The journal is partially supported by the Ministry of Scientific Research and Information Technology

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**Publisher**  - Wrocław University of Technology, Wybrzeże Wyspiańskiego 27, 50-370 Wrocław, Poland  
Optica.Applicata@pwr.wroc.pl  
www.if.pwr.wroc.pl/optappl  
tel. 48 71-320-23-93  
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*Optica Applicata* has been published since 1971 in a non-periodical form. Starting from 1973 it is published quarterly.

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Scalar and vectorial approaches to cavity modes of the GaAs-based 1.3-µm oxide-confined edge emitting diode laser

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The modelling of optical fields within cavities of GaAs-based oxide-confined edge-emitting diode lasers is analysed treating the 1.3-µm InGaAs/GaAs quantum-well laser as an example of a typical device. Usability of two different optical approaches is compared. While in the first approach, based on the scalar wave simplification, optical fields within laser resonators are found to be composed of the TE modes, an alternative, more precise vectorial approach leads to the hybrid modes: EH and HE. Advantages and disadvantages of both methods are discussed and their validity limits in determination of mode intensities are compared. Simplified scalar approaches have often happened to be surprisingly exact, except for their weaker guidance occurring for higher-order modes, narrower aperture widths and/or thinner oxidation layers, when more exact but also more time-consuming vectorial approaches should be exclusively used.

Keywords: laser simulations, laser modes, edge emitting diode lasers.

1. Introduction

Recently, resonators of semiconductor lasers have often been reduced to relatively small dimensions to enable reduction of both their threshold currents and an increase in operation temperature. To design such lasers ensuring even better performance, an exact simulation of simultaneous interrelations between electrical, thermal, gain, stress and optical phenomena taking place within the device volumes when in operation is essential.

The modelling of optical field appears to be a very challenging problem since Maxwell’s equations, treated rigorously, are not separable within the semiconductor laser domain. Moreover, wavelengths of the propagating light within modern
structures become often comparable with their sizes, which makes the common use of plane waves in scalar optical models unjustified. In such a situation, it seems necessary to employ the full vectorial theoretical model describing optical phenomena, instead of its simplified scalar approach. However, the computational experiment shows that, surprisingly, the scalar approach gives sometimes quite satisfactory results even beyond limits of its confirmed validity, e.g., in some modern micro-cavity devices. Therefore, it is necessary to examine the performance of both optical models when these are used to simulate an operation of specified laser devices to determine cases where less time-consuming and simpler scalar approaches may be used. In the present work, a comparison is made of the scalar and vectorial models represented by the effective index method and the method of lines, respectively, applied to the optical simulation of standard GaAs-based oxide-confined edge emitting diode lasers.

2. The scalar approach – the effective index method

The effective index method is the most commonly used scalar approach to the optical phenomena within semiconductor lasers [1–4]. The model assumes that changes of the refractive index are relatively small and the propagating wave is the plane one. Therefore, the electromagnetic wave may be treated as a scalar field, which may be separated into independent functions describing the electromagnetic wave separately along each direction.

Let us assume here that the $z$-axis of the Cartesian coordinate system is parallel to the direction of wave propagation and the $p$-$n$ junction of the structure lies in the $x$–$z$ plane where $x = 0$ corresponds to the active-region centre (Fig. 1). Taking advantage of the concept of the effective refractive index as well as making use of the assumptions that the wave propagates within the charge-free medium and the time dependence of the solution has a form of the function of type $\exp(i\omega t)$, the set of Maxwell’s equations may be reduced to the Helmholtz equation [5]:

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + k_0^2 \left[ n^2(x, y) - N_{\text{eff}}^2 \right] E = 0$$

(1)

where $\partial_u = \partial/\partial u$, $u = x, y$, $E$ is the electric field vector of the electromagnetic wave, $n$ – the position dependent complex refractive index of the laser domain, $k_0$ – the vacuum wave number, and $N_{\text{eff}}$ – the structure complex effective index. The field distribution within the domain of the waveguide is assumed to be a product of two functions, which is true only for a uniform, free source domain:

$$E(x, y) = E_x(x)E_{xy}(x, y)$$

(2)

The function $E_x(x)$ is expected to approach precisely the solution in the $x$-direction, whereas the second one, $E_{xy}(x, y)$, is only weakly $x$-dependent. After some manipulations, one can get two plane wave equations:
Scalar and vectorial approaches to cavity modes...

\[ \partial_y^2 E_{xy} + k_0^2 \left[ n^2(x, y) - n_i^2(y) \right] E_{xy} = 0 \]  
(3)

\[ \partial_x^2 E_x + k_0^2 \left[ n_i^2(y) - N_{\text{eff}}^2 \right] E_x = 0 \]  
(4)

where \( n_i \) denotes the complex effective index for the region assumed to be uniform in the \( x \)-direction. From equation (3), one can find the effective index \( n_i \) for each region, next setting \( n_i \) to equation (4), one can find the structure effective index \( N_{\text{eff}} \) for the whole structure. The general solution of the wave equations (3) and (4) for the uniform region may be expressed as a superposition of two waves travelling in the opposite directions:

\[ E(u) = A \exp \left[ \gamma(u - u_j) \right] + B \exp \left[ -\gamma(u - u_j) \right] \]  
(5)

where \( u = x, y, \) and \( \gamma \equiv k_0 \sqrt{n_e^2 - n_m^2} \) (\( n_e \) and \( n_m \) for Eq. (3) are \( n \) and \( n_j \), respectively, and for Eq. (4) – \( n_i \) and \( N_{\text{eff}} \), respectively), \( u_j \) stands for the position of the edge of the region. The regions are assumed to be rectangularly shaped. Amplitudes \( A \) and \( B \) and parameter \( \gamma \) can be found using the transfer matrix method [6].

3. The vectorial approach – the method of lines

Hitherto known vectorial models used to simulate optical fields in diode lasers are relatively complex and they often need special very time-consuming calculation algorithms. To this end, a more efficient new vectorial approach, namely the method of lines, was developed by R. Pregla and his group [7].

The initial set of Maxwell’s equations is expressed in the form given, for example, by Saleh and Teich [5]. Assuming oscillating dependence on time and on direction of propagation of both the electric field and magnetic field vectors, one can get the following set:

\[ \partial_y \bar{E} = -i \eta_0 \begin{bmatrix} k_0 \left( 1 - \frac{N_{\text{eff}}^2}{n^2} \right) & iN_{\text{eff}} \frac{1}{n^2} \partial_x \\ iN_{\text{eff}} \partial_x \frac{1}{n^2} & \frac{1}{k_0} \frac{1}{n^2} \partial_x + \frac{1}{n^2} \partial_x + k_0 \end{bmatrix} \bar{E} = -i \eta_0 \bar{R}_E \bar{E} \]  
(6)

\[ \partial_y \bar{H} = -i \eta_0 \begin{bmatrix} \frac{1}{k_0} \partial_x^2 + k_0 n^2 & -i N_{\text{eff}} \partial_x \\ -i N_{\text{eff}} \partial_x & k_0 \left( n^2 - N_{\text{eff}}^2 \right) \end{bmatrix} \bar{H} = -i \eta_0 \bar{R}_H \bar{H} \]  
(7)
where, for simplicity, we take:
\[ n = n(x, y), \quad \vec{E} = \begin{bmatrix} E_z \\ -E_x \end{bmatrix}, \quad \vec{H} = \begin{bmatrix} H_x \\ H_z \end{bmatrix}, \quad \eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \]

A combination of the two above equations results in:
\[ \partial_y^2 \vec{E} = -R_H R_E \vec{E} \equiv -Q_E^2 \vec{E} \quad (8) \]

After discretisation of Eq. (8) in the \( x \)-direction [7], it can be solved by diagonalizing the matrix \( Q_E \). This enables us to find a characteristic value of the problem, which corresponds to the effective index value, as well as characteristic vectors which determine the distribution of the electromagnetic field within the structure. For Eq. (8) rewritten in the new base:
\[ \partial_y^2 \hat{E} + \Gamma_E^2 \hat{E} = 0 \quad (9) \]

where \( \hat{E} \equiv T_E^{-1} \vec{E} \), the matrix \( T_E \) diagonalizes \( Q_E \). The solution to Eq. (9) has the form of a standing wave:
\[ \hat{E}(y) = A \cosh(i \Gamma_E y) + B \sinh(i \Gamma_E y) \quad (10) \]

Some algebraic manipulations lead to the relation between the electric and the magnetic fields within the layer assumed to be uniform in the \( y \)-direction:
\[ \begin{bmatrix} \hat{H}_0^{(i)} \\ -\hat{H}_d^{(i)} \end{bmatrix} = \begin{bmatrix} Y_1^{(i)} & Y_2^{(i)} \\ Y_2^{(i)} & Y_1^{(i)} \end{bmatrix} \begin{bmatrix} \hat{E}_0^{(i)} \\ \hat{E}_d^{(i)} \end{bmatrix} \quad (11) \]

where \( \hat{H} \) is the vector of the magnetic field components transformed to the base in which the matrix \( Q_E \) can be diagonalized. The superscript indicates the number of the layer, and the subscript the edge of the same layer; \( y_1 \) and \( y_2 \) are defined as follows:
\[ \begin{aligned} y_1 &= \left( T_E^{-1} R_H T_H \right)^{-1} \Gamma_E \tanh^{-1}(i \Gamma_E d) \\ y_2 &= -\left( T_E^{-1} R_H T_H \right)^{-1} \Gamma_E \sinh^{-1}(i \Gamma_E d) \end{aligned} \quad (12, 13) \]

From Eq. (11) one can find the relation between the magnetic and the electric fields:
\[ \hat{H}_d^{(i)} = Y^{(i)} \hat{E}_d^{(i)} \quad (14) \]
where
\[ Y^{(i)} = -\left\{ 2 \left( \begin{array}{c} T_{H}^{(i)} T_{H}^{(i-1)} Y^{(i-1)} \left( T_{E}^{(i)} \right)^{-1} T_{E}^{(i-1)} \right)^{-1} Y^{(i)} - Y^{(i)} \right\}^{-1} Y^{(i)} + Y^{(i)} \right\}^{(15)} \]

The assumption of the electric field decaying on the borders of the simulated domain yields the eigenvalue equation for the vector of the electric field between arbitrarily chosen layers \( m \) and \( l \):
\[ T_{H}^{(m)} Y^{(m)} = T_{E}^{(l)} Y^{(l)}, \quad M \tilde{E} = 0 \]  (16)

The solution of the above equation determines the characteristic values which correspond to the complex effective refractive index \( N_{\text{eff}} \) and eigenvectors related to the electric field. The equation is not trivial only if \( M \) is singular, then at least one eigenvalue of \( M \) is equal to zero.

4. The results

Usually, in the simulation of a laser operation, the final task is to calculate its output power as a function of applied voltage (or supply current). This makes it necessary to model self-consistently a complex network of interrelations between electrical, thermal, gain and optical phenomena. All they interact with the optical model influencing both (real and imaginary) parts of the complex refractive indices of structure layers. But the focus of this paper is on purely optical determination of the optical modes with the help of both scalar and vectorial approaches. Taking additionally into account an impact of gain, thermal and electrical phenomena on optical ones, can make it difficult to unambiguously interpret the results of analysis. On the other hand, a purely optical comparison of scalar and vectorial modes may enable determination of their validity limits, which is the main goal of this paper. Therefore, details of recombination processes, carrier transport and thermal flow are intentionally omitted here by using a constant optical gain (a step-like gain distribution) within the active quantum wells (QW’s) of the structure. Our goal is to calculate the cavity modes using both approaches and to compare their distributions.

Let us consider a standard design of the highly strained, stripe-geometry 1.3-\( \mu \)m InGaAs/GaAs double quantum well edge emitting (Fabry–Perot) diode laser [8]. Its layer structure is shown in Fig. 1 and details are listed in the Table. Its active region consists of two strained \( In_{0.44}Ga_{0.56}As \) quantum wells separated with the GaAs barrier. The necessary lateral confinement of the current flow and the optical field is realized with the aid of a 0.15 \( \mu \)m oxidized layer.

The scalar modes are assumed to be polarized (TE polarization) whereas the vectorial modes have been found to be hybrid ones. This means that the vectorial modes are determined by all six components of the electromagnetic field contrary to the scalar
modes, which are defined by three components only. Figure 2 presents intensity profiles of three lowest-order modes for the laser structure with a 12-μm stripe width. One can observe a distinctly better confinement of the fundamental vectorial mode within the active region than its scalar counterpart (Fig. 2a). It becomes even more pronounced for higher order modes, especially in the x-direction. Distinct leakage of scalar modes brings out the lowering of the real and imaginary parts of the mode effective refractive index since their electromagnetic fields penetrate the lateral passive regions exhibiting lower index and high absorption. On the other hand, vectorial modes suffer from the diffraction losses, which are not included in the scalar approach.

Table. Construction details of the highly strained, double quantum-well edge-emitting diode laser [8]. The gain within the active region corresponds to the carrier concentration equal to 3×10^{18} cm^{-3}. The values of refractive indices and gain coefficients have been taken from [9, 10].

<table>
<thead>
<tr>
<th>Material</th>
<th>Thickness [nm]</th>
<th>Doping concentration [cm^{-3}]</th>
<th>Real refractive index</th>
<th>Optical gain coefficient [cm^{-1}]</th>
</tr>
</thead>
<tbody>
<tr>
<td>p-GaAs</td>
<td>500</td>
<td>10^{19}</td>
<td>3.453</td>
<td>-226.379</td>
</tr>
<tr>
<td>p-Al_{0.98}Ga_{0.02}As/Al_2O_3</td>
<td>150</td>
<td>10^{19}</td>
<td>3.453/1.75</td>
<td>-226.379/0</td>
</tr>
<tr>
<td>p-Al_{0.4}Ga_{0.6}As</td>
<td>300</td>
<td>4×10^{17}</td>
<td>3.239</td>
<td>-64.9326</td>
</tr>
<tr>
<td>p-GaAs</td>
<td>100</td>
<td>10^{16}</td>
<td>3.481</td>
<td>-58.374</td>
</tr>
<tr>
<td>In_{0.44}Ga_{0.56}As</td>
<td>7</td>
<td>0</td>
<td>3.533</td>
<td>1171.78</td>
</tr>
<tr>
<td>GaAs</td>
<td>20</td>
<td>0</td>
<td>3.482</td>
<td>0</td>
</tr>
<tr>
<td>In_{0.44}Ga_{0.56}As</td>
<td>7</td>
<td>0</td>
<td>3.533</td>
<td>1171.78</td>
</tr>
<tr>
<td>n-GaAs</td>
<td>100</td>
<td>5×10^{16}</td>
<td>3.481</td>
<td>-59.0466</td>
</tr>
<tr>
<td>n-Al_{0.4}Ga_{0.6}As</td>
<td>1500</td>
<td>10^{18}</td>
<td>3.237</td>
<td>-75.0228</td>
</tr>
<tr>
<td>n-GaAs</td>
<td>5000</td>
<td>10^{18}</td>
<td>3.478</td>
<td>-75.0228</td>
</tr>
</tbody>
</table>

Emitted wavelength 1.23 μm
Next important difference, which is more pronounced for higher order modes, is connected with different values of intensity maxima within the active region. For a uniform region, the scalar solution is given by the sum of exponential functions, as

Fig. 2. Intensity profiles of the three lowest order modes calculated with the scalar (solid lines) and the vectorial (dashed lines) approaches for the highly strained 1.3-μm InGaAs/GaAs double quantum-well edge-emitting (Fabry–Perot) diode laser [8], with the stripe active region defined by a 12 μm aperture width. The left column of the figures depicts profiles of the mode intensities along the x-direction within the active region, whereas the right one presents the analogous profiles along the y-axis on the symmetry plane. The figures correspond to: HE_{00} and TE_{00} modes (a), EH_{00} and TE_{01} modes (b), HE_{01} and TE_{02} modes (c). Profiles of the refractive index along the x- (within oxide layer) and the y-axis are additionally shown.
if the region were infinite. Hence, the intensity oscillations of scalar modes exhibit a constant amplitude (cf. Fig. 2c). Besides the differences in the intensity distributions of the modes in the $y$-direction express the essential difference between the models. The scalar approach separates the solution for $x$- and $y$-directions. Hence, the mode intensity profile in the $y$-direction remains completely independent of the $x$-directional solution. The non-separated solution of the vectorial approach predicts the shift of the higher-order modes towards the $p$-contact (Fig. 2c), since those modes are weaker confined by the oxidation, and finally the high refractive index of the broad $p$-contact layer may even attract the modes, as considered below.

Figure 3 presents the difference in the mode intensity profile along the $x$-direction between the results of the scalar and the vectorial optical models for a narrow stripe width of 4 $\mu$m. Compared to Fig. 2a, the narrowing of the oxide window causes a more pronounced penetration by the mode of the lateral passive regions placed out of the central active region. This finally leads to the mode leakage. Both approaches predict such a behaviour, however, vectorial model indicates the diffraction process as the main reason for the leakage. The vectorial mode penetrating the regions out of the active region starts to oscillate (Fig. 3), which reduces the guiding process enhanced by the oxidations. The scalar model predicts considerably less pronounced penetration of the lateral passive region out of the active region by the mode without appearance of any oscillations caused by the diffraction.

Figure 4 presents an interesting process of weak guidance. Two structures with the oxidation layer of different thicknesses are considered. Thick oxidation ensures a stable waveguide process, whereas thin one allows the wave leaking. This time the leakage occurs in the $y$-direction. As one can see, too thin an oxidation layer does
not protect a mode optical field from its migration towards the high refractive $p$-contact layer. For a narrower oxidation layer, the vectorial model reveals an essential mode leakage and its oscillations within the $p$-contact layer whereas the scalar one penetrates this layer without any oscillations. This means that too narrow an oxidation layer may lead to a considerable increase in optical losses, which is followed by an increase in a lasing threshold.

5. Conclusions

Two approaches intended for simulation of the electromagnetic field within the cavity of edge emitting (Fabry–Perot) diode lasers are presented. The less exact scalar approach owes its extremely short calculation time to the plane wave assumption. On the other hand, the vectorial models, of which the method of lines is currently the most effective one, need usually $10^2$–$10^3$ times longer calculation time. The comparison of the mode profiles determined using both approaches reveals a better confinement of vectorial modes within the active region. This is a consequence of the assumed plane wave solution used in simplified scalar approaches which has been taken from the infinite domain and set to the finite region. Nevertheless, profiles of the modes determined using both approaches are usually surprisingly close, except for the weaker guidance occurring for higher-order modes, narrower aperture widths and/or thinner oxidation layers. The above restrictions define validity limits of simple scalar optical approaches in modelling optical fields within cavities of standard edge-emitting diode lasers.

Acknowledgements – This work was supported by the Stipend for Young Scientists from the Foundation for Polish Science and by the Polish Ministry of Science and Information Society Technologies (MNiI), grant No. 3-T11B-073-29.
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Received July 10, 2006
Examination of air density fluctuations with the aid of laser beam

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The paper presents theoretical foundations and results of experimental verification of the possibility of laser beam application for registration of air density fluctuations caused either by mechanically forced air flow near obstacles which are not streamlined or by the presence of solid objects which produce thermal disturbance (temperature gradients).

Keywords: turbulence, density fluctuations, laser application.

1. Introduction

Measurements of density of matter, i.e., the number of molecules in unit volume, are constantly developed and refined for different experimental situations. Air density fluctuations caused either by mechanically forced gas flow near non-streamlined obstacles or by the presence of solid objects (buildings) which produce thermal disturbance (temperature gradients) deserve special interest. There are many different, indirect methods of detection of gas density fluctuations. It can be realized, just to mention only a few possibilities, by monitoring the temperature of heated probes (thermo-anemometric sensors [1]) or by determination of whirls frequency near some specially shaped, non-streamlined, sharp edged elements [2]. Each of those methods has its special features which make it most suitable for specific applications. These features are connected with the obtained precision, time delay or spatial resolution. It must be remembered that all these methods make use of different probes which, to some extent, always introduce certain disturbance of temperature and flow conditions, just by their presence.

It seems that the technique which makes use of laser beam and its speckles as a probe, does not introduce any disturbance of the observed process [1, 3]. Moreover, the time
delay in this case is incomparably smaller than for other methods, which makes it especially useful for the examination of rapid changes of gas.

The authors of this paper wanted to examine the effect of air density fluctuations caused by forced air flow or by temperature gradients on the intensity of laser beam which propagates in such conditions.

2. Theoretical foundations

The fact that the refractive index of gas, especially of air, depends on the number of molecules in unit volume is confirmed by the results of many experiments. As an example one can discuss the experiments with Michaelson interferometer and a tube of gas (carbon dioxide) placed in one of its arms. The gas pressure is changed for constant temperature and consequently the concentration of gas molecules changes, too. The change in concentration of gas molecules is followed by translation of the interference fringes in one direction (or switching of bright and dark fringes), which proves that the increasing density of the gas results in bigger values of its refractive index [4].

