1, 2, 3], [-1, 1, 3, 2], [-1, 5, 4, 3], [-1, 3, 1, 0].$$

**Comment:** The students made the fewest errors in their solutions of this problem.

6. Compute the double integral on a normal region  $D$ :

$$\iint_D (x^2 - xy) dx dy, \text{ where } D: y = \sqrt{x}, y = x^2.$$

**Comment:** Students were not acquainted with Fubini's theorem that was necessary to solve this problem, besides they had difficulty integrating, hence failed to write the root as the power.

Group 2 (Faculty of Management, Information Science and Finance) included 208 students, with 157, i.e. 75 per cent, of those who had acquired a good grade from their high-school exit examination in mathematics at an advanced level. Below we present and briefly comment on the examination tasks for students of the Faculty of Management, Information Science and Finance.

1. Linear transformation  $S$  from  $R^3$  to  $R^3$  is a composition of two linear mappings:  $T$ , a symmetry with respect to the  $OXY$  plane, and  $L$ :

$$L(x, y, z) = (3x - 2y, 2x - 3y, 4z).$$

Find the relevant invariant subspaces of  $S$ .

**Comment:** Students had mostly a difficulty with finding the symmetry matrix with respect to the  $OXY$  plane – only 12.5% of those examined completed a correct form. Unfortunately, more than half of the students did not even attempt this task. This fact confirms the claim that more advanced problems are not liked even by the best students.

2. Find the dimensions of a cuboid maximizing its volume, given that the sum of its edges is 12.

**Comment:** Similarly as with the students from the Faculty of Economic Sciences, the students from the Faculty of Management, Information Science and Finance had difficulty finding the function of the volume of a cuboid in terms of its edges, even if those students should be familiar with the formula for the volume of a cuboid. They also knew the method and tools to solve this problem.

3. Determine the inverse transformation of the linear transformation:

$$f(x, y, z, t) = (x + y, -x + y, z + t, -z + t).$$

**Comment:** All the students completed this task correctly, finding the transformation matrix, its inverse and the inverse transformation.

4. Find the vector of the gradient of the function  $f(x, y) = (x^2 + y^2)^{0.5}$  at the point  $P(3, 4)$ . Determine and sketch the contour curve passing through this point and the vector of the gradient.

**Comment:** The vector of the gradient was found correctly by all the examined students. However, finding and sketching the contour curve turned out to be a big problem, although everyone knew the equation of the circle and the theorem stating that the gradient is perpendicular to the contour line.

5. Find local extrema of the function:

$$f(x, y) = x^3 + 3xy^2 - 15x - 12y.$$

**Comment:** Although this function has four critical points, the students solved this problem correctly.

6. Compute the double integral on a normal region  $D$ :

$$\iint_D \sin(x+y) dx dy, \text{ where } D: y=0, y=x, x+y=\pi.$$

**Comment:** Errors in the solutions of this task demonstrated a disappointing proficiency of the students in trigonometry; most of them integrated over the sum of normal regions with respect to the  $OX$  axis, although there was only one normal region with respect to the  $OY$  axis.

#### 4. Conclusions

The computer-based creative problem solving survey (PISA) conducted on 15-year old pupils, assessed the level of this competence in simulated life situations, different from those experienced typically in a school environment. The educators in Poland should draw a lesson from analyzing relative performance in computer-based problem solving compared with the performance in mathematics, reading and science, assessed in traditional written tests. In this classification, Polish pupils ranked 3<sup>rd</sup> worst, with a negative score difference (-44). According to some PISA experts, a deficiency in computer skills is one of main causes of such a poor result.

PISA 2012 also asked students to evaluate their happiness at school: 68.4 per cent of Polish 15-year-olds agreed or strongly agreed with the statement "I feel happy at school", a percentage that is 7<sup>th</sup> lowest among 65 participating countries. The PISA results also revealed that pupils in Poland had a lower motivation to learn mathematics in 2012 than in 2003, and that 15-year-olds do not take advantage of their peer groups. Class sizes at most schools in Poland are too big.

The grade "good" or higher was assigned to twenty-three students of the Faculty of Economic Sciences, i.e. 22.8 per cent of one hundred and one students taking the mathematics examination. Fifty-two students (51.5 per cent) failed the examination and had to resit it. Five students received a pass mark and also five students completed problem task 2.

The grade higher than "good" was given to twenty-five students of the Faculty of Management, Information Science and Finance, i.e. 12.5 per cent of two hundred and eight students taking the mathematics examination, meaning that at least twenty-five students completed both problem tasks (1 and 2). Eighty students, that is 38.5 per cent, scored less than 60 per cent.

Unfortunately, these results confirm that we do not teach how to solve problem tasks, which encourage attitudes of innovativeness and raise the level of sound scepticism of young people. This means that a major programme overhaul is needed for managing public education and empowering all Polish schools to equip their students with computer skills and the competence to use them.

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