Theoretical dependence of the refractive index value on the density of molecules is described by Lorenz–Lorentz formula [5]

\[
\frac{n^2 - 1}{n^2 + 2} = c \text{ const}(\alpha)
\]

where: \(n\) – the refractive index of gas (air), \(c\) – density of gas molecules (the number of molecules per unit volume), \(\text{const}(\alpha)\) – the coefficient dependent on the mean polarizability of isotropic molecule.

Because \((n^2 - 1) = (n - 1)(n + 1)\) and assuming \(n = 1.0002929\) (for the air at 14.5 \(^\circ\)C and for \(D\) line of sodium), after simple transformation of this formula, one obtains:

\[
n \approx \frac{3}{2} c \text{ const}(\alpha) + 1
\]

Equation (2) shows that there is a linear dependence of the refractive index \(n\) for air on the concentration of molecules \(c\).

If there is no flow of gas, especially in conditions of thermal equilibrium, local fluctuations of density are relatively small. The situation changes when the gas flows, especially close to the objects obstructing its flow, or when heat is exchanged between the gas and these objects.

Clapeyron’s equation can be used as a starting point to describe qualitatively the influence of the above-mentioned factors [6]:

\[
pV = \frac{N}{N_A} RT
\]
Examination of air density fluctuations ...

where: \( R \) – gas constant, \( p \) – pressure of the gas, \( V \) – volume of the gas, \( N \) – number of molecules in the gas, \( N_A \) – Avogadro’s number, \( T \) – gas temperature in kelvins. Equation (3) transforms into:

\[
\frac{N}{V} = \frac{N_A p}{R T}
\]  
(4)

having in mind that:

\[
c = \frac{N}{V}
\]  
(5)

and making use of the relation between gas constant \( R \), Avogadro’s number \( N_A \) and Boltzmann constant \( k = R/N_A \) one obtains the dependence of the concentration of molecules on pressure and temperature:

\[
c = \frac{p}{k T}
\]  
(6)

To sum up the above derivations it can be stated that in any gaseous medium, even when as a whole it remains in thermal equilibrium, small fluctuations of density are always present as a result of random motions of molecules. Local disturbances caused by applying pressure difference or heating are the source of additional and much stronger stimuli generating new regions with changed density and (according to Eq. (6)) changed value of the refractive index. The local disturbance of pressure, mentioned above, can occur when, for instance, the air stream collides with an obstacle (or goes by). It is obvious that when the obstacle has different temperature than the gas, it can cause local density changes even without any gas movement.

The laser beam propagating in the medium with gradients of refractive index (connected with density fluctuations \( F \) – dark and bright regions in Fig. 1) undergoes
local distortions of its electric field strength vector amplitude and phase. It finally results in changes of light intensity distribution across the beam and temporal changes of light intensity at a chosen point of the beam’s cross-section observed on the screen S. The effects described are schematically illustrated in Fig. 1.

Analysis of the data obtained must make use of statistical methods because of the random nature of time and spatial fluctuations of air density which influence the propagation of the laser beam. Statistical parameters [7], which best render the character of the obtained results are ($X_M$ – mean value, $X_{RMS}$ – standard deviation):

$$X_M = \frac{\sum_{i=1}^{N_t} x_i}{N_t}$$

$$X_{RMS} = \sqrt{\frac{\sum_{i=1}^{N_t} (x_i - X_M)^2}{N_t}}$$

where: $x_i$ – a result of one measurement, $N_t$ – total number of measurements in a series.

3. Experimental setup

As a source of light the authors used a diode laser of power 14 mW and a wavelength of 640 nm with additional lenses to make the beam quasi-parallel (along a distance of about 1.5 m). Two photodiodes, type BPYP17, were applied for the measurements of light intensity. One of them (PD1) monitored the intensity of the beam emitted by the laser. It was placed in one casing together with the laser L and the beam-splitter plate BS. The other one, positioned behind a diaphragm of 0.1 mm in diameter measured the intensity of light beam which went across the tunnel.

The laser and the photodiode were aligned on the same, heavy optic bench to minimize mechanical vibrations. There was no mechanical contact between the laser, the photodiode PD2 and the wind tunnel to avoid influence of vibrations on the intensity measurements.

The laser beam crossed a wind tunnel perpendicularly to the tunnel axis passing through two small holes made in the wall of the tunnel at a distance $h = 4$ cm from its base and $L = 150$ cm from four fans which forced air to flow. The electric signal from the photodiodes was connected to a computer controlled 16 bit A-C converter with memory (ADC). This system registered the intensity of light sampled with constant interval of 50 ms and the duration of the whole measurement was 5 µs. A special program applied could store about 32 thousands of intensity values.

After the measurements data were sent via RS232 port to the computer. Figure 2 presents a diagram of the experimental system.
Examination of air density fluctuations ...

The idea of applying laser beam for the detection of local density fluctuations near the obstacles which change the conditions of flow is based on the assumption that density fluctuations generated in this situation are big enough to cause a detectable change of propagation in comparison with the propagation in still air in conditions of thermal equilibrium between the gas and the solid objects.

4. Results of the experiment

4.1. Examinations of the measuring system stability

In real situations, the fluctuations of air density are very small even for turbulent flows, so the measuring system must be characterized by a very stable and low noise operation. This requirement applies especially to the laser and the detecting system.

The upper curve in Fig. 3 presents the registered signal from the photodiode PD1 (intensity of the laser beam before it enters the wind tunnel), while the lower one shows the signal from the photodiode PD2 illuminated with the beam after it goes across the tunnel. Both intensities fluctuate but the air in the wind tunnel is still and there are no heat sources in its neighbourhood.

The upper curve is for the case when the photodiode was placed close to the laser and the lower one for the detector placed 1 m (width of the tunnel) from the laser at the point where it remains during the experiment. The time interval between subsequent measurements was 0.1 s. The lack of clear differences between two curves from Fig. 3 indicates that the air density fluctuations were negligible or too small to be detected by this experimental system.

Fig. 2. The diagram of the experimental system: L – diode laser with additional lenses, PD1 – photodiode, BS – beam splitter, PD2 – photodiode with the diaphragm D, WT – axial-flow fan of the wind tunnel, ADC – analog-digital converter with memory, PC – computer. The figure does not render real proportions for obvious reasons.
A discrete character of the results presented by the two curves is due to the properties of the applied A-C converter – its sampling time was 50 ms. This and the fact that the points for subsequent moments are not connected explains why there are apparent parallel lines on the diagram. Vertical spacing between these lines is determined by the A-C converter sensitivity (1 mV). This sensitivity proves to be adequate for the measurements described. Standard deviation $X_{RMS}$ for signals from photodiodes PD1 and PD2 is 0.6209 and 0.6208, respectively.

All the measurements were conducted after the instruments reached the conditions of thermally stable operation.

### 4.2. Unobstructed flow of air

Then, similar measurements were conducted with moving air for the velocity of the wind in the tunnel equal to $2 \text{ m/s}$ at the height $h = 4 \text{ cm}$ from its base and $L = 150 \text{ cm}$ from the fans, in the place where the light beam crosses the stream of flowing air.

Figure 4 presents the fluctuations of light intensity measured with the photodiode PD2 in this case. The value of standard deviation $X_{RMS}$ is 6.8966.

![Fig. 3. The dependence of the signals from the photodiodes, PD1 (the upper curve) and PD2 (the lower curve), as a function of time when the air in the wind tunnel is still.](image)

![Fig. 4. The signal from PD2 photodiode versus time, for the velocity of air in the tunnel $v = 2 \text{ m/s}$.](image)
4.3. Flow of air in the presence of obstacles

The next measurement was conducted for the situation when 20 cm high obstacle \( h_p = 20 \text{ cm} \) was placed at a distance \( d = 0.5 \text{ cm} \), perpendicularly to the tunnel axis and completely blocked the air flow in the lower part of the wind tunnel.

Intensity of the beam crossing the wind tunnel was measured every 50 ms. The results are presented in Fig. 5a. The value of standard deviation \( X_{\text{RMS}} \) is 8.128. The height of the laser beam above the tunnel’s base was the same as in Sec. 4.2.

Fig. 5. The dependence of the PD2 photodiode signal as a function of time when the air in the wind tunnel moves with velocity 2 m/s and: a 20 cm high obstacle in the tunnel placed 0.5 cm from the laser beam (a), a 20 cm high obstacle in the tunnel placed 10 cm from the laser beam (b), and a 40 cm high obstacle in the tunnel placed 94 cm from the laser beam (c).
Then, for unchanged path of the beam across the tunnel, the obstacle \((h_p = 20\, \text{cm})
\), perpendicular to the tunnel axis and completely blocking the air flow in the lower part of the wind tunnel) was placed at \(d = 10\, \text{cm}\) in front of the laser beam. The results of measurements are presented in Fig. 5\(b\). The obtained value of standard deviation was \(X_{\text{RMS}} = 6.822\).

In the third measurement, the obstacle was bigger \((h_p = 40\, \text{cm})\) and was placed 94 cm from the beam, still completely blocking the flow of air in the lower part of the tunnel. The results are presented in Fig. 5\(c\). This time standard deviation of registered fluctuations was \(X_{\text{RMS}} = 14.036\).

In the next step, for the same geometrical conditions, the flow of air was switched on every 20 s. The fluctuations of light intensity were measured every 50 ms and the results are presented in Fig. 6. The values of standard deviation in subsequent

Fig. 6. The dependence of the PD2 photodiode signal as a function of time when the air in the wind tunnel moves with velocity 2 m/s with periodic operation of the fans (every 20 s) and there is a 40 cm high obstacle in the tunnel, placed 94 cm from the laser beam.

Then, for unchanged path of the beam across the tunnel, the obstacle \((h_p = 20\, \text{cm})
\), perpendicular to the tunnel axis and completely blocking the air flow in the lower part of the wind tunnel) was placed at \(d = 10\, \text{cm}\) in front of the laser beam. The results of measurements are presented in Fig. 5\(b\). The obtained value of standard deviation was \(X_{\text{RMS}} = 6.822\).

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Fig. 6. The dependence of the PD2 photodiode signal as a function of time when the air in the wind tunnel moves with velocity 2 m/s with periodic operation of the fans (every 20 s) and there is a 40 cm high obstacle in the tunnel, placed 94 cm from the laser beam.

Fig. 7. The dependence of the PD2 photodiode signal as a function of time when the fans in the wind tunnel are switched off but a heater (36 °C) is placed near the laser beam.
time intervals (indicated in Fig. 6) are: 0.949 (1), 12.103 (2), 1.475 (3), 12.794 (4), 1.998 (5), 12.758 (6), 2.409 (7), 12.976 (8).

4.4. Heat source

The next measurements of light intensity fluctuations were conducted when the air in the wind tunnel was still but warm object (36 °C) was drawn near the laser beam and then removed. The temperature in the tunnel was 21 °C. The light intensity was measured every 50 ms. The values of standard deviation for the measured signal are: 6.023 (1) – the heat source close to the beam, 1.475 (2) – the heat source moved away from the beam, 4.212 (3) – the heat source close to the beam, 1.604 (4) – the heat source is moving away from the beam (Fig. 7).

5. Summary and conclusions

It is clearly seen from the measurements conducted with the air flowing in the wind tunnel that there are significant differences in the values of standard deviation for fluctuations of light intensity detected with PD1 and PD2 photodiodes.

Different values of standard deviation for the measurements presented on subsequent diagrams prove that $X_{\text{RMS}}$ value can be used in examinations of air movements at certain points. This is most clear when they are compared for still air and when air movements are induced mechanically or thermally.

The authors described introductory investigation into the possibility of applying lasers in examination of air density fluctuations close to various objects which obstruct the flow.

Results of the measurements presented prove with no doubt that with this method parameters of air flow can be examined with a short time delay and without any additional disturbance of the processes observed. It makes this method especially suitable for the measurements of air fluctuations and dynamics for various phenomena in the close vicinity of different buildings and constructions (which cause temperature gradients and density fluctuations of air). This gives the basis for better understanding (and then application) of the complicated processes of air flow and heat exchange.

References


Received July 19, 2006
Single particle analysis of soil dust by aerosol time-of-flight laser mass spectrometry

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A laboratory study was carried out to investigate the single particle soil dust using aerosol time-of-flight mass spectrometry. The presence of crustal elements is observed in the mass spectra of individual particles. Potassium and iron constitute the two most commonly detected cations. Other common cations observed in the mass spectra of soil particles include sodium and magnesium. According to a large number of single aerosol diameters, we obtained the size distribution. Peaks in the positive ion mass spectra enable us to identify individual dust particles in atmospheric samples and track chemically distinct dust particles in the atmosphere using aerosol time-of-flight mass spectrometry.

Keywords: mass spectrometry, aerosols, trace elements, particulate matter.

1. Introduction

Atmospheric aerosols are receiving increased attention by governmental, industrial, and scientific communities due to recent studies that report a correlation between high levels of particulate pollution and adverse health effects [1]. Both natural and anthropogenic sources contribute to atmospheric aerosols with concentration reaching up to $10^6$ particles/cm$^3$ in some polluted regions. Soil dust is a significant contributor to anthropogenic particulate matter throughout the world. On a global scale, it is estimated that production of dust is of the order of $250\times10^6$ tons annually, ranking it as the second largest primary particle source after sea salt [2]. The transport of soil dust over very long distances has been shown on numerous occasions. Further studies have shown that mineral dust can play a role in light scattering, and climate forcing [3]. In order to enhance our understanding of the role of aerosols in atmospheric processes and to identify aerosol sources, the ability to characterize ambient particles in real time is of primary importance.

Microanalysis techniques, such as laser microprobe mass analysis (LAMMA), are currently the most utilized methods for complete characterization of individual aerosol particles [1]. Current microanalysis tools include mass spectrometry [4], optical
spectroscopy [5], and electron microscopy [6]. The chemical composition of size segregated soil dust has been obtained for several soil sources. Typically, the chemical composition data obtained during such studies is acquired using bulk analysis of the samples by techniques such as proton induced X-ray emission (PIXE), or X-ray fluorescence (XRF) [7]. The use of bulk analysis techniques inherently complicates the possibility of assessing the contribution of dust particles originating from different soil sources, since all particles are collected and analyzed as an integrated sample. However, these techniques suffer from being off-line analysis methods, and the majority can only be used for quantitative determination of either particle size or composition. An ideal aerosol analysis instrument would allow simultaneous determination of all physical properties of an aerosol. The size and composition of an individual particle not only reflects the nature of the source of the particle but ultimately determines the particle’s environmental and biological effects.

Source identification is a major goal of aerosol characterization studies. Specific chemical compounds associated with certain size modes allow sampled particles to be traced back to a single emission source. The purpose of this study is to characterize individual particles resulting from the suspension of soil dust. Data on the size and sets of chemical markers are used for the identification of individual soil dust particles in the atmosphere and monitoring their temporal profiles using the real-time capability of aerosol time-of-flight mass spectrometry. The temporal profiles obtained can then be correlated with meteorological factors such as wind speed and direction to assess the relative influence of transport. In addition, source apportionment of soil dust on an individual particle basis will allow for better quantification of the relative contribution of suspended soils to ambient particulate matter and the effects that ensue from their presence in the atmosphere.

2. Experiment – materials and methods

Soil samples were collected from a number of locations in China for the purpose of determining the size and chemical composition of dust particles derived from different areas. Approximately 10 g of each soil sample were introduced into a glass sampling bottle. Introduction of the sample in the container causes the suspension of dust particles in the air space above the soil.

The particle sample is then transferred to an aerosol time-of-flight mass spectrometer through an outlet valve and a plastic transfer line. A HEPA Capsule Filter is attached to an inlet valve on the glass container. The filter allows particle-free air to be pulled through the container to allow for a consistent flow of soil-entrained air into the instrument. This sample introduction procedure has also been employed for studies of automobile exhaust and biomass burning particles [8].

An aerosol time-of-flight mass spectrometer (AOTFMS) provides size and chemical composition information for individual particles. It operates in a continuous
Single particle analysis of soil dust ...

sampling manner, allowing for real-time analysis of particles with high time resolution. The operating principles of ATOFMS have been described in detail before, so only a brief description is presented here [9]. The principle is shown in Fig. 1. Particles are introduced into the instrument through a nozzle and undergo a supersonic expansion. The particles are accelerated to a terminal velocity that is proportional to their aerodynamic diameter. The particles then enter a sizing region where they pass through two continuous wave laser beams. Scattering signals are collected as each particle passes through each laser beam. A particle time-of-flight is measured that can be converted to aerodynamic size with a calibration curve. After being sized, the particle travels to the ion source region of a time-of-flight mass spectrometer. Chemical species in the particle are desorbed/ionized, using the pulsed output from an ultraviolet laser, and separated and detected using a time-of-flight mass spectrometer equipped with multichannel plate (MCP) for ion detection.

The experiments described in this paper were performed using the laboratory-based ATOFMS. An red, diode-pumped laser (operating at 650 nm) was used for the aerodynamic sizing region of the instrument, with size calibrations indicating a lower size limit of 0.3 µm for these experiments. Laser desorption/ionization of chemical species in the particles was accomplished using a Nd:YAG laser operating at 266 nm. Positive and negative ions were acquired separately by changing the applied voltages for the source of the time-of-flight mass spectrometer.

The instrument has proven useful for determining size and chemical correlation information of individual particles for complex atmospheric samples, for tracking particles with distinct chemical signatures in the atmosphere, and for observing meteorological effects on particulate pollution.

Fig. 1. Schematic diagram of aerosol time-of-flight mass spectrometer (ATOFMS).
3. Results and discussion

3.1. Particle size distributions

Analysis of ATOFMS data shows that suspension of soil dust results in the formation of mostly large-sized particles. This is expected, since it is known that large-sized particles tend to be derived from mechanically generated processes including wind blown dust and sea spray [10]. Figure 2 shows a size histogram for the particles generated by the suspension of a soil sample. This distribution is not corrected for instrumental biasing or sampling efficiencies. The dust particles analyzed exhibit a broad size distribution, measuring from approximately 0.4 µm to about 2.2 µm in aerodynamic diameter. In the ATOFMS system, few particles are observed above 2.2 µm due to losses in the current sampling inlet system. This size distribution is typical of the soil samples that were analyzed. It reflects the known contribution of dust as a dominantly coarse mode particle source that occasionally dominates mass distributions.

3.2. Chemical composition

Figure 3 shows the laser desorption/ionization (LDI) mass spectra from four individual particles showing representative positive ions common to soil dust. All the soil was used to cultivate baccy. The chemical heterogeneity of particles resulting from the suspension of soil dust is comparable to the measured composition of bulk soil dust observed in previous systematic studies [1]. The four spectra shown in Fig. 3 are the most typical types observed out of the particles in the soil samples that were analyzed.

In Figure 3a, a positive ion mass spectrum of a 1.2 µm particle is shown. This particle resulted from the suspension of a soil sample acquired from the Si Chuan

![.size_distribution](image-url)  
Fig. 2. Size distribution of particles resulting from the suspension of soil.
province. In this spectrum, dominate peaks are due to the presence of sodium, potassium, and iron. However, it also clearly contains peaks due to the three isotopes of magnesium (mass-to-charge ratio $m/z = 24, 25$). The peaks is very distinct, and seems to be more indicative of dust from sand sources, rather than fine soils. This particle type was observed frequently in the other three sand samples from other locations. The distinctive feature in this spectrum is the peaks at mass-to-charge ratio ($m/z$) equal to 156. This cluster of peaks appears to have isotope features that would be indicative of its being a silicon-containing cluster.

Figure 3b displays the positive ion mass spectrum of a 1.6 $\mu$m particle resulting from the suspension of a soil sample from the Nei Meng-gu province. The spectrum contains peaks due to sodium, potassium, magnesium and iron. The distinctive feature in this spectrum, however, that is, the relative ratios appear to vary considerably from one particle to another, indicating that several species may be responsible. Figure 3b shows the most common types of positive ion mass spectra observed during analysis of soil dust. However, the composition of soil particles shows considerably greater variability than those three mass spectra. In Figure 3, four more positive ion mass spectra are shown that display the diversity of the composition of soil particles. These particles are minor types that are observed in the soil samples analyzed. The chemical heterogeneity observed in soil dust particles is a result of the large number of mineral species that are present in soil, and thus one particle can be enriched with an element that is not present in another particle.

Figure 3c shows a positive ion mass spectrum for a 2.1 $\mu$m particle. This particle resulted from the suspension of soil from Hei Long-jiang province. In this spectrum,
several elements common to soil are observed, with the mass-to-charge ratio \((m/z)\) being equal to 23, 27, 39 and 56. These peaks are due to the most common isotopes of sodium, aluminum, potassium and iron. Two peaks are also observed at \(m/z\) equal to 49 and 66, probably corresponding to the presence of carbonate ion. Mass-to-charge ratios of 49, 66 are observed in particles generated from all four soil samples studied, although they are detected less frequently than the more common cations, such as sodium, aluminum, potassium, and iron.

In Figure 3d, a positive ion mass spectrum for another soil particle from the An Hui province is shown, and it measures 1.4 \(\mu m\) in diameter. The presence of sodium, potassium, and iron in the spectrum is observed. Both elements are observed in particles from several soil samples, typically, in over 56% of all the particles.

The aerosol time-of-flight mass spectrometry is capable of providing real-time data on atmospheric particles over extended periods of time [11]. And it has been used to monitor real-time changes in the observed composition of sea-salt particles. The data acquired using ATOFMS show a variation in sensitivity toward different elements. Such differences in chemical sensitivity due to the laser desorption/ionization process have been reported previously for the analogous off-line technique, LAMMA [8]. In current studies, we have determined the ATOFMS instrumental response factors for a number of elements. Since sodium is the most commonly observed element in the soil particles, all response factors are scaled relative to the measured sensitivity of sodium. Furthermore, because it is an alkali metal, sodium is relatively easy to ionize, making ATOFMS sensitive to its presence in particles.

4. Conclusions

This paper demonstrates how ATOFMS can be used for characterization of individual particles from the soil dust. ATOFMS measures both the size and chemical composition for single particles. In general, the particles that result from suspension of dust are large (> 1 \(\mu m\)). Although the chemical composition observed in the particles is very heterogeneous, it is possible to identify several broad classes of soil dust particles based on the combination of elements present in the mass spectrum. These differences in composition can then be used to monitor the presence of those particle types in the atmosphere in real-time.

The presence of cations due to sodium, potassium, and iron is typically observed in positive ion mass spectra, except in potassium-rich particles, where potassium clusters dominate. Using these markers, particles resulting from the suspension of soil dust can be identified in complex ambient samples. Ultimately, by providing sufficient characteristics of soil sources, it will be possible to measure atmospheric contributions of dust particles from different sources or locations on an individual particle basis.

Acknowledgements – This work was financed by the National Science Foundation of China (20477043) and Knowledge Innovation Foundation of the Chinese Academy of Sciences (KJCX2-SW-H08).
Reference


Received February 10, 2007
in revised form July 5, 2007
Investigation of strained InGaAs layers on GaAs substrate

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A set of In₁₀.₁₃Ga₀.₈₇As layers of various thicknesses on GaAs substrate has been grown by low pressure metalorganic vapour phase epitaxy (LP MOVPE). The initial stage of relaxation process has been investigated and critical layer thickness (CLT) has been determined. The investigations were performed by applying atomic force microscopy (AFM), high resolution X-ray diffractometry (HR XRD) with conventional and synchrotron radiation. The value of CLT determined by AFM observations agrees with that obtained from diffuse scattering measurements. The value is in agreement with HR XRD results.

Keywords: low pressure metalorganic vapour phase epitaxy (LP MOVPE), strained InGaAs layer, critical layer thickness, high resolution X-ray diffractometry (HR XRD), diffuse scattering, atomic force microscopy (AFM), misfit dislocation, plastic relaxation.

1. Introduction

The growth of strained epitaxial films is of fundamental importance to the fabrication of modern electronic devices. The AlGaAs/InₓGa₁₋ₓAs/GaAs structures are the material system preferred for higher frequency and lower-noise field effect transistors over AlGaAs/GaAs system due to its band construction and superior transport properties [1, 2]. The increase in the indium content x in those structures leads to a larger conduction band discontinuity and, therefore to improved electron confinement in the channel [3]. However, the lattice mismatch between InGaAs and GaAs materials can generate defects in the strained InGaAs layer. Above the critical layer thickness, the lattice strain is not accommodated elastically and misfit dislocations are introduced at the interface [4, 5]. The knowledge of critical layer thickness and the onset of misfit dislocations generation is necessary to achieve high performance of the devices.

The onset of misfit dislocation generation varies with growth conditions, such as growth temperature and growth rate. The aim of the paper is to compare the sensitivity of AFM to HR XRD, concerning the determination of the onset of misfit dislocation
generation. Our work was concentrated on the growth of $\text{In}_{0.13}\text{Ga}_{0.87}\text{As}$ layers of various thicknesses on GaAs substrate. The lattice mismatch was $\Delta a/a = 9.3 \times 10^{-3}$ and the calculated critical layer thickness (CLT) was $d_C = 20$ nm.

2. Experiment

The $\text{In}_{0.13}\text{Ga}_{0.87}\text{As}/\text{GaAs}$ epitaxial layers of various thicknesses were grown, using low pressure metalorganic vapour phase epitaxy (LP-MOVPE) on exactly oriented GaAs(001) substrates. A horizontal quartz reactor (AIX 200) and IR heated graphite susceptor were used. Trimethylindium (TMIn) and trimethylgalium (TMGa) were used for In and Ga sources, respectively, and 100% arsine (AsH₃) and phosphine (PH₃) were applied for group V elements with palladium-purified hydrogen carrier gas. During the layer growth, the reactor pressure and temperature were maintained at 20 mbar and at 700 °C, respectively. The V/III ratio was unchanged during the growth and amounted to 87. The growth rate of $\text{In}_{0.13}\text{Ga}_{0.87}\text{As}$ layer was equal to $r = 2.7 \text{Å/s}$.

The structural properties of $\text{In}_{0.13}\text{Ga}_{0.87}\text{As}$ layers were investigated by two techniques: atomic force microscopy (AFM) and high resolution X-ray diffractometry (HR XRD) with conventional and synchrotron radiation. The investigation of sample surfaces by AFM allowed us to notice the onset of misfit dislocation generation and the type of growth. HR XRD was used to obtain information about composition, thickness and layer crystalline quality – the presence of misfit dislocation in the interface.

3. Results and discussion

The $\text{In}_{0.13}\text{Ga}_{0.87}\text{As}$ layers of various thicknesses (10, 30, 58, 80 and 100 nm) were observed under Nomarski microscope. The layer surfaces of thicknesses of 10 nm and 30 nm were flat, without any defects. Starting from the layer thickness of 58 nm, a cross-hatch pattern was visible, which resulted in the onset of misfit dislocation generation. The following figures show images of layer surfaces with thicknesses: 10 nm (Fig. 1a), 58 nm (Fig. 1b) and 80 nm (Fig. 1c), respectively. The surface image of 30 nm InGaAs layer (not shown in Fig. 1) is similar to the one of 10 nm and the 100 nm InGaAs surface image is like the image of 80 nm layer.

The surface morphology of these films was studied by AFM (Fig. 2). The surface of 10 nm $\text{In}_{0.13}\text{Ga}_{0.87}\text{As}$ layer shows a regularly repeated terraces finished with monolayer steps (2D growth mode) – Fig. 2a. The average terrace width was 550 nm and surface roughness was $R_a = 0.85$ Å. With layer thickness increasing to 30 nm, the elastic strain energy builds up to the point, where it becomes energetically favourable to form misfit dislocation at the interface. Low ridge density appeared on the 30 nm $\text{In}_{0.13}\text{Ga}_{0.87}\text{As}$ layer surface. Average geometrical parameters of these ridges are: the width – 400 nm and the height – 4.5 Å. The distance between two neighbouring ridges was 2000 nm.
Fig. 1. Nomarski microscope images of In$_{0.13}$Ga$_{0.87}$As layer surfaces of various thicknesses: 10 nm (a), 58 nm (b), 80 nm (c); (×500).

Fig. 2. AFM images of In$_{0.13}$Ga$_{0.87}$As layer surfaces of various thicknesses: 10 nm (a), 58 nm (b), 80 nm (c).
Two-dimensional growth mode is still maintained, which is shown in Fig. 2b, but the average terrace width decreases in comparison with the terrace width on 10 nm layer surface which is 281 nm. Finally, high ridge density can be seen on the surface at the layer thickness of 58 nm, as seen in Fig. 2b. The average distance between two neighbouring ridges is 830 nm. As the layer thickness increases further (80 nm and 100 nm), the ridges density becomes higher. The occurrence of ridges on the surface is connected to the appearance of high misfit dislocation density in the interface. Hence one can conclude that the layer thickness, when dislocations start to appear, is below 30 nm.

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The In$_{0.13}$Ga$_{0.87}$As layer thicknesses in the range of 10–80 nm were also investigated by HR XRD method with conventional and synchrotron radiation. The (004) rocking curves of In$_{0.13}$Ga$_{0.87}$As layers for three values of thickness measured by means of conventional radiation were shown in Fig. 3.

It results from Scherrer effect that the reflection width from the layer increases when the thickness decreases. The layer reflection from InGaAs thickness of 10 nm is very wide and invisible when the conventional radiation is applied for measurements.

During the epitaxial growth, due to the lattice mismatch, the grown layers exhibit laterally compressive strain and consequently vertically tensile strain. Thus, the vertical lattice parameter is larger than the lattice constant of a strain-free In$_{0.13}$Ga$_{0.87}$As layer. The strain energy accumulated in the layer increases with the increase in the layer thickness. Above the critical thickness the layer stress is released by generation of misfit dislocations. As a result, the average vertical lattice spacing is decreasing. In this state the shift between the substrate and the epilayer reflections is measurable. In Figure 3 one can see that the shift is observable for the In$_{0.13}$Ga$_{0.87}$As
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layer thickness of 80 nm and that it is equal to 0.6016 in the $2\theta$ scale. Up to the layer thickness of 58 nm the shift was not observed. Hence, at which the lattice relaxation starts to appear for the epilayer thickness above 58 nm.

From the literature [4] it is known that the diffuse scattering is connected to misfit dislocations present in the interface. In order to confirm dislocation presence, we investigated the distribution of the intensity around the reciprocal lattice points 004 for layer thicknesses of 30, 58 and 80 nm, respectively. Reciprocal lattice maps 004 of epitaxial layers measured by means of synchrotron radiation were shown in Fig. 4.

Diffuse scattering distributed on both sides of vertical band is visible besides of the vertical band with interferential maximums. This diffuse scattering is connected to misfit dislocations (Figs. 4a and 4b). Figure 4c presents the distribution of scattered X-rays in the vicinity of 004 RLP for the 80 nm thick layer. In this case the diffuse component of the scattering which comes from the huge density of dislocations located in interface is dominant. The coherent strains of the layer are relaxed by dislocations which strongly contribute to the diffuse scattering. The strain relaxation causes the shift of the layer peak in the $\theta/2\theta$ scan in Fig. 3.

Examining of the distribution of the diffuse scattering intensity allows to state that the low misfit dislocation density is present even in the 30 nm layer. Thus, the misfit dislocations at the 30 nm layer heterointerface exist, but the shift between the substrate and epilayer is too small to be observed ($\Delta a/a = 1 \times 10^{-5}$).

The layer surface observed under Nomarski microscope has no cross-hatch at the small layer thickness (30 nm) when the linear density of misfit dislocation is low. About 100 of misfit dislocation ($\approx 100 b_r$, where $b_r$ – normal component of the Burger vectors) is needed to see one dislocation jog on the surface.

Fig. 4. Reciprocal lattice maps of In$_{0.13}$Ga$_{0.87}$As layers: 30 nm (a), 58 nm (b), and 80 nm (c).
Stress relaxation process is advanced for In$_{0.13}$Ga$_{0.87}$As layer of 80 nm thickness and the shift between this layer and 58 nm one is visible when we analyze of rocking curves (Fig. 3). Dislocations density was calculated on the basis of the shift value in the map and it is $1 \times 10^{-2}$ Å$^{-1}$ [6]. Indium content $x$ in In$_x$Ga$_{1-x}$As layer was determined from rocking curves of strained layers with thicknesses of 30 nm and 58 nm.

On the basis of these investigations, the generation dislocations point was assessed to be lower than 30 nm and to equal the one obtained from AFM observations.

4. Conclusions

We have examined the onset of dislocations generation in the In$_{0.13}$Ga$_{0.87}$As layers grown by LP MOVPE. The investigated layers thickness was in the range from 10 to 100 nm. Investigations were performed, applying two techniques: AFM, HR XRD with conventional and synchrotron radiation. The value of the layer thickness, when the misfit dislocations generation process starts (determined by AFM observations), is in agreement with the one measured by HR XRD with synchrotron radiation. This confirms high precision of AFM method. However, the main relaxation process, which should be considered as far as the layer structural quality is concerned, is related to the shift of the epilayer reflection.

Acknowledgments – The heterostructures were grown at the Institute of Electronic Materials Technology (Warsaw, Poland). The authors would like to thank dr. F. Eichhorn from ESRF in Grenoble for the help with synchrotron measurements.

References


Received March 7, 2007
in revised form June 15, 2007
Soliton pairing of two coaxially co-propagating mutually incoherent 1-D beams in Kerr type media

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In this paper, we have developed a theory (using parabolic equation approach) of coupled propagation of two coaxially co-propagating and mutually incoherent bright 1-D beams in Kerr type media. We have provided a detailed account of the propagation behavior and condition of formation of spatial soliton pairs for various coupling coefficients ($\kappa = 1, 2/3, 2$) when wavelengths and widths of the beams are the same/different. We have also identified conditions for a distinct type of coupled propagation. Our simple and straightforward theory presents many features of co-propagating beams which are in agreement with the features reported earlier using coupled nonlinear Schrödinger equation (NLSE). The paper adds to the understanding of coupled propagation by revealing many additional features not reported earlier.

Keywords: Kerr media, self-focusing, spatial solitons, soliton pairing.

1. Introduction

Formation of optical spatial soliton has attracted a lot of attention following the progress on photorefractive solitons [1], quadratic solitons [2] and solitons in saturable nonlinear media [3]. Investigation of soliton formation, interaction and soliton induced waveguide is of high interest due to their potential applications in all-optical switching and all-optical interconnects [4, 5], as well as waveguide applications [6–8]. Coupled spatial soliton pairs obtained using two co-propagating beams in nonlinear media are a special case of multicomponent solitons being studied starting from the early 1970’s [9] and are important in all-optical switching devices (see, for example, references [10–13]) and therefore, such pairing has always been an intriguing issue among spatial soliton interactions. Interaction of two spatial/temporal co-propagating solitons in bulk/waveguide media, and the possibility of formation of bright and/or dark soliton pairs have already been discussed in many papers, for example, in [14–20].

In the present paper, we have extended the work of [19, 21] to obtain general coupled propagation equations for two co-propagating 1-D beams. The theory presented is based on WKB and paraxial ray approximation and the assumption that the beams maintain their Gaussian shape while the widths vary along propagation.
length. Using these equations, we have provided a detailed explanation of the coupled propagation of two bright beams in Kerr media. We have identified conditions for distinct types of coupled propagation. Considering propagation of beams in all possible physical situations and parameters, we have obtained solitonic solutions for various coupling coefficients ($\kappa = 1, 2/3, 2$) when wavelengths and widths of the beams are the same/different.

It is worth mentioning here that a similar theory can be found in [19] for two co-propagating 2-D beams (whereas the present theory is for 1-D beams). However, in that theory, the choice of constants $\varepsilon_{ij}$ is not obvious and becomes very difficult particularly when coupling coefficient is not equal to unity. All results given in [19] could be reproduced by considering 2-D beams and coupling coefficient equal to unity in the present theory. In addition, the present theory is also capable of dealing with other possible cases like coupling coefficient other than unity and different/same beam widths. Therefore, the present theory is more versatile and simple.

The chief aim of this paper is to provide a simple and straightforward theory of coaxially co-propagating 1-D beams in nonlinear media. Another goal is to provide a physically intuitive understanding of the coupled propagation of coaxially co-propagating 1-D beams in all possible physical situations and parameters.

2. Theory of coupled propagation

2.1. One dimensional (1-D) bright Gaussian beams

A Gaussian beam of elliptical cross-section could, in general, be expressed as

$$A_1^2(z) = \frac{E_{01}^2}{f_x f_y} \exp \left(-\frac{x^2}{r_x^2 f_x^2} - \frac{y^2}{r_y^2 f_y^2}\right)$$

where $A_1$ is the real amplitude of the electric vector of the beam, $f_x$ and $f_y$ are the dimensionless beam width parameters with the initial value 1, and $r_x$ and $r_y$ are the initial widths of the beam along the $x$- and $y$-directions, respectively.

A 1-D optical beam could be viewed as a beam of elliptical cross-section with finite minor axis and infinite major axis, i.e., $r_y \to \infty$, therefore

$$A_1^2(z) = \frac{E_{01}^2}{f_x f_y} \exp \left(-\frac{x^2}{r_x^2 f_x^2}\right)$$

In the above equation, the dimensionless beam width parameter $f_y$ remains constant at its initial value 1 as the beam does not diffract along the $y$-axis (as $r_y \to \infty$), therefore,
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we rename $x$ as $r$ in the above equation and replace the subscript $x$ by 1, i.e.,

$$A_1^2(z) = \frac{E_{01}^2}{f_x} \exp \left( -\frac{x^2}{r_x^2 f_x^2} \right)$$

Similarly, the second 1-D beam may be expressed as:

$$A_2^2(z) = \frac{E_{02}^2}{f_1} \exp \left( -\frac{r_1^2}{r_1^2 f_1^2} \right)$$

or

$$A_j^2(z) = \frac{E_{0j}^2}{f_j} \exp \left( -\frac{r_j^2}{r_j^2 f_j^2} \right), \quad j = 1, 2. \quad (1)$$

2.2. Dielectric constant of the medium

We consider coaxial co-propagation of the above-mentioned two 1-D Gaussian beams of frequencies $\omega_1$ and $\omega_2$, respectively, along the $z$-axis. These beams modify dielectric constant of the medium as [19, 21, 22]:

$$\varepsilon(\omega_1) = \varepsilon_{10} + \varphi_1(A_1, A_2) \quad (2)$$

$$\varepsilon(\omega_2) = \varepsilon_{20} + \varphi_2(A_1, A_2) \quad (3)$$

where $\varepsilon_{10}$ and $\varepsilon_{20}$ are the dielectric constants at frequencies $\omega_1$ and $\omega_2$, respectively, and $\varphi_1$ and $\varphi_2$ are the nonlinear dielectric constants. For the Kerr type nonlinear medium, $\varphi_1$ and $\varphi_2$ may be expressed as:

$$\varphi_1 = \alpha_1 A_1^2 + \kappa \alpha_2 A_2^2 \quad (4)$$

$$\varphi_2 = \kappa \alpha_1 A_1^2 + \alpha_2 A_2^2 \quad (5)$$

In equations (4) and (5), $\alpha_1$ and $\alpha_2$ are nonlinear coefficients of the medium at frequencies $\omega_1$ and $\omega_2$, respectively, $\kappa$ is the coupling coefficient of the two beams that depends on the experimental conditions, $\alpha_j A_j^2$ (for $j = 1, 2$) is the dimensionless axial electric field intensity.
In paraxial ray approximation, one can, in general, expand $\varphi_1$ and $\varphi_2$ around their value at $r = 0$. Employing $A_j$ from Eq. (1), $\varphi_i$ can be expanded by Taylor’s expansion and terms except square ones can be neglected, so

$$
\varphi_1(A_1, A_2) = \varphi_1\left[\left(\frac{\alpha_1 E_{01}^2}{f_1} + \frac{\kappa \alpha_2 E_{02}^2}{f_2}\right) - \left(\frac{\alpha_1 E_{01}^2}{r_1^2 f_1^3} + \frac{\kappa \alpha_2 E_{02}^2}{r_2^2 f_2^3}\right) r^2\right]
$$

Here \(\frac{\alpha_1 E_{01}^2}{f_1} + \frac{\kappa \alpha_2 E_{02}^2}{f_2} \gg \left(\frac{\alpha_1 E_{01}^2}{r_1^2 f_1^3} + \frac{\kappa \alpha_2 E_{02}^2}{r_2^2 f_2^3}\right) r^2\), therefore, one can write:

$$
\varphi_1 = \varphi_1\left(\frac{\alpha_1 E_{01}^2}{f_1} + \frac{\kappa \alpha_2 E_{02}^2}{f_2}\right) - \left(\frac{\alpha_1 E_{01}^2}{r_1^2 f_1^3} + \frac{\kappa \alpha_2 E_{02}^2}{r_2^2 f_2^3}\right) \varphi_1\left(\frac{\alpha_1 E_{01}^2}{f_1} + \frac{\kappa \alpha_2 E_{02}^2}{f_2}\right)
$$

where prime over $\varphi_i$ denotes derivative with respect to the argument. On simplifying the above equation, we get

$$
\varphi_1 = \varphi_1\left(\frac{\alpha_1 E_{01}^2}{f_1} + \frac{\kappa \alpha_2 E_{02}^2}{f_2}\right) - \left(\frac{\alpha_1 E_{01}^2}{r_1^2 f_1^3} + \frac{\kappa \alpha_2 E_{02}^2}{r_2^2 f_2^3}\right) \varphi_1\left(\frac{\alpha_1 E_{01}^2}{f_1} + \frac{\kappa \alpha_2 E_{02}^2}{f_2}\right)
$$

(6)

since $\varphi_1\left(\frac{\alpha_1 E_{01}^2}{f_1} + \frac{\kappa \alpha_2 E_{02}^2}{f_2}\right) = 1$.

Similarly, one can obtain

$$
\varphi_2 = \varphi_2\left(\frac{\kappa \alpha_1 E_{01}^2}{f_1} + \frac{\alpha_2 E_{02}^2}{f_2}\right) - \left(\frac{\kappa \alpha_1 E_{01}^2}{r_1^2 f_1^3} + \frac{\alpha_2 E_{02}^2}{r_2^2 f_2^3}\right) \varphi_2\left(\frac{\kappa \alpha_1 E_{01}^2}{f_1} + \frac{\alpha_2 E_{02}^2}{f_2}\right)
$$

(7)

2.3. Coupled propagation of beams

In a medium described by Eqs. (2) and (3), the electric vector of the waves are governed by Maxwell’s equations, which in WKB approximation reduce to the wave equation

$$
\nabla^2 E - \frac{1}{c^2} \frac{\partial^2 D}{\partial t^2} = 0
$$

(8)

where $D = \varepsilon E$ is the electric displacement vector. For slowly converging or slowly diverging beams, Eq. (8) can be satisfied by the following solutions

$$
E = E_1 \exp\left[i(\omega_1 t - k_1 z)\right] + E_2 \exp\left[i(\omega_2 t - k_2 z)\right]
$$

(9)
where $E_1$ and $E_2$ are the space dependent complex amplitudes and $k_1 = \frac{\omega_1}{c}\sqrt{\varepsilon_{10}}$ and $k_2 = \frac{\omega_2}{c}\sqrt{\varepsilon_{20}}$ are the propagation constants.

Eikonal can be introduced to describe $E_1$ and $E_2$ as

\begin{equation}
E_1 = A_1 \exp[-ik_1S_1]
\end{equation}

\begin{equation}
E_2 = A_2 \exp[-ik_2S_2]
\end{equation}

Here, $A_1$ and $A_2$ are the space dependent real amplitudes. On substituting Eqs. (9)–(11) in the wave equation, we get the following set of equations:

\begin{equation}
2 \frac{\partial S_j}{\partial z} + \left(\frac{\partial S_j}{\partial r}\right)^2 = \frac{\phi_j}{\varepsilon_{j0}} + \frac{1}{k_j^2 A_j} \frac{\partial^2 A_j}{\partial r^2}
\end{equation}

\begin{equation}
\frac{\partial A_j^2}{\partial z} + A_j^2 \frac{\partial^2 S_j}{\partial r^2} + \frac{\partial S_j}{\partial r} \frac{\partial A_j^2}{\partial r} = 0
\end{equation}

With subscript 1 or 2 in the above equations, we get the relevant equations for the first or the second beam. To solve Eqs. (12) and (13) we assume that the nonlinear part of the dielectric constant is much smaller than the linear part, and therefore, nonlinearity may be treated as perturbation. One may, therefore, assume generalized spherical wave solution for Eqs. (12) and (13):

\begin{equation}
S_j = \frac{r_j^2}{2} \beta_j(z) + \eta_j(z)
\end{equation}

\begin{equation}
A_j^2 = \frac{E_{0j}^2}{f_j} \exp\left(-\frac{r_j^2}{2 r_j^2 f_j^2}\right)
\end{equation}

\begin{equation}
\beta_j = \frac{1}{f_j} \frac{\partial f_j}{\partial z}
\end{equation}

It can be noted that $\beta_j$ represents the inverse of radius of curvature of the beams’ fronts, and $r_j f_j$ stand for the widths of the beams.

Using equations (14)–(16) in equation (12) and using paraxial ray approximation, i.e., $(r/r_j f_j)^4 \ll 1$, we obtain

\begin{equation}
r_j^2 \left(\frac{1}{f_j} \frac{\partial^2 f_j}{\partial z^2}\right) + 2 \frac{\partial \eta_j}{\partial z} = \frac{1}{k_j^2 A_j} \left(\frac{A_j}{r_j^2 f_j^2} - \frac{A r_j^2}{r_j^4 f_j^4}\right) + \frac{\phi_j(A_1, A_2)}{\varepsilon_{j0}}
\end{equation}
On substituting $\varphi(A_1, A_2)$ from equation (6), equation (17) takes the form (for the first beam)

$$r^2 \left( 1 \frac{\partial^2 f_1}{\partial z^2} \right) + 2 \frac{\partial \eta_1}{\partial z} = \frac{1}{k_1 A_1} \left( \frac{A_1}{r_1 f_1^2} + \frac{A_1 r^2}{r_1 f_1^4} \right) + \frac{1}{\varepsilon_{10}} \varphi \left( \frac{\alpha_1 E_{01}^2}{f_1} + \frac{\kappa \alpha_2 E_{02}^2}{f_2} \right)$$

$$- \frac{1}{\varepsilon_{10}} \left( \frac{\alpha_1 E_{01}^2 r^2}{r_1 f_1^3} + \frac{\kappa \alpha_2 E_{02}^2 r^2}{r_2 f_2^3} \right)$$

Equating the coefficients of $r^2$ on both sides of the above equation, one obtains the following propagation equation that governs the beam width parameter of the first beam with the propagation distance

$$\frac{\partial^2 f_1}{\partial z^2} = \frac{1}{k_1^2 r_1^4} - \frac{C}{\varepsilon_{10} r_1^2 f_1^3} - \frac{D f_1}{\varepsilon_{10}^2 r_2^3 f_2^4}$$

(18)

where: $C = \alpha_1 E_{01}^2$ and $D = \alpha_2 E_{02}^2$.

Similarly, the propagation equation for the second beam could be obtained as

$$\frac{\partial^2 f_2}{\partial z^2} = \frac{1}{k_2^2 r_2^4} - \frac{D}{\varepsilon_{20}^2 r_2^2 f_2^3} - \frac{\kappa C f_2}{\varepsilon_{20}^2 r_2^4 f_2^4}$$

(19)

The set of coupled Eqs. (18) and (19) governs the evolution of widths of the two beams with the propagation distance.

For self-trapped beams (spatial solitons), we must have $\partial f_j/\partial z = \partial^2 f_j/\partial z^2 = 0$. One can assume $\partial f_j/\partial z = 0$ as the initial condition of the beams. To have $\partial^2 f_j/\partial z^2 = 0$, we correspondingly need:

$$D = \frac{\varepsilon_{10}^2 r_2^2}{\kappa k_1^2 r_1^4} - \frac{C r_2^2}{\kappa r_1^2}$$

(20)

$$D = \frac{\varepsilon_{20}^2 r_2^2}{k_2^2} - \frac{\kappa C r_2^2}{r_1^2}$$

(21)

3. Numerical appreciation and discussion

It is worth mentioning here that coupling coefficient $\kappa$ depends on the experimental conditions. In the present paper, we have investigated coupled beam propagation for coupling coefficients used in earlier literature, i.e., $\kappa = 2$, $\kappa = 2/3$ (see, e.g., [14]) and $\kappa = 1$ [10–12].
3.1. Coupled propagation when coupling coefficient $\kappa = 2$

3.1.1. Case I: beams of the same frequency and the same widths

For the purpose of numerical evaluation of Eqs. (18)–(21), we choose the following set of parameters: $\omega_1 = \omega_2 = 2.7148 \times 10^{15}$ rad/s, $\epsilon_{10} = \epsilon_{20} = (1.6276)^2$, and $r_1 = r_2 = 10 \mu$m. The approach given here is valid for any other set of parameters.

In Figure 1, we plot $D$ with $C$ using equations (20) and (21) for $\kappa = 2$ and for the above mentioned parameters. In the figure, the solid line represents the solution of Eq. (20) while dashed line is the solution of Eq. (21). Point $P_1$ is also a solution of Eq. (20), which corresponds to $D = 0$ (zero power) of the second beam, therefore, the power of the first beam corresponding to $P_1$ is its self-trapped power. Similarly, the power corresponding to point $P_2$ is the self-trapped power of the second beam. The point of intersection $S$ is the common solution of Eqs. (20) and (21). Therefore, values of $C$ and $D$ at point $S$ correspond to the powers of the two beams for mutual self-trapping. In other words, if two beams are coaxially propagated in the nonlinear medium with their power corresponding to point $S$, both will simultaneously be self-trapped or they will form a spatial soliton pair. To verify mutual trapping, we choose $C$ and $D$ from the point of intersection $S$, i.e., $C = D = 4.0743 \times 10^{-5}$ and obtain the evolution of the beams’ width with the propagation distance using Eqs. (18) and (19), as shown in Fig. 2. We have plotted $0.9 \times f_2$ just to resolve $f_1$ and $f_2$. It is clearly observable that both beams are mutually self-trapped or they form a spatial soliton pair. It can be observed in Fig. 1 that the power of each beam (corresponding to point $S$) required for soliton pair is one third of the self-trapped power (corresponding to points $P_1$, $P_2$). It is also obvious that there exists only one solution for soliton pairing.

Same features have been revealed in investigation of co-propagating beams using coupled nonlinear Schrödinger equations (NLSE) [14], however, one can note that
the present treatment is much simpler and provides an intuitive picture of solitonic solution.

We go further and, to the best of our knowledge, we are the first to report conditions for distinct types of coupled propagation. We intuitively draw an arc of the circle that passes through $P_1$, $S$ and $P_2$, as shown in Fig. 3. We have identified this arc as the existence curve of rhythmic breather pair (out-of phase width oscillations of the two beams). To confirm our claim, breather pairs are obtained in Figs. 4–6 using beam powers corresponding to points $u$ ($C = 0.69 \times 10^{-4}, D = 0.2 \times 10^{-4}$), $v$ ($C = 0.53 \times 10^{-4}$),
$D = 0.3 \times 10^{-4}$) and $w \ (C = 3.7705 \times 10^{-5}, \ D = 4.3705 \times 10^{-5})$, respectively. One can notice an out-of-phase rhythm in beam width oscillations in all these figures. Similar breather pairs could be obtained from the entire arc. It can be seen that amplitude of width oscillations of the breather pair is smaller if chosen point on the arc is nearer to $S$. In fact, at point $S$, the amplitude of width oscillation becomes zero and soliton pair is formed.

On the basis of our investigations, we have divided Fig. 3 into two regions, identifying them with two distinct types of coupled propagation: region I (below the dashed arc) and region II (above the dashed arc).

Fig. 4. Rhythmic breather pair is obtained using beam powers corresponding to point $u \ (C = 0.755 \times 10^{-5}, \ D = 1.0 \times 10^{-4})$ of Fig. 3.

Fig. 5. Rhythmic breather pair is obtained using beam powers corresponding to point $v \ (C = 2.111 \times 10^{-5}, \ D = 6.9853 \times 10^{-5})$ of Fig. 3.
Breather pair obtained from region I (\(C = 0.47 \times 10^{-4}, D = 0.3 \times 10^{-4}\)) is shown in Fig. 7 by dashed lines. One can notice that the pair first defocuses and then focuses while breathing and widths of beams oscillate about a width larger than the initial one. We confirmed that all breather pairs obtained from region I exhibit the same features. Breather pair obtained from region II (\(C = 0.67 \times 10^{-5}, D = 0.3 \times 10^{-5}\)) is shown by solid line. One can notice that the pair first focuses and then defocuses while breathing and widths of beams oscillate about a width smaller than the initial one.

One can notice a remarkable difference in the rhythmic breather pairs obtained from the arc, breather pairs of region I and breather pairs of region II.
3.1.2. Case II: beams of different frequencies but the same width

We choose different frequencies than in the earlier set of parameters, \( \omega_1 = 2.7148 \times 10^{15} \text{ rad/s} \) and \( \omega_2 = 2.5148 \times 10^{15} \text{ rad/s} \), keeping the rest of the parameters unchanged. For those parameters, Fig. 1 gets modified to Fig. 8. The solitonic solution shifts towards the \( x \)-axis with \( C = 5.42 \times 10^{-4} \), \( D = 3.39 \times 10^{-4} \) (for \( \omega_1 < \omega_2 \), solitonic solution shifts towards the \( y \)-axis). It can be easily seen from Eqs. (20) and (21) that solitonic solution exists in the \(+x, +y\) quadrant only for the frequency

![Graph](image1)

Fig. 8. With beams of the same width but different frequencies, \( \omega_1 = 2.7148 \times 10^{15} \text{ rad/s} \), \( \omega_2 = 2.5148 \times 10^{15} \text{ rad/s} \), and \( r_1 = r_2 = 10 \mu m \), Fig. 3 is modified as shown. The solitonic solution shifts towards the \( x \)-axis with \( C = 5.42 \times 10^{-4} \), \( D = 3.39 \times 10^{-4} \).

![Graph](image2)

Fig. 9. With beams of different widths but the same frequency, \( \omega_1 = \omega_2 = 2.7148 \times 10^{15} \text{ rad/s} \), \( r_1 = 10 \mu m \) and \( r_2 = 9.5 \mu m \), Fig. 3 is modified as shown. The solitonic solution shifts towards the \( x \)-axis with \( C = 5.92 \times 10^{-4} \), \( D = 2.84 \times 10^{-4} \).
ratio within the range $\sqrt{1/\kappa} < \omega_1/\omega_2 < \sqrt{\kappa}$. Beyond this range, solitonic solution goes beyond the $+x, +y$ quadrant. Physical interpretation of the above is that no soliton pair of bright beams exists for the beam frequency ratio beyond the aforementioned range. The same results have been obtained in [14] using comparatively complex NLSE. Through numerical investigations, we confirmed here also that one can have regions I and II by drawing an arc of the circle that passes through $P_1, S$ and $P_2$ (see Fig. 8). The behavior of coupled propagation in this case in different regions is similar to that of the previous case.

### 3.1.3. Case III: beams of the same frequency but different widths

Before discussing this case it is worthwhile to mention here that in coupled NLSE equations, the widths of two beams must be postulated to be identical to have solitonic solution, whereas in reality, those are often far from being equal [18]. The present theory provides solitonic solutions for coupled beams of unequal widths.

We choose the same frequency $\omega_1 = \omega_2 = 2.7148 \times 10^{15}$ rad/s and different widths $r_1 = 10 \, \mu m$ and $r_2 = 9.5 \, \mu m$ of the two beams, the other parameters being the same as in of Section 3.1.1. For the parameters chosen, Fig. 1 gets modified to Fig. 9, i.e., solitonic solution shifts towards the $x$-axis with $C = 0.592 \times 10^{-4}$, $D = 0.284 \times 10^{-4}$. For $r_2 > r_1$, solitonic solution shifts towards the $y$-axis. It could be easily shown using Eqs. (20) and (21) that solitonic solution exists in the $+x, +y$ quadrant only if the beam width ratio lies within the range $(1/\kappa)^{1/4} < r_2/r_1 < \kappa^{1/4}$, in other words, no solitonic pair of two bright beams exists for the beam width ratio beyond this range.

### 3.2. Coupled propagation when coupling coefficient $\kappa = 2/3$

We choose $\kappa = 2/3$ keeping other parameters the same as those of Section 3.1.1. For the parameters chosen, Fig. 3 is modified to Fig. 10. In this case, the power of each beam required for soliton pair is 60% of the self-trapped power of the individual beam, moreover, only one solution exists for soliton pairing.

We have also confirmed here through numerical investigation that the existence curve of rhythmic breather pair is the arc of the circle that passes through $P_1, S$ and $P_2$, as shown in Fig. 10, and no soliton pair of the same beam widths exists for the frequency ratio beyond the range $\sqrt{1/\kappa} < \omega_1/\omega_2 < \sqrt{\kappa}$. Propagation characteristics of the co-propagating beams in this case in regions I and II are the exactly the same as in the case of $\kappa = 2$.

### 3.3. Coupled propagation when coupling coefficient $\kappa = 1$

#### 3.3.1. Case I: two beams of the same frequency

For the set of parameters used in Section 3.1.1, an interesting and important situation arises when the coupling coefficient is unity, i.e., $\kappa = 1$. The solutions of Eqs. (20) and
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Fig. 10. For parameters of Fig. 3 and \( \kappa = 2/3 \), Fig. 3 is modified as shown. In this case, the power of each beam required for soliton pair is 60% of the power of the beam required to form a single soliton. Only one solution exists for soliton pairing.

(21) merge and form a single line, as shown in Fig. 11. Every point of this line corresponds to the powers of the two beams required to from one soliton pair as every point is the common solution of Eqs. (20) and (21). One such soliton pair is shown by solid lines in Fig. 12, where 0.95\( f_2 \) has been plotted just to resolve \( f_1 \) and \( f_2 \). If beam powers are chosen from a point below the existence line of Fig. 11, both beams mutually defocus and then focus as shown by dotted lines in Fig. 12, and if those are

Fig. 11. An interesting situation arises for \( \kappa = 1 \). For the parameters of Fig. 3 and \( \kappa = 1 \), solutions of Eqs. (20) and (21) merge and form a single existence line of soliton pair as shown. Every point of this line corresponds to the powers of beams of one soliton pair.
3.3.2. Case II: two beams of different frequencies

We choose different frequencies of the two beams $\omega_1 = 2.7148 \times 10^{15}$ rad/s and $\omega_2 = 2.5148 \times 10^{15}$ rad/s, the other parameters being the same as in Section 3.3.1. For the parameters chosen, solutions of Eqs. (20) and (21) form two parallel lines, as shown in Fig. 13, in other words, no solitonic pair exists of different frequencies and the same width (when $\kappa = 1$). However, if we equate Eqs. (20) and (21) with $\kappa = 1$, we get a condition for the beam widths to have soliton pairs, which is $r_2/r_1 = (\omega_1/\omega_2)^{1/2}$. If we use this beam width ratio in Eqs. (20) and (21), both solutions merge, as shown in Fig. 14. We have confirmed that every point of this line provides powers of beams for one soliton pair. In summary, when the coupling coefficient is equal to unity, soliton pairs of different frequencies and the same width do not exist. Pair formation becomes possible only if the width ratio is chosen as mentioned above.

4. Conclusions

Using parabolic equation approach, we have developed a theory of coupled propagation of two coaxially co-propagating and mutually incoherent bright 1-D beams in Kerr media. Propagation behavior and condition for the spatial soliton pairs to be formed have been investigated in detail for all possible situations and parameters,
Soliton pairing of two coaxially co-propagating mutually incoherent 1-D beams ...

and conditions for distinct types of coupled propagation have been identified. It is shown that:

– only one solution exists for solitonic pairing when coupling coefficient is different from unity;
– if coupling coefficient is different from unity, solitonic pairing is possible with same/different beam widths and/or same/different frequency of the beams, while, in
case of coupling coefficient equal to unity, solitonic pairing with different frequencies is only possible with different beams' widths;
– infinite solutions for solitonic pairing exist when coupling coefficient is unity and beam widths and frequencies are the same.

Acknowledgments – S. Medhekar and P.P. Paltani acknowledge the Board of Research in Nuclear Sciences (BRNS), Government of India, for financial assistance throughout the research project no. 2003/34/19/BRNS. R.K. Sarkar thanks Birla Institute of Technology for being granted a fellowship. Moral support and encouragement from H.C. Pande, S.K. Mukherjee and P.K. Barhai are gratefully acknowledged.

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Received April 15, 2007
in revised form June 29, 2007
The behavior of the Poynting vector in the area of elementary polarization singularities

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The behavior of the Poynting vector in the area of elementary polarization singularities with one or two C-points, which are bounded by regular shape s-contour is considered. It has been shown that C-points are associated with the “vortex” kind singularities of the averaged transversal component of the Poynting vector if the handedness factor and topological charge of C-point are characterized by different signs. “Passive” Poynting singularities arise in the area if the signs are the same. It has been shown that the positions of the Poynting singularities shift relatively to the C-points under the phase and amplitude asymmetry of orthogonal components of the resulting field. The results of the computer simulation are presented.

Keywords: angular momentum, Poynting vector, polarization, vortex, polarization singularities, s-contour, C-point.

1. Introduction

One of the theoretical aspects of the rapidly developing area of modern optical technology – elaboration of new kinds of optical tweezers [1] – is connected with the fact that vortex beams, polarized waves (both homogeneous and heterogeneous ones) possess an angular momentum [1–3]. The appearance of a controlled angular momentum provides the possibility of the controlled rotation of the micro objects locked by corresponding optical traps. An angular momentum of a field may be considered in each space point. An averaged angular momentum may be also considered for some space area. As it is well known the angular momentum may be decomposed into the spin angular momentum associated with elliptical polarization and the orbital one that is produced by the structure of a beam (see, for example, [2, 4]). However, such angular momentum is characterized by not only the magnitude, but also by the point of “applying”. As a result some ambiguity appears. At the same time, another physical quantity closely connected with the angular momentum, namely the space distribution of the Poynting vector characteristics (or rather its transversal component) is a univocal function of the coordinates of each field point.

Distribution of the Poynting vector parameters for Laguerre–Gaussian beams was considered in [5, 6]. However, only the behavior of an averaged Poynting vector for
a homogeneously polarized field and “symmetrical” beams was analyzed [2, 5]. Therefore, in our opinion the problem must be investigated in more details. The reasons of this are the following:

1. The angular momentum of the heterogeneously polarized field of more general kind must be investigated.

2. The behavior of an instantaneous Poynting vector must be considered because the analysis of the averaged one has some sense only in the case when the field influences the physical system with relaxation time which is significantly longer than the vibration period of a wave. Such condition is practically always satisfied for optical waves, whereas for a radio wave band it becomes problematic.

3. In our opinion, at least, for the heterogeneously polarized fields, the decomposition of the total angular momentum into the spin angular momentum, and the orbital one has no physical sense.

4. As the space distributed volumes, Poynting vector parameters have singular and stationary points. Information about the system of such points, their topological characteristics, and relationships between them, give us the possibility to predict the qualitative behavior of the Poynting vector at each field point and make the influence of the electromagnetic wave on the physical system clear to us.

The characteristics of the distributions of the Poynting vector of relatively simple heterogeneously polarized fields are considered in this paper. Instantaneous and averaged Poynting vectors, their singularities “determining” the point of “applying” of the averaged angular momentum, regularities of their singularities system formation and their relationship with conventional polarization singularities (C-points and s-contours) [7] are analyzed.

2. General assumptions

Let us assume that the paraxial approximation is satisfied. Only the fields which contain the minimal number (one or two) of C-points (points where a field is circularly polarized [7]) will be considered. Heterogeneously polarized area is limited by s-contour (line along which a field is linearly polarized [7]) of a relatively regular shape. Such field may be formed by the superposition of the vortex circular polarized beam and of the orthogonally polarized smooth one [3, 8, 9]. C-points are located in the vortex centers positions and s-contour is formed along the line where beams have equal intensities. C-point topological charge of the vibration phase \( S_C \), its Poincare index \( I_C \) (or simply C-point index) are defined by the topological charge of the “forming” vortex [10]:

\[
\begin{align*}
S_C &= \frac{1}{2} S \\
I_C &= h S_C
\end{align*}
\]
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where $h$ – the handedness factor which is equal to $\pm 1$ for right-hand and left-hand polarization regions, respectively; $S$ – the topological charge of the vortex in the vortex circularly polarized beam.

One of the possible behaviors of the superposing beams intensities is illustrated in Fig. 1. As it follows from the figure, the C-point is positioned at the point where the vortex component has exact zero of its amplitude [7].

It has been shown [3] that the instantaneous components of the Poynting vector may be written as the following relations:

$$
\begin{align*}
P_x &\approx -\frac{c}{4\pi k} \left( E_x T_2 - E_y T_1 \right) \\
P_y &\approx -\frac{c}{4\pi k} \left( E_y T_2 + E_x T_1 \right) \\
P_z &\approx \frac{c}{4\pi} \left( E_x^2 + E_y^2 \right)
\end{align*}
$$

(2)

where

$$
\begin{align*}
T_1 &= E_x \Phi_x^y - E_y \Phi_y^x + \frac{e_x^y}{\pi} E_x \frac{1}{\pi 2} - \frac{e_y^x}{\pi} E_y \frac{1}{\pi 2} \\
T_2 &= E_x \Phi_y + E_y \Phi_x^y + \frac{e_y^x}{\pi} E_x \frac{1}{\pi 2} + \frac{e_x^y}{\pi} E_y \frac{1}{\pi 2}
\end{align*}
$$

(3)

and

$$
\begin{align*}
E_i &= e_i \cos(\omega t + \Phi_i - k z) \\
E_i \frac{1}{\pi 2} &= e_i \sin(\omega t + \Phi_i - k z)
\end{align*}
$$

(4)
\( e_i, \ \Phi_i \) – the amplitudes and component phases, respectively; \( e'_i, \Phi'_i \) – their derivatives and \( i, l = x, y; \ k = 2\pi/\lambda \) – the wave number; \( \omega \) – the circular frequency of light vibration; \( c \) – light speed.

The computer simulation was performed on the basis of these relations and on the basis of the averaged in time version of them.

3. Polarization cell with one C-point.
Angular momentum in the area of C-point

The polarization cell with one C-point may be obtained in the following way. Let us assume that the vortex beam is the circular polarized isotropic vortex [11] and the smooth beam is the orthogonally polarized plane wave.

The complex amplitudes of the vortex beam and the smooth beam (in terms of linearly polarized orthogonal components) may be represented in the following form, respectively:

\[
\begin{align*}
U_{Vx} & = \rho \exp(jS\varphi) \\
U_{Vy} & = \rho \exp\left[j\left(S\varphi + h\frac{\pi}{2}\right)\right] \\
U_{Rx} & = 1 \\
U_{Ry} & = \exp(-jh\frac{\pi}{2})
\end{align*}
\]

where \( \varphi \) and \( \rho = \sqrt{x^2 + y^2} \) are the polar coordinates whose origin coincides with the vortex center; \( S \) – the topological charge of the vortex.

It can be shown that after little algebra on the basis of Eqs. (2)–(4) the instantaneous transversal components of the Poynting vector may be written as [3]:

\[
\begin{align*}
P_x & = -\frac{c}{4\pi k} \left\{ \frac{1 - Sh}{S - h} \sin\left[2(\omega t - kz)\right] - y \right\} (S - h) \\
P_y & = -\frac{c}{4\pi k} \left\{ \cos\left[2(\omega t - kz)\right] + x \right\} (S - h)
\end{align*}
\]

It can be seen that \( P_x, P_y = 0 \), when \( S = h \). In the opposite case \( (S = -h) \) we obtain:
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\[
\begin{align*}
P_x &= -\frac{c}{4\pi k} 2S \left\{ \sin \left[ 2(\omega t - kz) \right] - y \right\} \\
P_y &= -\frac{c}{4\pi k} 2S \left\{ \cos \left[ 2(\omega t - kz) \right] + x \right\}
\end{align*}
\]  

(8)

Accordingly [2, 4], z-component of angular momentum density is given by the following relation:

\[ j_z = x P_y - y P_x \]  

(9)

It can be shown that the following relation for the averaged angular momentum for the case \( S = -h \) may be obtained on the basis of Eqs. (8) and (9):

\[ \overline{M} = -\frac{2S \epsilon^2}{\alpha} J \]  

(10)

where \( J = \int_0^{\rho_0} \rho^2 d\rho \) is defined by the “power” of the vortex beam in the actual area of the cell with diameter \( 2\rho_0 \). Thus, the averaged angular momentum appears in the C-point vicinity when the signs of the vortex topological charge and handedness factor are different. In other words, the following relation must be satisfied for the angular momentum appearance with the point of application in the C-point position:

\[ S = 2S_C = -h \]  

(11)

The Poynting singularity may appear in two cases:
- all three components vanish simultaneously; this case corresponds to the appearance of the Nye’s disclination [7, 12, 13];
- only transversal component vanishes; this case corresponds to simultaneous vanishing of \( T_1 \) and \( T_2 \) (see Eq. (2)). Really, in this case, the orientation of the transversal component of the Poynting vector (its azimuth \( \theta = \arctan(P_y/P_x) \)) is indeterminate.

Figure 2 presents the behavior of the instantaneous transversal component of the Poynting vector obtained for polarization cell, defined by Eqs. (5) and (6). Distributions were calculated for the set of moments with the step equal to 1/8 of vibration period.

It can be seen, from Figure 2, that the Poynting singularity corresponding to the disclination moves along a circular \( s \)-contour. C-point is positioned in the center of the area bounded by it. It has been noted that the maximal angular momentum averaged over small time \( \delta t \) and over small area in the vicinity of this singularity is the angular momentum with the application point just in this singularity. The Poincare
index (calculated for the azimuth changes of the vector) characterizes this singularity. Let us call such singularity a “vortex” one due to the similarity of the Poynting vector “circulation” around the center of a common phase vortex [2]. It has been noted that the index equal to +1 corresponds to both possible different directions of vector circulation. Therefore, the additional parameter like chirality must be introduced for complete characterization of such Poynting vector azimuth singularity.

Fig. 2. The behavior of the instantaneous transversal component of the Poynting vector. Orientation of the component is represented by arrows. The modulus of this vector corresponds to the length of arrows.

Singularity moves along s-contour in a counterclockwise direction.

Fig. 3. Circulation of the Poynting vector transversal component around the vortex singularity.

Fig. 4. The behavior of the transversal component for different moments and for points localized along one of the diameters of s-contour. The orientation of the component is represented by arrows. The modulus of this vector corresponds to the length of arrows.
Let us assume that the field propagates toward the observer. Let the positive chirality $V = +1$ (cf. Fig. 3a) correspond to the clockwise vector circulation, and the negative chirality $V = -1$ (cf. Fig. 3b) correspond to the counterclockwise vector circulation.

The behavior of the transversal component for different moments and for points localized along one of the diameters of $s$-contour is illustrated in Fig. 4. Obviously, the averaged transversal component of the Poynting vector will be equal to zero in the C-point position and its magnitude will increase toward the $s$-contour.

The behavior of averaged transversal Poynting vector component is illustrated in Fig. 5. It can be seen that the averaged vector behavior presented is very similar to the one associated with the Poynting vector circulation in the vicinity of the common phase vortex. However, in the vortex center all three components of the averaged Poynting vector are equal to zero, whereas $z$-component is non-zero in our case. Thus, the energy current is absent along the zero line (3-D loci of vortex center) and it is maximum, as a rule, along the $z$-axis in our case.

It has been noted that the light optical trap with orbital angular momentum may be formed by focusing of such beam superposition [3].

4. Polarization cell with two C-points.

C-points of the same signs

Polarization cell with two C-points of the same signs may be obtained by superposition of a circularly polarized vortex beam with two identical vortices and an orthogonally polarized plane wave.

Figure 6 illustrates the structure of the phase of the vortex beam. The $s$-contour forming under superposition is also denoted in the figure by a white closed curve.

Accordingly [12, 13], two disclinations appear on the $s$-contour (see Fig. 7). It can be shown that the structures of these disclinations are absolutely the same. As a result, the structures of the Poynting singularities must be also the same.
Two vortex singularities associated with disclinations move along the $s$-contour. It has been noted that an additional Poynting singularity arises in the geometrical center of the area. Such additional singularity topologically connects Poynting vortices together. This is a saddle-like singularity of the Poynting vector azimuth. We will call it a “passive” singularity, because the angular momentum, averaged over small time $\delta t$, is equal to zero in its small area.

The behavior of the transversal component of the Poynting vector in the area of such singularity is illustrated in Fig. 8. Negative Poincare index characterizes this vector azimuth singularity.
The behavior of the averaged Poynting vector is illustrated in Fig. 9. It can be seen that such behavior (see Fig. 9a) differs slightly from the one corresponding to the cell with one C-point. Such azimuth distribution is practically the same if the space between C-points is small (see Fig. 9b). Nevertheless two instantaneous Poynting vortices are observed for the temporal behavior of the instantaneous Poynting vector. Corresponding distributions calculated for the set of moments with the step equaled to 1/8 of vibration period are presented in Fig. 10. The magnitude of the vector modulus is represented by the length of arrows.

5. Polarization cell with two C-points.

C-points of different signs

Polarization cell with two C-points of different signs may be obtained by the superposition of a circularly polarized vortex beam with two vortices with opposite signs and an orthogonally polarized plane wave.
The phase structure of the circularly polarized vortex beam and the position of $s$-contour of the resulting field are illustrated in Fig. 11. It can be seen that $s$-contour remains in the location denoted in Fig. 6.

Two Poynting singularities appeared, like in the case of the $C$-points of the same signs. However, they have different chiralities and these singularities move in opposite directions. From Fig. 12 (temporal step is the same like for Figs. 7 and 10) it can be

![Fig. 11. A phase map of the vortex beam. Different colors correspond to the different (within $2\pi$) phases. The position of the $s$-contour is denoted by the white line.](image)

![Fig. 12. Distributions of the instantaneous Poynting vector azimuth for the field, which contains two $C$-points with different signs. The magnitude of the vector module is represented by the length of arrows. Direction of the singularities motion is represented by thick arrows.](image)

![Fig. 13. The behavior of the transversal component of the averaged Poynting vector for the cell with the $C$-points of different signs.](image)
seen that the birth and annihilation events are observed at some moments and only when Poynting singularities are absent in the area.

The behavior of the azimuth of the averaged Poynting vector is illustrated in Fig. 13. The magnitude of the vector modulus is represented by the length of arrows. It can be seen that the modulus of the Poynting vector is practically zero in the area of C-point which has the sign of the topological charge of the vibration phase equal to the sign of the handedness factor $S_C = h/2$ (right C-point), while the vortex singularity is observed in the left C-point position.

6. “Non symmetrical” distributions

From the performed analysis it can be assumed that the positions of vortex singularities associated with the transversal component of the averaged Poynting vector coincide with positions of C-points. As a result, the points of applying the averaged field angular momentum also coincide with these points. However, this statement is not always true. This fact takes place only in the case, namely when interfering beams have symmetrical (relatively to the center of the resulting beam) distributions of amplitudes and phases.

Let us show that perturbation of amplitude and phase symmetry leads to the shift of the singularity point relatively to the C-point position. It can be shown that a principal factor is not associated with the magnitude of asymmetry between single beams, but with the mutual changes of phase difference and amplitude ratio gradients. Similarly, as it is well known, the identical changes of phases of the interfering beams (even significant ones) do not result in bending of interference fringes of the resulting pattern in a conventional interference experiment. Thus, let us assume that asymmetry is introduced only in one beam, namely in the reference one.

It can be shown that in this case (the complex amplitude of the vortex beam is given by Eq. (5)) the averaged components of the Poynting vector are given by the following relations:

$$
\begin{align*}
\mathcal{P}_x &= -\frac{c}{4\pi k} A^2 \left[ \frac{h - S}{A^2} y + \Phi_y - h \frac{A_y}{A} \right] \\
\mathcal{P}_y &= -\frac{c}{4\pi k} A^2 \left[ \frac{h - S}{A^2} x + \Phi_x - h \frac{A_x}{A} \right]
\end{align*}
$$

(12)

where $\Phi_x, \Phi_y$ – the phase derivatives; $A$ – the amplitude of the reference beam; $A^x/A$ and $A^y/A$ – the relative rate of amplitude changes.

Thus, the shift of the Poynting zero of the transversal component is determined by the following factors: the phase changes gradient of the referent beam, the gradient
of relative changes of its amplitude, and by the ratio of the intensities of vortex and reference waves.

The influence of the asymmetry was considered in the area which coincides with the vortex core [7]. Therefore, the linear approximation was satisfied for the amplitude and phase changes of a beam. Figure 14 illustrate such changes of the amplitude. The phase changes of a reference beam have a similar character.

Note that the goal of this paper is to recognize general tendencies of the Poynting vector behavior in the area of elementary polarization singularities. Therefore, strict estimations will be done in future papers. Here we used rather “indistinct” notions for the description of a reference beam like “insignificant” and “significant” asymmetries of its parameters. As “insignificant” asymmetry of a phase and an amplitude we took the magnitudes of the phase and amplitude gradients, which are less than $\pi/2$ and less than 0.5, respectively.

As it follows from Eq. (12), the location of the singularity depends also on the intensity of a reference beam. More exactly, the location of such point depends on the ratio between the intensities of a vortex and reference beams. However, accordingly to Eq. (5), the intensity of the vortex beam permanently increases if the point of interest is moved away from the center of the vortex. Therefore, we
characterize the ratio of intensities of the superposing beams by the average intensity along $s$-contour where the amplitudes of both waves are equal. The dimension of $s$-contour increases when such parameters increase too.

The results of computer simulation are presented in Figs. 16–19. Both types of the asymmetry (phase and intensity) were introduced in $x$-direction alone. The results of computer simulation for the beams without asymmetry are presented in Fig. 15 for comparison.

The singularity point coincides with the position of C-point. The $s$-contour is the regular circle. Figure 15a corresponds to the distribution of the transversal component modulus of the Poynting vector. Figure 15b illustrates the behavior of its azimuth. Different colors correspond to different (within $2\pi$) azimuths. Figure 15c presents the joint behavior of the modulus and azimuth of the Poynting vector. Orientation and magnitude of the vector are represented by the direction of arrows and their length, respectively.

Figure 16 illustrates the shifts of the Poynting singularity under the influence of the phase asymmetry. The shift magnitude increases if the phase gradient of the reference beam also enlarges. The shape and the dimension of $s$-contour and the location of C-point remain without any changes in respect to the previous case.

![Fig. 16. Shifts of the Poynting singularity under the influence of the phase asymmetry of the reference beam; a, b, c – the “insignificant” phase asymmetry of the referent beam (modulus of the phase gradient is equal to $\pi/4$); d, e, f – the “significant” phase asymmetry of the reference beam (modulus of the phase gradient is equal to $3\pi/4$); a, d correspond to the distribution of the transversal component modulus of the Poynting vector; b, e illustrate the behavior of its azimuth (different colors correspond to different, within to $2\pi$, azimuths); c, f illustrate the joint behavior of the modulus and azimuth of the Poynting vector (orientation and magnitude of the vector are represented by the direction and length of arrows, respectively).](image-url)
As it is seen from the figures, the shift of the singularity position is observed in “y-direction” (accordingly to Eq. (12)), due to the fact that phase asymmetry was introduced in “x-direction”.

Figure 17 presents the influence of the phase asymmetry for the same magnitudes of the phase gradient (modulus of phase gradient is equal to $3\pi/4$) and different ratios between vortex and reference beams. Intensity of the reference beam in the case d–f exceeds intensity of the regular beam (case a–c) by 4 times. Case a and d correspond to the distribution of the transversal component modulus of the Poynting vector. Case b and e illustrate the behavior of its azimuth. Different color correspond to different (within $2\pi$) azimuths. Case c and f illustrate the joint behavior of the modulus and azimuth of the Poynting vector. Orientation and magnitude of the vector are represented by the direction and length of arrows, respectively.

As it is seen from the figures, the shift of the singularity position is observed in “y-direction” (accordingly to Eq. (12)), due to the fact that phase asymmetry was introduced in “x-direction”.

Figure 17 presents the influence of the phase asymmetry for the same magnitudes of the phase gradient (its magnitude is equal to $3\pi/4$) and different ratios between the vortex and reference beams. Figures 17d–17f correspond to the intensity of the reference beam, which exceeds 4 times the intensity of the regular wave used in the case a–c. The dimension of s-contour increased twice. At the same time, the singularity shift in the case d–f exceeds the shift associated with the case a–c 4 times. It is interesting that for the second intensity level of the reference beam the position of the Poynting singularity is shifted to the area with another rotation direction of a field vector (handedness factor $h$ changes its sign under the crossing of the s-contour).

The influence of the intensity asymmetry on the position of the Poynting singularity is illustrated in Fig. 18. It is seen that the intensity asymmetry of the reference beam results in the transformation of the s-contour shape, depending on the intensity modulation of the regular wave. The shift of singularity is observed in the direction (x-direction), which corresponds to the direction of the intensity modulation of the reference beam. Figures 18d–18f correspond to the intensity gradient of the
Fig. 18. Shifts of the Poynting singularity under the influence of the amplitude asymmetry of the reference beam; a, b, c – the “insignificant” amplitude asymmetry of the referent beam (modulus of the amplitude gradient is equal to 0.5); d, e, f – the “significant” amplitude asymmetry of the referent beam (modulus of the amplitude gradient is equal to 1). Cases a and d correspond to the distribution of the transversal component modulus of the Poynting vector. Cases b and e illustrate the behavior of its azimuth. Different colors correspond to different (within $2\pi$) azimuths. Cases c and f illustrate the joint behavior of the modulus and azimuth of the Poynting vector. Orientation and magnitude of the vector are represented by the direction and length of arrows, respectively.

Fig. 19. Shifts of the Poynting singularity under the influence of both kinds of asymmetry (phase and amplitude) simultaneously.
reference beam, which exceeds 4 times the intensity of the regular beam used in
the case a–c. At the same time, the singularity shift in the case d–f increases only
twice in respect to the shift associated with the case a–c.

Figure 19 presents the shifts of the Poynting singularity under the influence of
both kinds of asymmetry (phase and amplitude) simultaneously. Due to such changes
of the reference beam, the shift of the singularity point takes place in both x- and
y-directions. It has been noted that the magnitude of this “joint” shift is more than
“summary” shift associated with the vector sum of the corresponding shifts resulting
from “pure” phase and amplitude asymmetries. It is seen from Figs. 19d–19f that
the singularity point shifts even to the area with another rotation direction of a field
vector.

7. Conclusions

The following conclusions can be derived from conducted studies:

1. Averaged angular momentum appears in the C-point vicinity when the signs of
the topological charge of the vibration phase and handedness factor are different and
it is equal to zero when the signs are the same. The position of vortex singularity
associated with the transversal component of the averaged Poynting vector coincides
with the position of C-point.

2. Generally the positions of the singularities of the transversal Poynting vector
component do not coincide with the C-points location. The shift of the Poynting
singularities relatively to C-points is defined by the ratio between phase gradients and
the ratio of amplitude gradients associated with the orthogonally polarized components
of a field.

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pp. L745–51.


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Received February 2, 2007
Loss analysis of single mode telecommunication fiber thermally-diffused core areas

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In this work, diffusion processes in thermally connected cylindrical fibers with weakly guiding and circular cross-section, that is, telecommunication fibers, have been presented. There have been discussed diffusion distributions of the core dopant of fibers spliced at \( T \approx 2000 \, ^\circ \text{C} \). Gaussian approximations of the core dopant concentration distribution and refractive index in the connecting area of single mode telecommunication fibers have been presented. Theoretical analysis of propagation and loss characteristics for thermally-diffused expanded core (TEC) of single mode telecommunication fibers has been performed, as well. Consistence of theoretical calculation results with experimental data, achieved on the basis of connecting telecommunication fibers with significantly different parameters, has been proved.

Keywords: single mode fibers, thermally-diffused expanded core (TEC), dopant distribution, refractive index profile, Gaussian approximation.

1. Introduction

Waveguide fibers are very thin in relation to their length. If in these types of fibers refractive index coefficients of the core \( (n_r) \) and cladding \( (n_p) \) differ insignificantly they are considered weakly guiding. Waveguide fibers with weakly guiding and circular symmetry are called telecommunication fibers.

Optimization of spliced joints of telecommunication fibers of different types involves thermal diffusion of the connected fibers core dopant (most often GeO_2) in such a way that the mode field radii in the thermally-diffused expanded core (TEC) – the splice intermediate area, will equalize [1]. It can be proved that this area with length \( 2L \) (Fig. 1) retains its single mode character as the amount of dopant in the process of thermal diffusion remains the same [2]. Normalized frequency \( V \) in the transit area is smaller than 2.405.

The present work aims at an analysis of diffusion processes within the splice, their approximation by Gaussian distributions, and evaluation of diffusion optimal times,
as well as mainly theoretical analysis of losses occurring in the transit area resulting from mismatch between the mode fields and TEC area dimensions, and finally, comparison of the analysis results with experimental data.

2. Diffusion processes in the transit area of the connected fibers

Proper matching of mode field diameters (MFD) within the thermal joint of telecommunication fibers of different types reduces considerably loss of such joints [3, 4]. Matching is the result of the core dopant thermal diffusion.

Diffusion coefficients $D$ of the core dopant GeO$_2$ in SiO$_2$ for $T \approx 2000$ °C presented in [3] were in the range $D = 10^{-11} - 10^{-10}$ m$^2$/s. However, approximating the values of $D(T)$ taken from [5] for $T \approx 2000$ °C yielded $D = 5 \times 10^{-13} - 10^{-12}$ m$^2$/s, and these were accepted in the present work.

In Figures 2 and 3, diffusion distributions are presented, with their Gaussian approximation of GeO$_2$ concentration in SiO$_2$ being given for five splice times: $t_1 = 2$ s, $t_2 = 3$ s, $t_3 = 5$ s, $t_4 = 7$ s, $t_5 = 10$ s, and fusion splicing temperature $T = 2000$ °C.

For distributions of Fig. 2, there was assumed the diffusion from a layer with limited thickness [6], that is, $h = 2a = 8$ µm, dopant concentration (before diffusion) $N_0 = 6.79 \times 10^{26}$ m$^{-3}$ (3.1 at%), which corresponds to the refractive index coefficient in the core $n_r = 1.45149$, and the refractive index coefficient in the cladding (undoped SiO$_2$) was $n_p = 1.44680$, with $n_r$ and $n_p$ calculated using the Sellmeier formula [7]. The above parameters correspond to the standard single mode telecommunication fiber with step refractive index profile (SI SMF) G.652 [8] with $\Delta\% = \left(\frac{(n_r^2 - n_p^2)}{2n_p^2}\right)\times100\% = 0.32\%$ and numerical aperture NA = 0.116 (calculations were made for $\lambda = 1.31$ µm). For calculation purposes, a diffusion coefficient $D = 5 \times 10^{-13}$ m$^2$/s was accepted.

For distributions of Fig. 3, also the diffusion from a layer with limited thickness [6] was assumed, but the core thickness was $h = 2a = 6$ µm, dopant concentration...
(before diffusion) $N_0 = 1.27 \times 10^{27} \, \text{m}^{-3}$, which corresponds to the refractive index coefficient in the core $n_r = 1.45569$, and the refractive index coefficient in the cladding (undoped SiO$_2$) was $n_p = 1.44680$ ($n_r$ and $n_p$ were calculated with the use of the Sellmeier formula [7]). The above parameters correspond to single mode telecommunication fibers with step refractive index profile SI SMF of type G.653 or G.655 [9, 10] with $\Delta% = 0.61\%$ and numerical aperture NA = 0.161 (the calculations were made for $\lambda = 1.31 \, \mu\text{m}$). In this case, diffusion coefficient $D = 10^{-12} \, \text{m}^2/\text{s}$ was accepted as along as with higher dopant concentrations an increase of the diffusion coefficient can be observed [11, 12].

Diffusion distributions were calculated with the use of an expression of diffusion from a layer with limited thickness (core diameter) $h = 2a$:

$$N(r, t) = \frac{N_0}{2} \left[ \text{erf} \left( \frac{r + h}{2 \sqrt{Dt}} \right) - \text{erf} \left( \frac{r}{2 \sqrt{Dt}} \right) \right]$$ (1)

where: $N_0$ – core dopant concentration before diffusion, $h$ – layer thickness (core diameter), $D$– diffusion coefficient, $t$ – diffusion time, $r$ – distance from the core center.

Gaussian distributions were calculated using the expression:

$$N(r, t) = N_0(0, t) \exp \left( -\frac{r^2}{A_{\text{diff}}^2} \right)$$ (2)

here: $N_0(0, t)$ – concentration in the core center decreasing with time, $A_{\text{diff}}$ – the core diameter after diffusion where the dopant concentration decreases by $e$ times $A_{\text{diff}}^2 = a^2 + 4Dt$.

Fig. 2. Dopant diffusion distributions and their Gaussian approximation for SI SMF of G.652 type.
A good consistency of diffusion and Gaussian distributions was found, which were the better the longer the diffusion time and the greater diffusion the coefficient (Figs. 2 and 3).

2.1. Refractive index profiles

It is obvious that refractive index profiles reflect well the dopant distributions. Gaussian function describes well the fundamental mode field distribution LP\(_{01}\) for the step refractive index profile [13, 14]. Attempts were made to describe the fundamental mode field distribution LP\(_{01}\) for fibers with random profile of refractive index by the Gaussian function [13–15]. It turns out that the Gaussian approximation of field distributions which significantly simplifies calculations of the connected fibers losses, is specially useful in the cases of refractive index profiles for which there is no analytic solution of a scalar wave equation [15]. Gaussian profile of refractive index is such an example. This form of profile is of great practical importance as, first of all, it reflects diffusion processes of dopants between the core and the cladding in the production process of fibers, and besides, it is possible to approximate refractive index distributions in the transit area of thermally connected telecommunication fibers of different types.

Values of GeO\(_2\) concentration were assigned corresponding values of refractive index coefficient, calculated using the Sellmeier formula [7]:

\[
n^2 = 1 + \sum_{i=1}^{3} \frac{a_i \lambda^2}{\lambda^2 - b_i^2}
\]  

(3)

Fig. 3. Dopant diffusion distributions and their Gaussian approximation for SI SMF of G.653 or G.655 type.
where $a_i$ and $b_i$ (in micrometers) are constants determined experimentally for different dopants and doped levels.

Examples of the levels of GeO$_2$ doping and the corresponding refractive indices in the core are presented in Tab. 1.

Dopant distribution after diffusion is consistent with Gaussian distribution, which leads to Guassian distribution of the refractive index:

$$n^2(r) = n_p^2 + \left( n_r^2 - n_p^2 \right) \exp \left( \frac{-r^2}{A_{\text{diff}}^2} \right)$$

where: $n_r$ is the value of the refractive index in the core center ($r = 0$) after diffusion, $A_{\text{diff}}$ is the core diameter calculated as half width on the height $1/e$ from the dopant maximum concentration $A_{\text{diff}} = \sqrt{a^2 + 4Dt}$.

---

**Table 1.** GeO$_2$ doped levels of the core and equivalent values of refractive index for $\lambda = 1.31$ µm.

<table>
<thead>
<tr>
<th>Dopant concentration [at%]</th>
<th>0</th>
<th>3.1</th>
<th>5.8</th>
<th>7.9</th>
<th>13.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dopant concentration [m$^{-3}$]</td>
<td>0</td>
<td>$6.79 \times 10^{26}$</td>
<td>$1.27 \times 10^{27}$</td>
<td>$1.73 \times 10^{27}$</td>
<td>$2.95 \times 10^{27}$</td>
</tr>
<tr>
<td>Refractive index $n_r$</td>
<td>1.44680</td>
<td>1.45149</td>
<td>1.45568</td>
<td>1.45904</td>
<td>1.46807</td>
</tr>
</tbody>
</table>

---

Fig. 4. The core refractive index profiles in SI SMF of G.652 type for five diffusion (splicing) times, $\lambda = 1.31$ µm.
Refractive index profiles for five diffusion times (splicing), calculated according to expression (4) and corresponding to the dopant distributions of Figs. 2 and 3 are presented in Figs. 4 and 5.

2.2. Matching mode field radii

Optimization of the process of thermally connecting telecommunication fibers with different parameters, involves matching mode fields of the connected fibers by a method of diffusing the core dopant [3] and minimizing the TEC area loss, see Fig. 1.

Figure 1 shows an ideal optimization when the dopant diffusion in the area of splice caused equalization of the dopant concentration and the core diameters in the connection place. This means equalization of the mode fields. It is possible at the same temperature and heating (splicing) time as for a higher dopant concentration (smaller core diameter) the diffusion coefficient is of higher value [11, 12].

However, if the mode fields of the connected fibers are not totally equalized, then, apart from transmission losses of TEC area, there also occur losses λ due to mismatch of the mode fields (see Eq. (5)). With the assumption that the fundamental mode field distribution $LP_{01}$ can be approximated with the Gaussian distribution, which is true for step and Gaussian refractive index profiles [1, 15, 16], we obtain:

![Fig. 5. The core refractive index profiles in SI SMF of G.653 or G.655 type for five diffusion (splicing) times, $\lambda = 1.31 \mu m$.](image)
where $\omega_{\text{diff}1}$, $\omega_{\text{diff}2}$ are the mode field radii of the connected fibers in the connection place (here, after diffusion, though the formula is of general character), in TEC.

The practice of splicing connecting fibers with significantly different parameters, e.g., G.652 and G.655, indicates that it is possible to arrive at a loss in those fibers lower than 0.1 dB [17] using dopant diffusion within the splice (TEC).

It is optimal, however, to rely on calculations, thus with the above given diffusion coefficients and thermal connecting temperature it is advisable to match optimal splicing times after diffusion on the basis of the core radius values, mode fields radii and TEC transmission parameters.

In Tables 2 and 3, the calculated values of the core and mode field radii for different diffusion times and two wavelengths $\lambda = 1.31 \, \mu m$ and $\lambda = 1.55 \, \mu m$, for fibers G.652 and G.655, are presented. For the Gaussian mode field distribution LP$_{01}$ [5, 16]: radii of the mode field before diffusion $\omega = a / \sqrt{\ln V}$, where $a$ – core before diffusion, and because $V = (2\pi/\lambda) a NA$ remains constant after the diffusion process [2, 15], then, $\omega_{\text{diff}} = A_{\text{diff}} / \sqrt{\ln V}$. For the accepted G.652 fiber parameters, $V = 2.23$ for $\lambda = 1.31 \, \mu m$, and $V = 1.88$ for $\lambda = 1.55 \, \mu m$. Whereas, for G.655, $V = 2.32$ for $\lambda = 1.31 \, \mu m$, and

$$A = -10 \log \left( \frac{2 \omega_{\text{diff}1} \omega_{\text{diff}2}}{\omega_{\text{diff}1}^2 + \omega_{\text{diff}2}^2} \right)^2$$

(5)
$V = 1.96$ for $\lambda = 1.55 \, \mu m$. The core and mode field dimensions before diffusion for G.652 are: $a = 4 \, \mu m$, $\omega = 4.66 \, \mu m$ for $\lambda = 1.31 \, \mu m$ and $\omega = 5.53 \, \mu m$ for $\lambda = 1.55 \, \mu m$, and for G.655: $a = 3 \, \mu m$, $\omega = 3.36 \, \mu m$ for $\lambda = 1.31 \, \mu m$ and $\omega = 3.98 \, \mu m$ for $\lambda = 1.55 \, \mu m$.

The values $\omega_{\text{diff}}$ presented in Tabs. 2 and 3 indicate that splicing times of fibers G.652 and G.655, at $T = 2000 \, ^\circ C$, optimal for equalization range from 3 to 7 seconds. Then, the losses of TEC area resulting only from the mismatch of mode field radii (5) are the smallest. In Table 4, losses in the TEC area for different splicing times due to the mismatch of mode field radii are presented.

Total losses of the TEC area are the sum of losses due to the mismatch of the mode field radii and the transit area size, that is, first of all, its length $2L$ and $\gamma_{\text{max}}$. Analysing these two components makes it possible to match the optimal thermal connecting time. It should be emphasized that connecting the analyzed fibers without diffusion results in a big splice loss (see Eq. (5)) $A = 0.457 \, \text{dB}$ for $\lambda = 1.31 \, \mu m$ and $A = 0.465 \, \text{dB}$ for $\lambda = 1.55 \, \mu m$.

### 3. Analysis of the transit area transmission properties of the spliced fibers

Even with the assumption of an ideal matching of $\omega_{\text{diff1}}$ and $\omega_{\text{diff2}}$ there will still occur the TEC transmission losses [2, 5, 15].

Transmission power coefficient of the TEC for the general propagation beam model [5] expressed by:

$$T_P = \frac{\int_0^\infty E_o(r)E_i^*(r)r \, dr}{\left(\int_0^\infty |E_i(r)|^2 r \, dr\right)^2}$$

Here, $E_i(r)$ is the field electrical component in the input TEC, $E_o(r)$ is the field electrical component in the output TEC.
Dependency curves $T_p(L, \gamma_{\text{max}})$ \cite{5} show the maximum for the TEC area characteristic length $L_c$:

$$L_c = \left[\gamma_{\text{max}}^2 (\gamma_{\text{max}} - 1)\right]^{1/2} \frac{\pi n_r \omega}{\lambda} \quad (7)$$

Splicing fibers of the same type has been assumed here, that is, with the same $n_r$ and $\omega$ (parameters before diffusion), Fig. 6. Lengths $L_c$ as a function of diffusion time ($\gamma_{\text{max}} = A_{\text{diff}}/a$) for fibers G.652 and G.655 are presented in Tab. 5.

Loss of the transit area is of course lower than the maximum if its length $L > L_c$ or $L < L_c$, and significantly smaller if $L \gg L_c$ or $L \ll L_c$ \cite{5}. When the TEC area length is $L \gg L_c$ or $L \ll L_c$ (Fig. 6), then the transmission loss is negligibly small, which is the most desirable case for optimization of thermal fibers connecting process, e.g., fusion splicing. The choice of temperature and thermal connecting times in such a way that good matching of the mode field radii and simultaneously the smallest losses connected with TEC dimensions are due to occur, finishes the process of thermal connecting optimization.

The fact that loss is small if $L \gg L_c$ or $L \ll L_c$ causes that the transmission losses in TEC area can be described by two different processes – transmission models. When

![Fig. 6. Linear TEC area with a step refractive index profile – joining fibers of the same type.](image-url)
the transit area is an equivalent of the gap (Fig. 7) between connected fibers [4, 5], when $L \gg L_c$ a phase front transformer model is used [5, 15].

### 3.1. Gap model

With a simplified assumption of connecting the same type of fibers and an assumption that the transit area changes linearly, i.e., the refractive index profile remains of step character and the core diameter changes linearly (normalized frequency remains stable in the TEC area) – Fig. 7 based on the gap model [4], the transmission coefficient of the TEC area can be expressed by:

$$ T_g = \left[ 1 + \frac{\lambda L}{\pi n_c \omega^2} \right]^{-1} $$  \hspace{1cm} (8)

The loss $-10\log T_g$ of the TEC area as a function of length $L$ for a gap model and joints G.652-G.652 with parameters accepted in the work, is presented in Fig. 8. The loss in this model does not depend on $\gamma_{\text{max}}$. This means that it does not depend on the diffusion
time and temperature during thermal fiber connecting. If we assume \( L \rightarrow 0 \),\n\( i.e., \ T_g \rightarrow 1 \), then while connecting fibers with different \( \omega \), there will occur independently diffusion, big losses resulting from the mismatch of \( \omega \) fibers “see” each other without the TEC area, Fig. 7. Thus, optimization of losses due to different \( \omega \) is practically impossible, which does not comply with experimental data \([1]\). So, the model is useless in the case of optimization by thermal diffusion method. Therefore, it is necessary to conduct the diffusion process so that the TEC area length will be \( L \gg L_c \).

### 3.2. Phase-front transformer model

Under simplified assumptions of connecting fibers of the same type and linear transformation of the transit area, \( i.e., \) the refractive index distribution retains its step character and the core diameter changes linearly, then based on a phase-front transformer model \([5,15]\), the TEC area transmission coefficient (Fig. 6) can be written as:

\[
T_f = T_A T_B T_C
\]

(9)

here,

\[
T_A = T_C = \left\{ 1 + \left[ \frac{1}{2} \gamma_{max} - 1 \right] \pi R \omega \frac{2}{\lambda L} \right\}^{-1}
\]

(10)

\[
T_B = \left\{ 1 + \gamma_{max} \left( \gamma_{max} - 1 \right) \pi R \omega \frac{2}{\lambda L} \right\}^{-1}
\]

(11)

\( T_A, T_B \) and \( T_C \) are optical power transmission coefficients at points \( A, B, C \) (see Fig. 6).

Loss \( T_f \) of the TEC area as a function of length \( L \), for a phase-front transformer and G.652-G.652 joints models with parameters accepted in this work, and five diffusion times are shown in Fig. 8; \( L_c \) can be defined approximately from the intersection of curves \( T_f(L) \) and \( T_g(L) \).

In the case of connecting fibers of different types the core dopant diffusion process should be conducted in such a way that equalization of the mode field radii is due to occur (in our case, at \( T = 2000 \degree C \), best for diffusion times from 3 to 7 seconds) and so that the transmission coefficient should be the highest.

Assuming the mode field radii equalization and linear change of the transit area, \( i.e., \) refractive index distribution retains its step character and the core diameter
changes linearly (Fig. 9), the lowest value of transmission coefficient $T_f$ – the highest loss of the transit area – will be achieved, $T_f$ being presented in the form:

$$T_f = T_A T_{B1} T_{B2} T_C$$  \hspace{1cm} (12)$$

Here,

$$T_A = \left\{ 1 + \left[ \frac{1}{2} \left( \gamma_{\max1} - 1 \right) \frac{\pi n_{r1} \omega_{1}^2}{\lambda L} \right]^2 \right\}^{-1}$$  \hspace{1cm} (13)$$

$$T_C = \left\{ 1 + \left[ \frac{1}{2} \left( \gamma_{\max2} - 1 \right) \frac{\pi n_{r2} \omega_{2}^2}{\lambda L} \right]^2 \right\}^{-1}$$  \hspace{1cm} (14)$$

$$T_{B1} = \left\{ 1 + \left[ \frac{n_{r1}^2 \omega_{1}^2}{\lambda L} \gamma_{\max1} \gamma_{\max1} - 1 \right]^2 \right\}^{-1}$$  \hspace{1cm} (15)$$

$$T_{B2} = \left\{ 1 + \left[ \frac{n_{r2}^2 \omega_{2}^2}{\lambda L} \gamma_{\max2} \gamma_{\max2} - 1 \right]^2 \right\}^{-1}$$  \hspace{1cm} (16)$$

In Equations (13)–(16), index 1 stands for the parameters for an input fiber (type G.655) and index 2 for an output fiber (type G.652), Fig. 9. Figure 10 shows loss of
TEC areas for connecting fibers of G.652 and G.655 types at five thermal connecting times ensuring good match of mode fields, i.e., 2, 3, 5, 7 and 10 seconds.

From the results presented in Fig. 10 it follows that neglecting the mode field matching, the shortest the diffusion time, the smaller the TEC area loss.

Length $2L$ of the TEC area during optimization of the splicing process (splicer Ericsson FSU 925TC) was evaluated on the basis of thermoluminescence of the spliced fibers. Figure 11 presents a thermoluminescence intensity profile, along the cores, of the fusion spliced fibers G.652 and G.655. The splicing temperature in the center was $T \approx 2000$ °C, splicing time $t_2 = 3$ s. The temperature was decreasing from the splice center until it reached room temperature, where the thermoluminescence line changes into a horizontal one from the left- to the right-hand side of the splice. The distance which can be identified as $2L$ is 675 µm and is considerably longer than $L_c$, Tab. 5. Thus, the phase-front transformer model is justified in this case.
Values of $T_f$ (12) and loss $-10\log T_f$ for $2L = 675$ µm, $\lambda = 1.31$ µm and $\lambda = 1.55$ µm, respectively, are presented in Tab. 6.

<table>
<thead>
<tr>
<th>Diffusion time [s]</th>
<th>$T_f (\lambda = 1.31 \text{ µm})$</th>
<th>$-10\log T_f (\lambda = 1.31 \text{ µm})$ [dB]</th>
<th>$T_f (\lambda = 1.55 \text{ µm})$</th>
<th>$-10\log T_f (\lambda = 1.31 \text{ µm})$ [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.995</td>
<td>0.022</td>
<td>0.993</td>
<td>0.031</td>
</tr>
<tr>
<td>3</td>
<td>0.989</td>
<td>0.048</td>
<td>0.983</td>
<td>0.074</td>
</tr>
<tr>
<td>5</td>
<td>0.964</td>
<td>0.159</td>
<td>0.951</td>
<td>0.218</td>
</tr>
<tr>
<td>7</td>
<td>0.926</td>
<td>0.334</td>
<td>0.899</td>
<td>0.462</td>
</tr>
<tr>
<td>10</td>
<td>0.852</td>
<td>0.696</td>
<td>0.801</td>
<td>0.964</td>
</tr>
</tbody>
</table>

Tab. 7. Values of $A_\Sigma$ for $\lambda = 1.31$ µm and $\lambda = 1.55$ µm.

<table>
<thead>
<tr>
<th>Diffusion time [s]</th>
<th>$A_\Sigma (\lambda = 1.31 \text{ µm})$ [dB]</th>
<th>$A_\Sigma (\lambda = 1.55 \text{ µm})$ [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.070</td>
<td>0.084</td>
</tr>
<tr>
<td>3</td>
<td>0.058</td>
<td>0.087</td>
</tr>
<tr>
<td>5</td>
<td>0.163</td>
<td>0.220</td>
</tr>
<tr>
<td>7</td>
<td>0.362</td>
<td>0.486</td>
</tr>
<tr>
<td>10</td>
<td>0.768</td>
<td>1.027</td>
</tr>
</tbody>
</table>

Values of $T_f$ (12) and loss $-10\log T_f$ [dB] for this length and for splicing times of 2–10 seconds and $\lambda = 1.31$ µm and $\lambda = 1.55$ µm, respectively, are presented in Tab. 6.

Total losses of the TEC area are the sum of losses due to the mismatch of mode field radii and losses resulting from transmission. In Table 7, summary losses

$$A_\Sigma = -10\log \left[ \frac{2 \phi_{\text{diff1}} \phi_{\text{diff2}}}{\phi_{\text{diff1}}^2 + \phi_{\text{diff2}}^2} \right]^2 - 10\log T_f$$

for the assumed by the author diffusion times are presented.

Theoretical calculations presented in Fig. 10 and Tab. 7 confirmed data from [1, 17], where diffusion times were matched experimentally, that optimal fusion times for connecting fibers with parameters such as fibers of the type G.652 and G.655 at $T \approx 2000$ °C are times ranging from 2 to 5 seconds. At the same time it should be noted that the increase of thermal connecting loss with the diffusion time (within rational limits) is mainly due to transmission properties of the TEC area rather than mismatch of the spliced fiber mode field radii.

Consideration similar as for the TEC linear areas can be given to areas with Gaussian refractive index distribution in the fibers and along TEC. Gaussian distribution assumption leads to slightly smaller loss $T_f$ of the TEC area –
propagation beam model [5] which does not change but rather confirms the above presented results and conclusions.

4. Conclusions

In the process of thermal fiber connecting there occurs the core dopant diffusion. The consistence of the dopant diffusion and Gaussian distributions was found the better the longer the diffusion time (thermal connecting) and the better the consistence. Thus, the dopant diffusion distributions and in effect the refractive index profile can be approximated with Gaussian distributions. Gaussian field distribution approximation which significantly simplifies calculation of losses due to the mismatch of mode field radii of the spliced fibers is specially useful in those cases of refractive index profiles for which there is no analytic scalar wave equation solution. Gaussian refractive index profile is such an example. This form of profile is of great importance from the practical point of view as, first, it reflects dopant diffusion processes between the core and cladding in the fiber production process, second, it can be used for approximation of refractive index in the transit areas of thermally connected telecommunication fibers of different types. In so connected fibers it is possible, thanks to proper dopant diffusion, to achieve a thermally expanded-core transit (TEC) area and join fibers with different values and refractive index coefficients almost without any losses.

The above theoretical analysis of the TEC transmission area properties showed that it is possible to achieve loss below 0.1 dB for a thermal joint (splice) of fibers such as: G.652 and G.655, whose parameters differ significantly. The results of analysis are consistent with experimental data obtained by the author.

References


Received June 7, 2007
An effective iris location method with high robustness

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Now, iris recognition is a topical issue. This paper focuses on iris location, which is an important stage in iris recognition, and develops a new method for that purpose. Use has been made of dilation operator in order to suppress the interference of eyelash, and support vector machines (SVM) classifier has been employed to identify the iris boundary. The results of experiment carried out on iris image database showed the method proposed to have an encouraging performance. Compared with the traditional methods, those of Daugman and Wildes, this method offers a higher correct location ratio and is more computationally efficient.

Keywords: iris location, boundary recognition, support vector machines (SVM).

1. Introduction

In recent years, iris recognition has become a hot area of the biometrics technology due to its high reliability, uniqueness, noninvasiveness and stability for personal identification [1–7]. The human iris is an annular part between the pupil (the black central area in the eye) and sclera (the white area in the eye). In general, the iris recognition includes iris location, feature extraction and iris matching. The literature published shows that iris location is one of the key steps in iris recognition system because it not only consumes about 20%–40% of the entire time needed for iris recognition, but directly affects the result of the latter [2, 4, 6].

This paper develops a new method for iris location aimed to save location time and improve correct location ratio. The remainder of this paper is organized as follows. Section 2 briefly reviews some existing location methods. Section 3 provides detailed descriptions of theory and steps of our method. Experimental results and discussion are provided in Section 4. Finally, Section 5 concludes the paper.
2. Related work

The iris is an annular portion between the pupil (inner boundary) and the sclera (outer boundary). Both of the boundaries can be taken as circles which are usually not concentric [1]. So, the task of iris location is to find these two circles and compute their exact parameters. Now, much work has been done on iris location, the methods of Daugman [1] and Wildes [4] being the most well-rounded ones. Daugman’s method used integrodifferential operator as circle search operator to search over the image domain for the maximum with respect to increasing radius, of the normalized contour integral along a circular arc of radius and center coordinates. Wildes’ method converted the eye image into a binary edge-map via gradient-based edge detection, then voted to get the parameters of iris boundaries by Hough transforms. However, both of them have two disadvantages:

– computationally, they are tedious. Especially, Wilde’s method [4] is very computationally demanding because it introduces lots of edge points of other objects, such as eyelashes and eyelids, in Hough transform;
– they are lacking in robustness. The correct ratio of location of them seriously drops followed the increasing eyelash or eyelid.

In order to overcome the limitation mentioned above, a new location method is developed as below. The method uses the dilation operator of morphological to locate the iris inner boundary, and the support vector machines (SVM) to gain the iris outer boundary.

3. Introduction of our approach

3.1. Iris inner boundary location

Although the iris inner boundary is easily distinguishable and can be located correctly by Daugman’s and Wildes’ methods, both of them are computationally tedious. The proposed method can reduce the expenditure of time without inducing any negative effect to location result. Considering the shape of eyelash as the fine nondirectional slit in the background of eye image, the elimination of the eyelash interference can be seen as a process filling up these fine slits by mathematical method. So, the dilation operator, is introduced into our method for inner boundary location,

\[(f \oplus b)(s, t) = \max_{(x, y) \in b} \left\{ f(s-x, t-y) + b(x, y) \mid (s-x), (t-y) \in f \right\} \]

where \( \oplus \) denotes the dilation operation, \( f \) is the eye image, \( b \) is the structuring element.

As mentioned above, the detail stage for iris inner boundary location using morphological can be listed as below, and the part of result of the stage are shown as in Fig. 1.
An effective iris location method with high robustness

1. Suppressing eyelash interference by dilation operator:

\[ I_1(x, y) = I_0(x, y) \oplus B \]  

where \( \oplus \) and \( I_0(x, y) \) denote the dilation operation and original eye image, respectively, \( B \) is a 3×3 square structuring element.

From Figure 1b, the most of eyelash melt into image background. Obviously, the dilation operator suppresses the eyelash interference effectively.

2. Binarization and edge detection. Because the gray-level value of eyelash becomes whiter, the iris inner boundary can be get easily and quickly by the binarization operation and edge detection operator in morphological. The result of edge detection is shown in Fig. 1c;

\[ I_2(x, y) = \text{Bin}_r(I_1(x, y)) - \left[ \text{Bin}_r(I_1(x, y)) \ominus B \right] \]  

where \( \text{Bin}_r \) is the binarization of image with threshold \( r \), \( \ominus \) denotes the eroding operator of morphological, \( I_2(x, y) \) is the edge detection image.

3. Obtaining the circle centre and radius of inner boundary by Hough transform:

\[ (x_p, y_p, r) = H[I_2(x, y)] \]  

where \( H \) is the Hough transform, \( (x_p, y_p, r) \) is the circle center and radius of the iris inner boundary.

As the number of pixel which votes to iris parameters through Hough transform is reduced largely, it makes Hough transform more computationally efficient. Otherwise,
because Hough transform is hardly affected by local information, the robustness of the proposed method does also enhance. Figure 1d shows the result of location.

3.2. Iris outer boundary location

From Figure 1a, we can see that the iris outer boundary is not as sharp clear as inner boundary, and can be interfered easily by eyelash, etc. The experiment shows that the location correct ratio of Daugman’s and Wildes’ methods drops down seriously following the increasing eyelash interference. In order to resolve this problem, the proposed method treats the boundary location as classification of pixels.

Support vector machine (SVM) provides an efficient approach to the problem of classification by the kernel mapping technique. The SVM can always find the global and unique minimum, and shows excellent performance in nonlinear and higher dimensional applications. Let \( S = \{(x_1, y_1), \ldots, (x_N, y_N)\} \) be a linear non-separable set. Using the Karush–Kuhn–Tucker (KKT) condition, the classification function can easily gain by the SVM [8]:

\[
    f(x) = \text{sgn} \left[ \sum_{i=1}^{N} y_i \alpha_i K(s_i, x) + b \right]
\]

where \( K(s_i, x) = \psi(s_i) \cdot \psi(x) \) is the kernel function which is the dot product of \( \psi(s_i) \) and \( \psi(x) \), \( \alpha_i \) is the Langrange multiplier, \( s_i \) is the support vector, \( b \) is constant.

Because gray-level changes or gradient of eyelash, and iris boundary are different in different directions, the iris outer boundary can be found by classification. The detail stage is shown as below and in Fig. 2.

1. Let \( T\{t_1, t_2, \ldots, t_N\} \) be the training set, \( t_i = (x_i, y_i) \) the training data composed of feature vector \( x_i \), including the gradient in a 3×3 region of an pixel \( p(m, n) \) from an sample eye image which chosen prior, and scalar \( y_i \), which is

\[
    \begin{cases}
    y_i = +1 & p(m, n) \text{ is boundary pixel} \\
    y_i = -1 & \text{otherwise}
    \end{cases}
\]

The proposed method chooses polynomials, \( K(x_i, x_j) = (x_i \cdot x_j + 1)^d \), as kernel function. Then, the SVM classifier is

\[
    f(x) = \text{sgn} \left[ \sum_{i=1}^{N} y_i \alpha_i K(s_i, x) + b \right]
\]

2. For location, the gradient information of every pixel of eye image \( P_0(m, n) \) constructs as same as the feature vector which be used in training process. Then, the vector \( x \) is identified by the SVM classifier expressed by Eq. (6). The result of classification is the iris outer boundary image \( P_1(m, n) \).
An effective iris location method with high robustness

Note that the image $P_0(m, n)$, is obtained as a result of dilation operation of the origin eye image, because the dilation operation can suppress the majority of eyelash interference and make classification more accurate.

The experiments show that the proposed SVM classifier can remove the majority of other objects, and preserve a perfect outline of iris boundary. The result is shown in Fig. 2b.

3. For obtaining the parameters of iris outer boundary, the proposed method adopts the Hough transform to calculate

$$\begin{align*}
(x_0, y_0, r) &= H[P_1(m, n)]
\end{align*}$$

where $H$ is the Hough transform, $(x_0, y_0, r)$ is the circle center and radius. Figure 2c shows the location result.

Mostly, the contour of iris outer boundary which is obtained by SVM classifier is not intact, but applying the Hough transform we can calculate the parameters of circles by that information. Of course, the Hough transform is time-saving in this case because the most of the unrelated pixel is removed.

Note that if the iris region is roughly determined at first by the gray-level value, the SVM classifier will become more computationally efficient.

4. Experiments and results

To evaluate the performance of the proposed method, we apply it to the CASIA ver1.0 [9] database, and present a detailed comparison with the methods of Daugman and Wildes, implemented according to the published papers [2, 3] and the open codes.
of MASEK [7]. All of the algorithms are implemented in MATLAB 7.0 and executed on the same computer (Celeron 2.6 GHz, CPU 192M RAM). Note that the original eye images are classified into class I (lighter interfered) and class II (seriously interfered), according to the extent to which iris boundaries are overlapped by eyelash and eyelid. All of the experiments were completed in same environment.

4.1. Qualitative comparison

This experiment is designed to compare the location performance of the three methods. Figures 3 and 4 show the location result for class I and class II, respectively.

Obviously, Fig. 3 shows that all three methods can locate the iris boundaries correctly, and there are hardly any differences between the results. Certainly, it suggests that the proposed method can locate the iris, and the new idea of the proposed method is right.

From Fig. 4, one can see that Daugman’s and Wildes’ methods give incorrect results, but the result of the proposed method is satisfactory. The robustness of the proposed method is due to the dilation operator and SVM classifier, which have a powerful suppressing ability to the eyelash and eyelid. Thus, the proposed method is an iris location method which offers satisfactory performance.

This experiment shows that the proposed method can complete iris location even for a seriously interfered image. It is more robust than Daugman’s and Wildes’ methods.

4.2. Quantitative comparison

In the experiment a comparison was made of the correct location ratio and the expenditure of time of the three methods. Table 1 shows the results obtaining using
the three methods with reference to particular class, and Table 2 lists the results for both classes.

Table 1 is designed to test whether the proposed method can give perfect performance or not. By quantitative data, the difference of these methods can be illustrated clearly. From Table 1 it follows that, the proposed method has better performance than the other methods for both classes. Whether it is class I or class II, the proposed method not only reduces the location errors, but makes it more computationally efficient.

Especially, we can also see from Tab. 1 that the performance of proposed method is better than that of Daugman’s method and Wildes’ method when eye image is

<table>
<thead>
<tr>
<th>Method</th>
<th>Time [s]</th>
<th>CLR [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>Daugman</td>
<td>6.82</td>
<td>8.26</td>
</tr>
<tr>
<td>Wildes</td>
<td>13.75</td>
<td>16.6</td>
</tr>
<tr>
<td>Proposed</td>
<td>2.85</td>
<td>3.01</td>
</tr>
</tbody>
</table>

Table 2. Results obtained with the use of three methods for all images (CLR: correct location ratio).

<table>
<thead>
<tr>
<th>Method</th>
<th>CLR [%]</th>
<th>Time [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Daugman</td>
<td>93.85</td>
<td>7.52</td>
</tr>
<tr>
<td>Wildes</td>
<td>93.6</td>
<td>15.24</td>
</tr>
<tr>
<td>Proposed</td>
<td>97.17</td>
<td>2.94</td>
</tr>
</tbody>
</table>
interfered seriously. So, it can be concluded that the proposed method is more robust than the other methods so far as the interference of eyelash is concerned.

Table 2 is designed to compare the performance form whole view, and can give a global impression about all of the three methods. The data from Tab. 2 illustrate the satisfactory performance of the proposed method. Compared with Daugman’s and Wildes’ methods, the proposed method can make the correct location ratio increase by 3.28% and 3.57%, respectively. Similarly, the expenditure of time in the location process is reduced to 60.9% and 80.7%, respectively.

5. Conclusions

Based on the above comparison and data analysis, we can draw a number of conclusions:

1. Unlike Daugman’s and Wildes’ methods, the proposed method accomplishes the iris location task according the different characteristics of iris for different location stages. In other words, the proposed method cannot only make the computations more efficient, but also improve the correct location ratio at the same time. The experiments prove that the idea of the proposed method is quite feasible.

2. This paper develops an iris location method combining the morphological operator and SVM classifier. A series of experiments show that the proposed method has satisfactory performance. The method can suppress most interference coming from the eyelash and so on, and has higher stability characteristic.

3. Compared with Daugman’s and Wildes’ methods, the proposed method can make the correct location ratio increase by 3.28% and 3.57%, respectively, and reduce the location time expense to 60.9% and 80.7%, respectively.

Acknowledgements – In our research, use was partially made of the CASIA iris image database collected by Institute of Automation, Chinese Academy of Sciences. The work was supported by a grant from the National Natural Science Foundation of China (No. 60502021), the Ph.D. Programs Foundation of Ministry of Education of China (No. 20050698025).

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An effective iris location method with high robustness


Received July 7, 2007
This study demonstrates a novel athermal arrayed waveguide grating (AWG) which is composed of silica/polymer hybrid materials on a silicon substrate. The temperature-dependent wavelength shift of the AWG depends on the refractive indices of the materials and the size of the waveguide. The athermalization of the AWG can be realized by selecting the proper values of the material and structural parameters of the device.

Keywords: arrayed waveguide grating, temperature-dependent wavelength shift, athermalization.

1. Introduction

Because of their excellent features and potential applications, the arrayed waveguide gratings (AWG) have currently received considerable attention and have become key components in dense wavelength-division-multiplexing (DWDM) networks [1–4]. However, an AWG made of silica is very sensitive to temperature. Therefore, in order to realize the normal operation of the AWG, the temperature control unit is required, i.e., a heater or a Peltier cooler. To eliminate the undesirable temperature dependence, the athermalization should be realized in the operation of the AWG devices [5–10]. An excellent athermal AWG enables the performance of the AWG unaffected by ambient temperature variation.

Recently, a hybrid waveguide with a silica core and polymer overcladding is considered as the most attractive athermal structure because of its resistance to the thermo-optic sensitivity of the materials and its simple fabrication process [11].

In this study, such an athermal AWG is designed by theoretic simulation. First, the principle of the athermal AWG with silica/polymer hybrid materials is described, and the relative formulas are derived for analyzing the temperature dependence of the AWG. Then, the theoretical simulation and optimum design of the athermal
AWG are carried out. Finally, a conclusion is reached, based on the analysis and the discussion.

2. Principle of athermal AWG

In this section, we present the athermal condition and the relative formulas of silica/polymer hybrid material AWG on a silicon substrate. The temperature/wavelength tuning rate \( \frac{d\lambda_c}{dT} \) of the AWG is expressed as [12]

\[
\frac{d\lambda_c}{dT} = \frac{\lambda_c}{n_c} \left( \frac{dn_c}{dT} + n_c \alpha_{\text{sub}} \right)
\]  

where \( T \) is the temperature; \( \lambda_c \) is the center wavelength in free-space; \( n_c \) is the mode effective refractive index which is defined as \( n_c = (\lambda_c/2\pi)k_z \); here \( k_z \) is the mode propagation constant along the propagating direction \( z \); \( \frac{dn_c}{dT} \) is the thermo-optic (TO) coefficient of the waveguide; and \( \alpha_{\text{sub}} \) is the coefficient of thermal expansion (CTE) of the substrate. Integrating Eq. (1), we can obtain

\[
\lambda_c = C n_c \exp(\alpha_{\text{sub}} T)
\]  

where \( C \) is an integrating coefficient. Assume that \( \lambda_c = \lambda_0 \) and \( n_c = n_{c0} \) when \( T = T_0 \), then we can determine \( C \) as

\[
C = \frac{\lambda_0}{n_{c0}} \exp(-\alpha_{\text{sub}} T_0)
\]  

Substituting Eq. (3) into Eq. (2), we get

\[
\lambda_c = \frac{\lambda_0 n_c}{n_{c0}} \exp[\alpha_{\text{sub}}(T - T_0)]
\]  

Thus, from Eq. (4) we obtain the central wavelength shift caused by the temperature variation as

\[
\Delta \lambda = \lambda_c - \lambda_0 = \frac{\lambda_0}{n_{c0}} \left( n_c \exp[\alpha_{\text{sub}}(T - T_0)] - n_{c0} \right)
\]  

Taking \( \Delta \lambda = 0 \), from Eq. (5) we can obtain the athermal condition of the AWG as

\[
\alpha_{\text{sub}}(T - T_0) = \ln \left( \frac{n_{c0}}{n_c} \right)
\]
By differentiating Eq. (6), the athermal condition of the AWG can also be expressed in another form as follows

\[ n_c \alpha_{\text{sub}} = -\frac{dn_c}{dT} \]  

(7)

Because the effective refractive index \( n_e \) is dependent on the refractive indices of the materials and on the size and the shape of the waveguide, then by selecting proper material and structural parameters of the waveguide to satisfy Eq. (6) or (7), an athermal AWG can be designed.

3. Simulation and design of athermal AWG

In this section, we carry on the theoretical simulation and the optimum design of an athermal AWG with silica/polymer hybrid materials.

Figure 1a shows the scheme of an AWG device which consists of two focusing slab waveguides, \( 2N+1 \) input/output channels, and \( 2M+1 \) arrayed waveguides. The core cross-sections of the input/output channels and the arrayed waveguides are designed as rectangles with the core width \( a \) and the core thickness \( b \). Figure 1b shows the cross-section and the refractive index profile of the input/output channels and the arrayed waveguides. We design three layer hybrid waveguide structure which contains a silica substrate of a 500 \( \mu \)m thickness, a silica undercladding of a 15 \( \mu \)m thickness and with the refractive index \( n_2 = 1.445 \) [12], a silica core of the core size \( a = b = 5 \) \( \mu \)m (in width and in thickness) and with the refractive index \( n_1 = 1.454 \) [12], and a polymer overcladding of a 15 \( \mu \)m thickness and with the refractive index \( n_3 = 1.440 \) [12]. Assume that both the silica undercladding and the silica core have the same positive material TO coefficient, that is \( dn_1/dT = dn_2/dT = 1.0 \times 10^{-5} /\degree C \) [13].

![Fig. 1. Scheme of an AWG (a), and cross-section and refractive index profile (b) of the input/output channels and the arrayed waveguides.](image-url)
To reduce the temperature dependence of the wavelength shift, the polymer overcladding has a negative material TO coefficient of \( \frac{dn_3}{dT} = -1.1 \times 10^{-4} / {^\circ}C \) [13]. The center wavelength at temperature \( T_0 \) is selected to be \( \lambda_0 = 1550.918 \text{ nm} \) (or 193.3 THz for frequency), which is one of the standard wavelengths recommended by the International Telecommunications Union (ITU) [14]. This AWG device is made on the silicon substrate, having a CTE of \( \alpha_{\text{sub}} = 2.63 \times 10^{-6} / {^\circ}C \) [13].

Because the environmental temperature of an AWG is usually changed from 20 °C to 70 °C, we only discuss the central wavelength shift \( \Delta\lambda \) in this range of temperature variation. The subsequent relations between the wavelength shift \( \Delta\lambda \) and the refractive indices of the core, undercladding and overcladding \( n_1, n_2, n_3 \) (as well as the core width and core thickness \( a, b \)) are analyzed and discussed as follows.

First, it is necessary to investigate the behavior of the TO coefficient \( \frac{dn_c}{dT} \) of the waveguide. The values of \( \frac{dn_c}{dT} \) at different values of temperatures can be determined by using the finite difference method. The TO coefficient \( \frac{dn_c}{dT} \) can be approximately expressed as

\[
\frac{dn_c}{dT} \approx \frac{\Delta n_c}{\Delta T}
\]

(8)

where \( \Delta n_c = n_{c2} - n_{c1} \), \( \Delta T = T_2 - T_1 \), \( n_c = n_{c1} \) and \( n_c = n_{c2} \) when \( T = T_1 \) and \( T = T_2 \), respectively, and \( T_1 \) and \( T_2 \) are very close to each other, then \( \Delta n_c \) and \( \Delta T \) are very small quantities.

Figure 2 shows the dependence of the TO coefficient \( \frac{dn_c}{dT} \) on the temperature \( T \). We can find that \( \frac{dn_c}{dT} \) is not constant with the variation of the temperature which nonlinearly increases as the temperature increases, although \( \frac{dn_1}{dT} \), \( \frac{dn_2}{dT} \) and \( \frac{dn_3}{dT} \) are constants, respectively. Therefore, this behavior of \( \frac{dn_c}{dT} \) will obviously affect the shifts of the central wavelength and the transmission spectrum of the AWG caused by the variation of the temperature.
Design of athermal arrayed waveguide grating ...  

Figure 3 shows the dependences of the central wavelength shift $\Delta \lambda$ on the refractive indices of the core, undercladding and overcladding $n_1$, $n_2$, $n_3$ as well as the core width $a$ and core thickness $b$ for the designed athermal hybrid material AWG, which are calculated from Eq. (5). We can see that there exists an optimal operation condition of the AWG, which should guarantee the central wavelength shift to be small enough in a sufficiently large range of the temperature variation. To be precise, when we select $n_1 = 1.454$, $n_2 = 1.445$, $n_3 = 1.440$, and $a = b = 5.0 \mu m$ (the thick line in every sub-figure), the central wavelength shift is within the range of $-0.020 \sim 0.022 \text{ nm}$ as the temperature increases from 0 °C to 70 °C. In this case we can presume that the athermalization is realized in the designed AWG.

Figure 4 compares the central wavelength shift $\Delta \lambda$ of the conventional silica AWG with the designed athermal hybrid material AWG, which are calculated from Eq. (5). The refractive index and the TO coefficient of the silica overcladding of...
the conventional silica AWG are taken to be $n_3 = 1.440$ and $dn_3/dT = 1.0 \times 10^{-5} /{\circ}C$, respectively. We can observe that as the temperature increases from 20 °C to 70 °C, the central wavelength shift of the conventional silica AWG increases to 0.66 nm, while that of the designed athermal hybrid material AWG only increases to 0.025 nm. This indicates that the central wavelength shift of the designed athermal hybrid material AWG is much lower than that of the conventional silica AWG.

Figure 5 compares the transmission spectrum of the conventional silica AWG with the designed athermal hybrid material AWG, which are calculated from Eq. (4) in [15], the temperature $T = 25 \, ^\circ{C}$, 45 °C, 65 °C. The refractive index and the TO coefficient of the silica overcladding of the conventional silica AWG are taken to be

$$n_1 = 1.454, \quad n_2 = 1.445, \quad n_3 = 1.440, \quad dn_3/dT = 1.0 \times 10^{-5} /{\circ}C \quad \text{for conventional AWG}$$

and

$$dn_3/dT = -1.1 \times 10^{-4} /{\circ}C \quad \text{for designed athermal AWG}.$$
\[ n_3 = 1.440 \text{ and } dn_3/dT = 1.0 \times 10^{-5} /{}^\circ\text{C}. \] We can see that the transmission spectrum shift of the conventional silica AWG is about 0.30 and 0.60 nm, while that of the designed athermal hybrid material AWG is only about –0.019 and 0.011 nm, as the temperature increases from 25 °C to 45 °C and 65 °C. This indicates that the transmission spectrum shift of the designed athermal hybrid material AWG is much lower than that of the conventional silica AWG.

4. Conclusions

After the preceding analysis and discussion about the athermal hybrid material AWG, a conclusion is reached as follows.

In this paper, we present a novel technique for theoretical simulation and optimum design of the athermal AWG with silica/polymer hybrid materials. By selecting the proper values of the refractive indices of the materials and the size of the waveguide, the athermalization can be realized in the silica/polymer hybrid material AWG. To be precise, the center wavelength shift of the designed athermal hybrid material AWG only increases to 0.025 nm, while that of the conventional silica AWG increases to 0.66 nm, as the temperature increases from 20 °C to 70 °C. The transmission spectrum shift of the designed athermal hybrid material AWG is only about –0.019 and 0.011 nm, while that of the conventional silica AWG is about 0.30 and 0.60 nm, as the temperature increases from 25 °C to 45 °C and 65 °C.

This work, we think, is not only valid for the presented AWGs, but also helpful for other optical devices based on hybrid material waveguides to realize the athermalization.

Acknowledgments – The authors wish to express their gratitude to the National Science Foundation Council of China (the project no. 60576045) for its generous support to this work.

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Received May 10, 2007
in revised form July 31, 2007
Web camera-enabled material research: an acoustooptic example

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The applicability of standard PC web cameras to material testing has been discussed. A simple method for measuring the acoustic velocity and attenuation of transparent solid materials has been described as an illustrative example. It is based on the acoustooptic diffraction in the Raman–Nath limit and a digital readout and processing of the information obtained. Acceptable accuracy of the results is achieved. The method is particularly suitable for very quick testing of high loss materials.

Keywords: web camera, acoustooptic diffraction, material testing, acoustic velocity and attenuation.

1. Introduction

Material research methods often employ light as a probing tool to reveal macroscopic or microscopic properties. In various applications, the spatial distribution of the emitted or scattered light intensity is mapped electronically to obtain a desired feature. This trend has been pushed forward in recent decades by the advent of the optical detection matrices (or image sensors). The advantage offered by these matrices is basically twofold: i) they spare the effort required in older systems to displace individual diode detectors across the area under study; ii) when digitized, they are readily connectable to personal computers to perform on-line numerical processing of the data obtained.

The available numerous types of optical matrices, their operational regimes and measurement capabilities hardly need to be discussed in a contribution like this. We restrict ourselves here to the web cameras and by this term we mean the least expensive charge-coupled device (CCD) or complementary metal-oxide semiconductor (CMOS) cameras that are typically used as inherent parts of computers. We do not mean the much more expensive cooled CCD cameras whose performance is expectedly (but not necessarily) higher. What we consider as most important in this discussion is the fact that, with contemporary computing means, quite modest types of devices are
sometimes capable of delivering information that is adequate enough to ensure the receipt and processing of optical experimental data.

A stimulus for this study came from a recent article posted on the Internet that compares the quality of astronomical pictures taken by web and cooled CCD cameras [1]. Surprisingly, the observations made by the web camera turn out to be of even higher quality, raising a number of questions concerning, for example, the role of timing (sampling and image rates) in measuring unstable patterns. Other research applications of the web camera could be found in areas so much different as watermark detection [2] and robotics [3]. The present study illustrates its potential in classical optical research of transparent materials, specifically addressing the acoustooptic diffraction method.

2. Material properties studied by acoustooptic diffraction

The acoustooptic interaction has proved to be a powerful means for studying sound waves in transparent solids and liquids [4]. Although different aspects of this interaction have been known, the acoustooptic diffraction has occupied the major place in both research and applications. Depending on a number of parameters, mainly the acoustic frequency and the aperture of the sound beam within the solid sample, the diffraction displays two limit regimes: the Raman–Nath limit with a multitude of diffraction orders, and the Bragg limit with only one pronounced order. The Bragg diffraction is much more efficient but at the same time more difficult to realize because of the restrictive angular rules.

There are two integral acoustic parameters – the acoustic wave velocity and attenuation – that are directly related to the diffraction output. The attenuation affects the intensity of the diffracted light via the acoustic power flow in the sample at the cross point with the incident light beam. To measure the attenuation, either the light beam or the illuminated sample has to be shifted along the acoustic propagation path to monitor the decay of power [5, 6]. The velocity, on the other hand, determines the angular separation of the diffracted orders. Its measurement requires finding the position of an order with respect to that of the zeroth order in a plane normal to the acoustic wave vector [6]. Apart from this geometrical role, the velocity has a critical impact on the efficiency of the diffraction, reducing the diffracted intensity proportionally to its third power [7].

The propagation of an acoustic wave in a solid medium is accompanied by effective interaction of the acoustic phonons with defects and various elementary excitations such as thermal phonons, free electrons, excitons, etc. Thus, the obtained velocity and attenuation might provide information about the microscopic status of the material under investigation, the characteristics of its elementary excitations or, in general, its scope of applicability as acoustic material. A typical example is a recent acoustooptic study of the relaxation of the ionic motion in photochromic glasses [8].
In most acoustooptic studies the diffracted light has been captured by sensitive detectors, such as photo-multipliers or diodes, to monitor the position and intensity of order spots. The measurements require goniometry, cumbersome electronics and extensive analog information processing. The advent of the optical matrix detectors has provided the option of a simple digital readout of the diffracted intensity distribution. Similar procedures have recently been followed in a number of studies, concerning, for example, the surface profiling of thin films [9] or the ultrasound modulated optical tomography of living subjects [10].

3. A digital acoustooptic scheme for measuring sound velocity and attenuation

The experimental acoustooptic setup for the present study is illustrated in Fig. 1. The light beam from a 1 mW semiconductor laser with $\lambda = 635$ nm is directed at normal incidence to one of the sides of a parallelepiped sample made from the transparent solid material under study. The sample is placed in a holder mounted on a three-dimensionally moving bench with a step of 10 $\mu$m along each direction. The light interacts with a longitudinal acoustic wave continuously generated in the sample by a ceramic transducer of fundamental frequency 7.6 MHz, fed from a digital PC generator (ETC-M631) connected to the printer port of a computer (Pentium IV, 3 GHz, 1 GB RAM). Owing to the right angle between the two wave vectors and a relatively low frequency, the resulting diffraction of light is dominated by the Raman–Nath limit. The diffracted light is passed consecutively through an attenuating filter and a focusing lens, so that the first few diffracted orders of $+$ and $-$ directivity are sensed by the open active area of a standard web camera (QuickCam Logitech, 640×480 pixels, 7.5 $\mu$m per pixel, 30 frames per second). The output of the camera is fed to the USB port of the computer for image reconstruction and

![Fig. 1. Experimental setup for acoustooptic diffraction studies using web camera as light detector.](image-url)
processing of the data via Matlab. As the diffracted pattern seen by the camera is a size-contracted replica of the real pattern, it is possible to obtain an absolute estimate of the real order spacing. However, this approach requires, as in all classic studies, that the distance $L$ between the sample and the lens be precisely known (Fig. 2), which complicates the experiment. We adhere to a different approach where this requirement is removed by comparing the pattern generated by the sample being studied to that from a reference sample with known acoustic velocity.

The acoustic velocity $V$ can be deduced from the angular condition for the first Raman–Nath diffraction order [7]:

$$\sin \theta_0 = \frac{A}{\lambda n} = \frac{Af}{Vn}$$  \hspace{1cm} (1)

where $\theta_0$ is the diffraction angle in the sample, $A$ – the light wavelength in vacuum, $\lambda$ – the acoustic wavelength, $f$ – the acoustic frequency and $n$ – the refractive index of the sample. The angle $\theta$ which we measure in air is larger because of the refraction at the sample/air interface. In the general case, it involves a combination of the distance $L$, the sample width parallel to the propagation direction $d$, and the refractive index $n$, which ultimately makes the proposed comparative approach inapplicable. However, the simplifying condition of large $L$ compared to $d$ reduces this formula to:

$$\sin \theta \approx \frac{l}{2L} = \frac{A}{\lambda} = \frac{Af}{V}$$  \hspace{1cm} (2)

Here, $l$ denotes the spacing between the +1st and –1st diffraction orders; it should be measured upon screening out the zeroth order to reduce the noise.
Let us define two angles $\theta_s$ and $\theta_r$ corresponding to diffraction caused the sample under study and a reference sample, respectively. This sub-index convention holds for all quantities to be used in further calculations. In order to ensure equivalence in the experimental conditions, the two samples are provided with identical transducers, generating along parallel lines. The samples are placed side by side in the holder, so that the light can just be switched from one sample to the other at unchanged positions of the laser and the web camera. The two characteristic spacings $l_s = AD$ and $l_r = BC$ are projected on the focal plane as $l_s' = A'D'$ and $l_r' = B'C'$, see Fig. 2. The similarity of triangles indicates that:

$$\frac{l_s}{l_r} = \frac{l_s'}{l_r'} \quad (3)$$

From (2) and (3) it readily follows that:

$$V_s = \frac{l_r f_s}{l_s f_r} V_r = \frac{l_r' f_s}{l_s' f_r} V_r \quad (4)$$

To measure the acoustic attenuation the sample should be shifted parallel to the acoustic wave vector and the intensity of one of the diffraction orders should be measured as a function of distance $X$. Matlab permits digitization of the detected image pattern into 256 shades of gray that is quite a satisfactory resolution for the pixel intensity. The spatial intensity variation obtained is then exponential-fitted to give the attenuation:

$$I_d = I_{d0} \exp(-\alpha_l x) \quad (5)$$

A well-established fact for the Raman–Nath diffraction limit is the linearity between the diffracted light intensity and the acoustic power $P$ at non-elevated power levels [4]. Thus the optical intensity decay $\alpha_l$ is just the acoustic power attenuation, i.e., twice the acoustic amplitude attenuation:

$$\alpha_l = 2 \alpha_a \quad (6)$$

Apart from simplicity and computer-aided performance, this method has another major advantage over conventional pulse-echo and interference methods [11]. It is applicable to high loss materials that do not tolerate multiple echoes. With such materials even the detection of the first echo might be problematic. However, illuminating the near-field region close to the transducer should, in principle, provide a detectable diffraction regardless of the magnitude of attenuation. The accuracy of the velocity measurement depends on the error in measuring the order spacings and the reference velocity, and is typically of the order of a few percent. The accuracy of
the attenuation cannot be specified in rigorous terms because of the factors influencing the detection during the movement of the sample at long continuous exposures of the web camera. More details will be provided in the next section.

4. Experimental results and concluding remarks

Two experimental samples have been prepared in the form of parallelepipeds with optically flat sides. They are made of isotropic materials to avoid any side effects, such as beam steering, which could violate the experimental equivalence. The reference sample is of vitreous silica glass doped with lead inclusions to reduce the acoustic velocity and thus improve the diffraction efficiency. Its longitudinal velocity has been measured by a pulse-echo method: \( \nu_r = 3935 \text{ ms}^{-1} \pm 0.8\% \). The sample is made of polymethylmetacrylate (PMMA). The small bases of the parallelepipeds are provided with ceramic PzT transducers firmly attached by epoxy bond. As neither of the transducers is matched to its load, the frequency of operation should be adjusted to reach maximum power within the available frequency band. Therefore, the two frequencies involved in Eq. (4) are generally different. The holder carrying the two samples is located at distance \( L = 70 \text{ cm} \) from the focusing lens (focal length \( F = 10 \text{ cm} \)), much larger than the samples’ width \( d \) that is of the order of a few millimeters. Under these geometrical conditions and the operating frequency range used, the real order spacing is of the order of a few millimeters; this is contracted by the lens to hundreds of micrometers (tens of pixels) on the camera sensor plane. All experiments have been carried out at room temperature and have been compared to room temperature data from literature.

![Fig. 3. Plane distribution of the light intensity diffracted from the PMMA sample at 7.8 MHz.](image)
The experimentally registered diffraction caused the PMMA sample is illustrated in Figs. 3 and 4. Figure 3 is a view of the plane distribution of the diffracted intensity involving the first four Raman–Nath orders: ±1st and ±2nd (weakly expressed). The operating frequency is 7.8 MHz. In this figure, the strongly dominating background intensity of the incident laser beam (zeroth order) has been mathematically removed to clearly reveal the ±1st orders and make further calculations easier. This is done by subtracting the intensity distribution measured without sound excitation from the full diffraction picture. In Figure 4, a cross-section of the intensity surface with a vertical plane passing through the positions of the two maxima is presented. The resulting dependence is given a Gaussian fit to determine the characteristic spacing $l'_r = 100.3$ pixels. Similarly, the corresponding order spacing in the silica case is $l'_r = 79.3$ pixels at 9.2 MHz (not shown). Following Eq. (4), the resulting longitudinal velocity of PMMA is $V_s = 2630 \text{ m s}^{-1} \pm 3.2\%$. Besides the 0.8% tolerance for the reference velocity, the overall error of the result involves 1% for each of the two order spacings as following from the corresponding Gaussian confidence intervals. Another 0.4% comes from neglecting the finite width of the sample, while the acoustic frequency, measured to around 100 Hz, practically does not affect the accuracy. The mean velocity value obtained is very close to that measured by high-frequency time-resolved broad-band microscopy ($2640 \pm 4 \text{ m s}^{-1}$) [12]. However, it deviates significantly from higher values measured by pulse-echo techniques in [13, 14], which even fall out of our confidence interval. This inconsistency is another clear indication that polymer acoustic constants should be treated with much care concerning the experimental method and its inaccuracy. It is possible that some deviations might also be due to differences in the conditions of material polymerization.
The measurement results for the acoustic attenuation are shown in Fig. 5 (PMMA) and Fig. 6 (silica), respectively. During the experiments the sample is moved with a step of 1 mm and the evolution of a specified order is monitored at a constant position of the web camera. At each sample position the diffracted intensity is determined as the amplitude of the Gaussian function and the resulting sequence of peak intensity values is fitted exponentially to give the attenuation. The calculated value for the attenuation in PMMA is 133 Np m$^{-1}$. It should be compared to 106 Np m$^{-1}$ measured under similar conditions (room temperature and 10 MHz) in [14]. The value obtained for silica is 19 Np m$^{-1}$ (1.3 Np m$^{-1}$ for pure silica [15]). There is no reason for making a comparison with literature data in the latter case because the attenuation of these glasses is heavily dependent on the lead content and the preparation mode. As commented in the previous section, it is not possible to provide a rigorous analysis of the accuracy of the attenuation result. This is mainly due to fluctuations of the camera output, which limit the confidence of the exponential fitting. A plot of the time
dependence of the signal at a constant position of the sample is shown in Fig. 7. Apart from the considerable noise, there is a tendency towards casual floating of the average level that affects the attenuation estimate at longer exposures of the camera. The value and even sign of this instability arise from semiconductor surface state statistics at ambient temperatures and are difficult to predict. Another source of errors might be a departure from linearity of the camera response. However, as is evident from the measured characteristics of Fig. 8, this departure occurs above the working range of this study (0–100 arb. units). A rough estimate of the overall accuracy of the attenuation might be a 30% deviation from literature data in the PMMA case. In view of the simplicity and rapidity of the method the featured accuracies of velocity and attenuation might be considered as acceptable. The method is particularly applicable to very quick testing of the acoustic properties of transparent lossy materials.

Acknowledgments – This work has been partially supported by the Scientific Fund of Sofia University under grant No. 004/2007.
References


Received May 15, 2007
in revised form July 31, 2007
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PL ISSN 0078-5466